THE ROLE OF PHONOLOGICAL AND VISUAL WORKING MEMORY IN CARRY OPERATIONS OR INTERMEDIATE SOLUTIONS IN COMPLEX MENTAL ARITHMETIC

BY

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Doing a Ph.D. is a challenging experience, especially when working at the same time.

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ABSTRACT

The present research comprises four experiments designed to explore the role of visual and phonological working memory resources in carry operations or intermediate solutions in complex mental addition and multiplication. A special consideration was given to the effect of arithmetic operation on the relative involvement of visual and phonological resources in complex addition and multiplication.

A pilot study was conducted prior to the experiments, aiming to examine the suitability of visual and phonological stimuli for change detection and working memory capacity estimation. Two staff of Victoria University of Wellington with normal or corrected vision attended the pilot study as participants. Pilot Experiments 1 to 4 tested the suitability for probing visual working memory (VWM) capacity of two types of visual stimulus with different feature dimensions: bars of different orientations and Gabor patches with different orientations and spatial frequencies. A single-probe change-detection experimental paradigm was used, with participants making decisions about whether or not probe items were the same as memory items presented previously. Both presentation durations and set sizes were manipulated. Stable estimates of visual working memory capacities were found when Gabor patches with varied spatial frequencies were used, suggesting its utility as a probe for estimating visual working memory capacity. Pilot Experiment 5 was designed to examine the suitability of pronounceable consonant-vowel-consonant non-words as a probe of phonological working memory (PWM). Valid estimates of PWM capacity were found for both participants, suggesting the suitability of phonological non-words as phonological stimuli of assessing PWM capacities and interfering with information phonologically-represented and maintained in working memory.

Experiments 1 to 4 investigated the relative involvement of visual and phonological working memory resources in carry operations or intermediate solutions in mental addition and multiplication. Fifty-six undergraduate students of Victoria University of Wellington participated all experiments, and 48 of them provided valid data for final analysis. A dual-task interference paradigm was used in all experiments, with arithmetic tasks and visual/phonological change-detection tasks either performed alone,

or simultaneously. For arithmetic tasks, double-digit addition problems and multiplication problems comprising one single-digit and one double-digit were presented horizontally and continuously, and participants reported the final solutions verbally. For visual change-detection tasks, study items were visually presented to participants for 1,000ms before they disappeared. After a 4000ms retention interval, a probe item was presented and participants judged whether the probe item was the same as one of the memory items. For phonological change-detection tasks, phonological nonwords were verbally presented to participants sequentially. After a 4000ms retention interval, a probe nonword was presented to participants, and they indicated whether or not the probe was the same as one of the study non-words. Both numbers of carry operations involved in the arithmetic problems (zero, one, and two) and levels of visual/phonological loads (low, medium, and high) were manipulated in all experiments.

For all experiments, the effect of the number of carry operations on calculation performance was observed: arithmetic problems involving more carry operations were solved less rapidly and accurately. This effect was enlarged by concurrent visual and phonological loads, evidenced by significant interactions between task conditions and number of carry operations observed in the accuracy analyses of the arithmetic tasks in all experiments except Experiment 2, in which multiplication problems were solved under visual loads. These findings suggest that both visual and phonological resources are required for the temporary storage of intermediate solutions or carry information in mental addition, while for mental multiplication, only evidence for a role of phonological representations in carry operations was found.

For all experiments, the greater performance impairment of carry problems than no-carry problems associated with the presence of working memory loads was not further increased by increasing load level: There were no significant three-way interactions between task conditions, number of carry operations and load levels in accuracy analyses of arithmetic tasks. One possible explanation for this absence of significant three-way interactions might be attributable to some participants switching between phonological and visual working memory for the temporary storage of carrier information or intermediate solutions as a result of decreasing amount of available phonological or visual working memory resources.

In conclusion, the findings of the present research provide support for a role of both visual and phonological working memory resources in carry operations in mental addition, and a role of phonological working memory resources in carry operation in mental multiplication. Thus, it can be concluded that solving mental arithmetic problems involving carry-operations requires working memory resources. However, these results contradict the prediction of the Triple Code Model, which assumes addition mainly relies on visual processing, and multiplication mainly relies on verbal processing, while complex mental arithmetic is solved with the aid of visual processing regardless of the arithmetic operation. Thus, these results challenge the operation-specific involvement of working memory resources in complex mental arithmetic. However, it should be noted that the same arithmetic problems were solved three times by the same participants, which might have encouraged more activation in phonological processing than visual processing due to the practice effect.

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General Introduction

Mental arithmetic, as a phenomenon, follows relatively explicit and general rules, facts and principles, which are in wide use all over the world. A series of cognitive processes is required to solve any arithmetic problem, with longer series required for complex problems comprising multi-digit operands (Geary, 1994; Geary, Widaman, Little, & Cormier, 1987; Hope & Sherrill, 1987). For example, when calculating multi-digit addition problems involving carry operations, such as 37 + 96, retrieval of arithmetic facts stored in long-term memory might be used to determine the sum of the units (7 + 6 = 13). Following this step, there are at least two ways to complete the calculation. One relies on the short-term storage of the intermediate solution (13) while the sum of the decades (30 + 90) is determined. This process is normally referred as a *decomposition* strategy in mental arithmetic. The other is to remember a digital carrier (1) as well as the solution for the units (3) while retrieving the arithmetic fact (3 + 9 = 12). Simple addition of the partial results and the digital carrier can then be conducted to achieve the final solution (133).

No matter which calculation procedure is used, correctly remembering the digital carrier or intermediate solution is necessary to achieve final solutions successfully. This mnemonic process has been theorised to make substantial demands on a theoretical, capacity-limited cognitive system that temporarily stores and manipulates the information necessary for complex cognitive activities, such as reasoning, learning and comprehension. This system is referred to by many theorists as *working memory* (Baddeley & Hitch, 1974; Baddeley, 2002).

A particular focus of research on the cognition of mental arithmetic has been on the involvement of working memory resources (Fürst & Hitch, 2000; Heathcote, 1994; Hitch, 1978; Imbo & LeFevre, 2010; Logie et al., 1994; Trbovich & LeFevre, 2003). The theoretical foundation of a majority of research about the role of working memory in mental arithmetic is the multi-component working memory model proposed by Baddeley and Hitch (1974; 1997), which is theorised to comprise three components, including an attentional control component called the *central executive*, and two short-term storage components called the *phonological loop* and the *visuospatial-sketchpad*. This model emphasises the separate storage of modality-specific information from different but interrelated channels, such as vision and audition (Baddeley, 1986, 2000; Baddeley & Hitch, 1974). For example, in the domain of mental arithmetic, digital carriers or intermediate solutions could be temporarily stored visually or phonologically during the calculation process.

A *dual-task interference paradigm* (Kahneman, 1973) has most often been used to examine the role of working memory components in mental arithmetic. This paradigm involves the simultaneous performance of two tasks, each requiring maintenance and processing of information (e.g., Daneman & Carpenter, 1980; Oberauer & Gothe, 2006; Turner & Engle, 1989). The rationale of this paradigm is that, if the same working memory resources are required by both tasks, there will be less capacity available to allocate to each task when they are performed simultaneously. The resulting competition for resources will lead to performance impairment, such as longer response latencies and higher error rates, than those observed for the performance of either task alone (Wickens, 1981). In the domain of mental arithmetic, the dual-task paradigm often comprises a primary calculation task and a secondary task designed to load a particular working memory component (Otsuka & Osaka, 2014). Whether a working memory component is involved in mental arithmetic can therefore be estimated by the degree of disruption to performance of the calculation task in a dual-task condition compared to a control condition, in which the calculation task is performed alone.

The extent of the involvement of working memory in mental arithmetic has been extensively studied in terms of the effect of presentation format (horizontal vs. vertical, e.g., Imbo & LeFevre, 2010; Trbovich & LeFevre, 2003), strategy choice (counting vs. memory retrieval of arithmetic facts storing in long-term memory, e.g., Hubber, Gilmore, & Cragg, 2014; Imbo & Vandierendonck, 2008; Imbo, Vandierendonck, & Vergauwe, 2007) and proficiency in mental arithmetic skills (Thevenot, Fanget, & Fayol, 2007). However, the role of working memory resources in carry operations or storage of intermediate solutions has not been systematically investigated through a simultaneous manipulation on the number of digital carriers and the level of working memory load. The number of carry operations involved in an arithmetic problem has been theorised to be positively related to the relative involvement of working memory resources: With more digital carriers needing to be temporarily maintained, there is theoretically a greater demand on working memory resources (Imbo, Vandierendonck, & De Rammelaere, 2007). In dual-task conditions, this storage process might also be affected by the level of working memory load imposed by a concurrent working memory load task, as a higher load would leave fewer working memory resources available for the temporary storage of carrier information or intermediate solutions. Thus, a simultaneous manipulation of both numbers of carry operations and levels of working memory load is required to comprehensively examine the involvement of working memory resources in carry operations or the storage of intermediate solutions in mental arithmetic.

Although carrier information and intermediate solutions are theorised to be retained in phonological or visual working memory (see DeStefano & LeFevre, 2004 for a review), the relative involvement of visual and phonological working memory resources in carry operations in mental arithmetic has yet to be rigorously investigated. In particular, a lack of comparability of visual and phonological interference tasks might also have led to apparently inconsistent findings regarding the relative involvement of visual and phonological resources in carry operations in mental arithmetic. Only a few studies have investigated the pattern of relative involvement of phonological and visual working memory resources in mental calculation using equivalent and effective working memory interference tasks that can effectively interfere with both phonological and visual representations (e.g., Trbovich & LeFevre, 2003). Thus, comparable phonological and visual interference tasks are required to explore the involvement of phonological and visual resources in carry operations in mental arithmetic.

It has been theorised that different arithmetic operations (e.g., addition, multiplication, subtraction and division) also place different demand on visual and verbal processes in mental arithmetic (Dehaene, 1992; Dehaene & Cohen, 1997). For example, addition is assumed to require more visual resources, while multiplication is assumed to require more phonological resources. This is because different arithmetic operations are theorised under the *Triple Code Model* to be differently associated with three numerical processing codes: The *Analog Magnitude Code* is postulated to be used to compare numbers (e.g., which number is larger, 6 or 9?), *the Verbal Code* to facilitate fact-retrieval (e.g., 3 × 4 =?) and the *Visual Arabic Code* used to process visually presented Arabic number tasks (Dehaene, 1992; Dehaene & Cohen, 1995; 1997). However, in spite of have been investigated extensively in the area of neuro-imaging, studies of the operation-specific involvement of these numerical processing paths have heavily focused on simple (single-digit) problems, while complex (multi-digit) problems have received much less attention. Unlike simple problems, complex arithmetic problems not only require fact-retrieval, but also procedural numerical processing that relies heavily on working memory (Fürst & Hitch, 2000). Although the Triple Code Model predicts that complex arithmetic problems are solved using the analog magnitude code that activates visual processing (Dehaene & Cohen, 1995; 1997), few neuro-imaging studies have included complex arithmetic problems, and inconsistent findings regarding this prediction have been reported in behavioural studies. Therefore, the role of arithmetic operations in the relative involvement of phonological and visual working memory resources in complex mental arithmetic needs further investigation.

In the present research, using a concurrent interference task that can effectively interfere with both visual and phonological working memory, the relative involvement of working memory resources in *carry operations or intermediate solutions* of multi-digit mental addition and multiplication was investigated by simultaneously manipulating both the number of digital carriers and the level of working memory load. The role of arithmetic operations (addition, multiplication) in the relative involvement of working memory resources in complex mental arithmetic was also investigated.

This introductory chapter provides background information about working memory and evaluates empirical evidence in relation to its role in carry operations or intermediate solutions of mental addition and multiplication. The theoretical framework of The Triple Code Model and its prediction of operation-specific involvement of visual and verbal processes in addition and multiplication is also introduced. The Multi-Component Working Memory model (Baddeley & Hitch, 1974; 2002) and the Triple Code Model (Dehaene, 1992; Dehaene & Cohen, 1997) were used as theoretical frameworks for the present research.

Baddeley's Working Memory Model

The multi-component working memory model proposed by Baddeley and Hitch (1974), depicted in Figure 1, comprises an attentional control component with limited capacity, supplemented by two storage systems. The model is predicated on an assumption that working memory storage is capacity-limited. Instead of arguing for a domain-general limit on the information that can be stored in working memory, Baddeley and Hitch proposed a domain-specific view with two capacity-limited components: a *phonological loop* and a *visuo-spatial sketchpad*. An attentional control component, the *central executive,* was theorised to regulate encoding to and retrieval from the two subsystems, by allocating attentional resources and mediating the transformation to and from long-term memory (Baddeley & Hitch, 1974; Baddeley, Cocchini, Sala, Logie, & Spinnler, 1999).

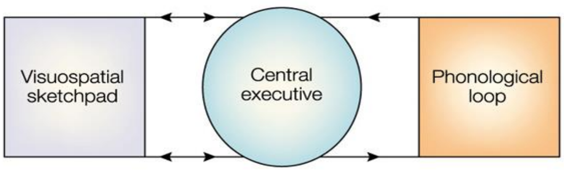


Figure 1. A diagrammatic depiction of working memory model of Baddeley and Hitch (1974). Adapted from "Working memory: looking back and looking forward" by A. Baddeley, 2003, Nature Review Neuroscience, 4, p. 830. Copyright 2003 by the American Psychological Association.

Since the original conceptualisation of the model, an additional component, an episodic buffer, has been theorised to coordinate the storage and integration of domain-specific representations into coherent episodes (Baddeley, 2000). This component was originally suggested to have direct access only to the central executive (Baddeley, 2000), whereas more recently, Baddeley (2007) proposed that the episodic buffer could also have direct access to the phonological loop and visuo-spatial sketchpad.

The Phonological Loop

The phonological loop has been theorised to be a "post-perceptual component of the cognitive architecture designed specifically for the temporary storage of abstract representations of verbal events" (p. 54, Jones, Hughes, & Macken, 2006), and to comprise two sub-components, a *phonological store* and an *articulatory rehearsal mechanism* (Baddeley, 1986). The latter was proposed as a mechanism to maintain information in the phonological store through recitation. In the absence of recitation, the persistence of representations in the phonological store is theorised to be around two seconds (Baddeley, 1983).

Auditory information, such as speech, is theorised to gain obligatory access to the phonological store, while information presented in other formats (such as in a visual form) must first be transformed into a phonological code through a process known as phonological or verbal recoding. This transformation is another function of the articulatory rehearsal mechanism. In particular, subvocalisation is theorised to be necessary for visual information that has a verbal label, such as a picture of an animal, to be recoded into speech-based information and then can be maintained in the phonological store (Baddeley, 1983). This phonological recoding process has been theorised to be relatively automatic, occurring in the articulatory rehearsal mechanism without attentional control being required.

Evidence for the phonological loop and its functioning has been explored extensively in empirical research. A range of laboratory-based findings support the concept of the model, with the phonological-similarity effect (Conrad & Hull, 1964) being taken to support the presence of the store itself, while evidence supporting the rehearsal process came from the word-length effect (Baddeley, Thomson, & Buchanan, 1975; Burgess & Hitch, 1999) the irrelevant-sound effect (Colle & Welsh, 1976), and the articulatory suppression effect (Baddeley, Lewis, & Vallar, 1984). However, a growing number of empirical findings challenge the validity of the concept of the phonological loop and provide alternative explanations for these effects traditionally classed as short-term memory phenomena addressed by the phonological loop hypothesis (e.g., Hughes & Jones, 2005; Hulme, Neath, Stuart, Shostak, Surprenant, & Brown, 2006; Hulme, Suprenant, Bireta, Stuart, & Neath, 2004; Lewandowsky & Oberauer, 2015; Lovatt, Avons, & Masterson, 2000; Jones, Macken, & Nicholls, 2004; Woodward, Macken, & Jones, 2005).

The Word-Length Effect

The *word-length effect* is the phenomenon that immediate recall performance for spoken word sequences is inversely related to the lengths of the words to be remembered (Baddeley et al., 1975). Baddeley et al., (1975) showed that performance in a word-recall task was weaker when long words (e.g., *characterisation*, *individualisation*) were used than when short words (e.g., *yes*, *item*) were used. They argued that the phonological loop model could accommodate these results in terms of pronunciation time; with longer pronunciation times slowing down rehearsal speed, leading to less rehearsal within a certain period of time, and therefore, greater loss of information due to insufficient reactivation. These results support the notion of articulatory processing, which is the proposed function of the articulatory rehearsal system. This notion was further supported by the study of Mueller, Seymour, Kieras, and Meyer (2003). Both the phonological similarity of words and their articulatory durations were assessed in their study, and the notion of trace decay was supported by observed effects of duration and phonological similarity.

However, an alternative interpretation has been proposed involving linguistic complexity: greater numbers of syllables make words more difficult to recall (Caplan, Rochon, & Waters, 1992; Hulme et al., 2004, 2006; Service, 1998, 2000). Hulme et al., (2004) compared immediate serial recall of pure lists of long words with five syllables, pure lists of short words with one syllable, and mixed lists comprising equal numbers of long and short words. Although the results of pure-list conditions supported the word-length effect (longer words were recalled less well than shorter words), the word-length effect was abolished in mixed lists, with both long and short words being recalled as well as short words in pure lists. These results could not be addressed by the word-length effect: If it is the overall duration of the list that determines the ease with which the list is recalled, then lists with equal proportions of both long and short words should be recalled at intermediate levels compared with pure lists of long or short words (Hulme et al., 2004). Thus, Hume et al., argued that the word-length effect can be explained on item complexity and item distinctiveness: the word-length effect depends on the phonological complexity of the items rather than on the time required to rehearse them, and the recall of a given item in the list depends on how distinctive the item is compared with the other items in the list. This explanation has been supported by a reverse word-length effect, reported by Hulme et al., (2006). Specifically, the recall of pure lists of long words, pure lists of short words and mixed lists of long and short words containing a single isolated word of a different length were compared. Contrary to the prediction of the explanation proposed by Baddeley et al., (1975), isolated words were recalled much better than the remaining words in the list, demonstrating the importance of distinctiveness as a critical determinant of the 'word-length' effect.

Articulatory Suppression

Articulatory suppression refers to the use of irrelevant utterances to suppress sub-vocal rehearsal. The word length effect has been shown to be eliminated when this method is applied (Baddeley, Lewis, & Vallar, 1984), presumably because the target material is prevented from being rehearsed. This suggests that articulatory rehearsal refreshes decaying memory in the phonological store. This notion was further supported by Gupta and MacWhinney (1995), who identified an articulatory component in rehearsal.

Articulatory suppression has also been found to interfere with the recoding of visual stimuli as phonological representations (Baddeley et al., 1984; Hanley, 1997; Murray, 1968; Salamé & Baddeley, 1982). These findings support the notion that a mechanism like the articulatory rehearsal component of the phonological loop is used to recode visually-presented stimuli into phonological representations.

Although there is ample evidence supporting the notion that rehearsal helps to prevent memory decay, the role of verbal rehearsal in working memory has been challenged in different ways. For example, Cowan (1992) suggested that there is an attentional maintenance mechanism that is responsible for refreshing and strengthening memory traces. The neural system of this mechanism has been shown to be different from the neural areas involved in verbal rehearsal (Raye, Johnson, Mitchell, Greene & Johnson, 2007).

However, the role of rehearsal (either verbal or attentional) in short-term memory maintenance was rejected by a recent model of working memory proposed by Lewandowsky and Oberauer (2012), referred to as the Serial Order in a Box-Complex Span (SOB-CS) model. This model assumes that no maintenance processes are required to counteract the decay of memory items because memory traces are theorised not to suffer decay but rather, interference from involuntarily encoded distractors. Based on this model, an alternative explanation to impaired performance caused by the articulatory suppression was proposed by Lewandowsky and Oberauer (2015): Rather than the suppression of rehearsal, it is the interference between the irrelevant material and representations in memory that leads to the adverse effects. Thus, verbal rehearsal is simply an epiphenomenon that does not have a role in memory maintenance (Lewandowsky & Oberauer, 2012).

Although predictions of the SOB-CS model have received repeated experimental support (e.g., McFarlane & Humphreys, 2012; Oberauer & Lewandowsky, 2008), they cannot account for the findings of the study of Lucidi, Langerock, Hoareau, Lemaire, Camos and Barrouillet (2016). In these experiments, verbal recall performance was impaired by increasing the rate of repeating articulation of the same syllable for a fixed duration, but no similar disruptive effect was observed for the maintenance of visuospatial information (Lucidi et al., 2016). These findings were consistent with the notion of an articulatory rehearsal system specifically devoted to the maintenance of verbal information. The authors thus concluded that “it is difficult to produce a plausible and coherent model of working memory that does not attribute to verbal rehearsal a casual role in verbal working memory” (p. 205, Lucidi et al., 2016).

The Irrelevant Sound Effect

The *irrelevant-sound effect* refers to an impairment of immediate recall for visually-presented verbal material brought about by a concurrent or subsequent presentation of irrelevant background sounds. This effect was initially reported by Colle and Welsh (1976), with recall performance for a list of visually-presented items greatly impaired by exposure to the irrelevant spoken material. The phonological loop model addressed this effect by stipulating obligatory access of irrelevant auditory material to the phonological short-term store, causing interference with the phonological recoding of visually-presented items by the articulatory rehearsal system. Thus, the phonological loop model predicts that an articulatory suppression task will impair the performance of immediate serial recall when the list is aurally presented but not when it is visually presented (Hanley & Broadbent, 1987) because the suppression task will limit the availability of the loop for recoding visually presented items.

However, evidence supporting this hypothesis is sparse and inconsistent (Jones et al., 2004). Only one study (Hanley & Broadbent, 1987) has shown the predicted effect with auditory presentation, and this research has been criticised on methodological grounds: Each irrelevant sound was presented in synchrony with an item in the memory list, so that the irrelevant sound effect might have been associated with the encoding effect (Hanley & Broadbent, 1987).

To avoid encoding effects, Nicholls and Jones (2002) compared the impact of irrelevant sounds on the immediate serial recall of auditory and visual lists under articulatory suppression by starting the irrelevant sounds before the presentation of each list. Identical irrelevant sound effects were found in auditory and visual modalities (Nicholls & Jones, 2002). This result was reinforced in the subsequent study of Jones et al., (2004), in which an interaction between articulatory suppression and presentation modality was investigated with three relevant sound conditions: silence, repeating the same letter and changing letters. An interaction predicted by the phonological loop hypothesis was obtained; however, it was found to be limited to the letters late in the sequences. Jones et al., therefore suggested that the concept of the phonological loop should be replaced by a hypothesis based on “a combination of acoustic organisational processes and ‘gestural’ rehearsal” (p. 499, Baddeley & Larsen, 2007).

The Phonological-Similarity Effect

The *phonological-similarity effect* refers to the phenomenon that sequences of dissimilar-sounding letters are easier to recall than sequences of similar sounding ones (Conrad, 1964; Conrad & Hull, 1964). Research has also shown that phonological similarity plays a crucial role in the number of words that can be recalled, while similarity of meaning is relatively unimportant (Baddeley, 1966). Because similar items have fewer distinguishable cues than dissimilar items, those with a high degree of similarity in respect of the code used to store them are more likely to be forgotten. The similarity effect, therefore, suggests that, at least for short-term storage purposes, items are phonologically coded rather than semantically coded. Baddeley thus argued that this effect reflects an important role for a phonological store in verbal working memory on the grounds that performance of a verbal short-term storage task appears to depend on the difficulty of translating verbal information into a sound-based code. Thus, the phonological loop hypothesis is supported by an interaction between phonological similarity, articulatory suppression and presentation modality: the phonological similarity effect should be suppressed by a concurrent articulatory suppression task when the target list is visually presented because the process of recoding visually-presented items is thereby impaired. When presentation is auditory, such an effect would survive due to the obligatory access to the phonological store of verbal stimuli (Baddeley & Larsen, 2007).

This notion was challenged by a series of experiments reported by Jones and colleagues (Jones et al., 2004; Jones, Hughes, & Macken, 2006, 2007). In the study of Jones et al., (2004), the interaction between phonological similarity, articulatory suppression and presentation modality was reassessed using seven-item lists with silence or concurrent whispered articulatory suppression. The interaction predicted by the phonological loop hypothesis was obtained, however, the survival of the phonological similarity effect was predominant for items late in the list, suggesting a recency (serial position) effect. Moreover, the interaction could be removed by introducing a redundant end-of-list auditory perceptual organisation task (verbalising a suffix). In a subsequent study (Jones et al., 2006), similar survival of the phonological similarity effect was observed using five-item auditory lists, and this survival could also be blocked by adding redundant acoustic items at the list-initial and list-end boundaries. Based on these results, Jones et al. challenged the idea of the phonological loop component of Baddeley's working memory model and proposed a hypothesis based on a combination of acoustic organisational processes and gestural rehearsal (Jones et al., 2004; Jones et al., 2006; 2007).

Baddeley and Larsen (2007) used the same repeated measures design as that in the study of Jones et al., (2004; 2006), but did not replicate their findings. Baddeley and Larsen compared the influence of phonological similarity, articulatory suppression and tapping rhythms on immediate serial recall in tasks involving both visual and auditory presentation of six-letter sequences. The phonological similarity effect survived suppression with auditory but not visual presentation, and the probability of survival was evenly distributed throughout the list rather than predominantly for late items (Baddeley & Larsen, 2007).

In conclusion, the phonological loop has been challenged in various ways, especially by the studies of Jones et al., (2004; Jones et al., 2006). However, these findings have been suggested to be an identification of a component of the interaction between modality, suppression and similarity, which only has a small impact on performance in the condition that the phonological loop is overloaded or abandoned (Baddeley & Larsen, 2007). The phonological loop hypothesis continues to receive extensive support from empirical studies (e.g., Baddeley, Gathercole, & Papagno, 1998), neuropsychology studies (e.g., Vallar, 2006) and action-control studies (e.g., Emerson & Miyake, 2003). Thus, the phonological loop hypothesis remains influential as a simple and applicable conceptualisation of verbal short working memory.

Visuo-spatial Sketchpad

The second subsidiary system in Baddeley's working memory model, the visuo-spatial sketchpad, is theorised to retain and process image-based information from the perceptual system, as well as similar information retrieved from long-term memory (Baddeley & Hitch, 1974; Baddeley, 1999; 2001). It has been theorised to be sub-divided in different ways based on various theoretical and empirical perspectives. For example, a dissociation between visual (*what*) and spatial (*where*) functions has been proposed (e.g., Chen, Myerson, Hale & Simon, 2000; Logie & Marchetti, 1991; Ungerleider & Mishkin, 1982; Van Essen, Anderson & Felleman, 1992), as has a dissociation between dynamic and static functions (e.g., Pickering, 2001).

Logie (1995) proposed a theoretical model of visuo-spatial working memory based on a distinction between visual and spatial functions. In his model, the visuo-spatial sketchpad comprises two functionally-distinct components: a *visual cache* and an *inner scribe*. The visual cache is proposed to be a passive visual storage component responsible for the temporary storage of visual features, such as shape and colour (Logie, 1999). The inner scribe is theorised to be a more dynamic spatial retrieval and rehearsal system responsible for storing movement-related information and reducing the decay of information stored in the visuo-spatial sketchpad through a rehearsal mechanism. Mental image manipulation is also theorised to be a function of the inner scribe (Logie, 1999).

Logie's view is supported by a study conducted by Quinn and McConnell (1996, Experiment 1), in which dynamic visual noise was found to interfere with visual imagery. Specifically, participants tried to memorise lists of words using either rote rehearsal or a visual mnemonic strategy. The former required the words to be remembered by sub-vocal repetition, and the latter required creating a mental image for each word in the list. A display of dots that changed randomly and continuously was presented simultaneously with the learning task as a secondary (interference) stimulus. The reported reason given for using this dynamic visual noise was that, unlike regularly-changing visual displays, it would not engage attention due to the change being "constant and evenly distributed throughout the whole display" (Quinn & McConnell, 1996, p. 204). Irrelevant pictures were found to interfere with the processing of words learned by both subvocal rehearsal and visual imagery strategies, while dynamic visual noise only interfered with the words learnt through the visual mnemonic strategy, suggesting that the visuo-spatial subsystem, but not the featural subsystem, is susceptible to interference from visual noise. These results provide further support to Logie's notion that visual information gains access to visuo-spatial sketchpad obligatorily, without higher-level cognitive analysis being required.

Findings from a number of experiments (e.g., Baddeley, 1996; Chen et al., 2000; Logie, 1986; Logie & Marchetti, 1991; Quinn & McConnell, 1996) and neuropsychological studies (Baddeley, Bressi, Della Sala, Logie, & Spinnler, 1991; Farah, Hammond, Levine & Calvanio, 1988) have also been interpreted as supporting a dissociation of visual and spatial components within visuospatial working memory. For example, in the study of Della Sala et al. (1999), a disassociation of visual and spatial span was observed. In their study, a *Corsi block* task (Kessels, Van Zandvoort, Postma, Kappelle, & De Haan, 2000) was used to measure spatial span, while visual span was measured with a pattern span task. In the Corsi block task, participants attempted to replicate the sequence tapped by the experimenter on an array of nine blocks. The sequence length increased with each correct response and the test terminated when participants could not imitate a sequence. In the visual-span task, matrices with randomly filled cells (approximately 50%) were presented to participants, who were asked to recall which cells were filled after the matrix had been removed. The test started with a 2 × 2 pattern, with the matrix size increasing until participants could no longer recall correctly. A performance decrement in the Corsi block-tapping task was observed when a concurrent spatial interference task was presented, while a concurrent visual interference task did not show any disruptive effect. The reverse was true for the pattern span task (Della Sala, Gray, Baddeley, Allamano, & Wilson, 1999).

However, the predominant view, that the visuo-spatial sketchpad comprises two specialised buffers (Baddeley & Logie, 1999), has been challenged by empirical studies indicating a distinction between temporary memory of static visual configurations and memory of movement sequences (e.g., Della Sala et al., 1999; Logie & Pearson, 1997; Logie & Marchetti, 1991; Pickering, Gathercole, Hall, & Lloyd, 2001). For example, Pickering et al., (2001) proposed that the visuo-spatial sketchpad should be conceptually fractioned between dynamic and static functions rather than visual and spatial functions. In this study, two tasks were administered to children between 5 and 11 years old, each involving visuo-spatial information presented in both static and dynamic formats. In the static version of both tasks, participants were presented with static images. The dynamic version of each task involved the presentation of two types of dynamic information: squares in a visual matrix presented in a spatial sequence and a moving route traced by the experimenter's finger. A developmental dissociation between the static and the dynamic version of each of the two tasks was observed. Thus, Pickering et al., (2001) argued that, rather than a visual and spatial distinction, a static and dynamic distinction should be considered regarding the structure of visuo-spatial working memory. Similarly, Logie and Pearson (1997) found that compared to memory for location sequences, children's memory for static visual matrix patterns developed much faster. These results further support a disassociation between static and dynamic components of visual short term memory.

Neuroimaging studies on patients with brain damage have also provided support for a distinction between static and dynamic visuo-spatial functions (e.g., Carlesimo, Perri, Turriziani, Tomaiuolo, & Caltagirone, 2001; Farah et al., 1988; Wilson, Baddeley, & Young, 1999). For example, Farah, Hammond, Levine and Calvanio (1988) reported on a patient with damage to the right temporal lobe and in the right inferior frontal lobe who performed well on mental rotation tasks (letter rotation, 3-D form rotation), whereas his ability to recall the shapes, colours and sizes of objects was severely impaired.

In summary, research on visuo-spatial working memory has progressed significantly in the past two decades; different theoretical frameworks in terms of dissociated functions have been proposed based on empirical and neuroimaging studies. However, there is still little consensus regarding the structure and the segregation of the visual component of the working memory system (see Logie & D'Esposito, 2007; Mammarella, Pazzaglia & Cornoldi, 2008). Although dissociations between object and location, as well as between dynamic and static functions have been suggested for visual working memory, in the present research, “visual working memory” refers to its static and visual aspects. This is because studies investigating the role of visual working memory in mental arithmetic have been concentrated on static and visual elements (Hubber, Gilmore, & Cragg, 2014). It is likely that, due to the nature of the stimuli, visually-represented numerical information in the problem-solving process is processed by the static and visual aspect, rather than by the dynamic and spatial aspect of visual working memory.

Although dynamic and spatial elements of visual working memory might also be involved in mental arithmetic (Reuhkala, 2001), there is little evidence supporting this notion. For example, in the study of Hubber et al., (2014), accuracy of addition problems was not affected by either static or dynamic visual load, and arithmetic performance did not differ between the static and the dynamic visual load conditions. Similarly, Otsuka and Osaka (2014) did not find a significant difference in error rates between addition tasks performed alone and those performed with a concurrent spatial tapping task, which is theorised to load the spatial and dynamic part of visual working memory. The results from studies mentioned above are consistent with the notion that static rather than dynamic visuospatial working memory accounts for variance in mathematics performance (Kyttälä & Lehto, 2008). These will be further discussed in the section “The Role of Visual Working Memory in Mental Arithmetic”.

Central Executive

In the multi-component working memory model proposed by Baddeley and Hitch (1974), a central executive sub-system is theorised to be an attentional control system responsible for allocating limited processing resources to the phonological loop and the visuo-spatial sketchpad. Initially, the central executive was a vaguely-defined, unitary structure. However, evidence supporting dissociations between different functions of the central executive has accumulated (e.g., Baddeley, 1996; Burgess & Shallice, 1994; Duncan, Johnson, Swales, & Freer, 1997; Friedman & Miyake, 2004; Fuster, 1997; Miyake, Friedman, Emerson, Witzki, Howerter, & Wager 2000; Shallice & Burgess, 1993; Smith & Jonides, 1999).

Baddeley (1986) adopted the attentional control model proposed by Norman and Shallice (1986) as the first model of the central executive, which was then known as the *supervisory activating system* (SAS). This system has been theorised to be a capacity-limited attentional mechanism, functioning to control the process of understanding novel situations and problems that cannot be resolved by previous experience (Norman & Shallice, 1980). The supervisory-activating-system model provided an explanation of results reported by Baddeley (1966) using a random letter-sequence generation task. In this study, when participants were asked to produce random letter sequences with short time intervals between the letters, the sequences generated were more random than those produced when there were longer intervals between letters (Baddeley, 1966). Under the supervisory-activating-system model, this is accounted for in terms of the selection of information requiring intervention or supervision from an attentional control system: When participants have a shorter time to generate a letter, the supervisory activating system has less time to intervene in the selection process, allowing information sequences with more 'randomness' to be generated (Norman & Shallice, 1986). However, this original unitary model of central executive was vaguely defined.

Better understanding of central executive was achieved through growing number of empirical studies investigating the cognitive processes carried out in the following decade. Evidence that the central executive is not unitary in architecture, but comprises a number of sub-systems has been found (e.g., Shallice & Burgess, 1994; Burgess, 1997; 1998). For example, results of single-case analyses of brain-damaged patients have shown normal performance on tasks requiring inhibitory processes but impaired performance on tasks involving rule detection (Shallice & Burgess, 1994). These findings challenged the unitary model of the central executive and attempts have been made to further fractionate the central executive component into sub-components (e.g., Baddeley et al., 1986).

Based on the findings of dual-task experiments, Baddeley (1996) divided the central executive into several theoretical components. Subsequently, an architecture of the central executive with four different processes attributed to this attentional control system being specified (Baddeley, 1996): focused attention, co-ordination of the two subsystems (phonological and visual), selective attention, and selection and manipulation of information related to the content of working memory retrieved from long-term memory.

Miyake, Friedman, Emerson, Witzki and Howerter (2000) showed that executive functioning comprises diverse functions. They examined the separability of three different executive functions (the ability to shift between tasks, updating representations in working memory, and inhibition of automatic responses) using a latent variable analysis. Although largely independent, these executive functions were also found to be moderately correlated with each other, suggesting that the three executive functions are not completely independent, but share some common underlying mechanisms. The authors therefore concluded that the central executive comprises both unitary and diverse functions.

Recently, the notion of the central executive as a capacity-limited but flexible attention system comprising multiple specialized functions has been challenged by evidence from structural and functional brain imaging (e.g., Nijboer, Borst, Rijn, & Taatgen, 2014; Parra, Della Sala, & Logie, 2014). For example, in the study of Nijboer et al., (2014), participants performed cognitive tasks alone or concurrently. Impaired performance under dual-task compared with single-task conditions was only observed when the brain areas activated by tasks overlapped. Thus, they concluded that the reduction in dual-task performance is closely related to interactions between brain networks and the extent to which the activated brain areas overlap. Therefore, it is suggested that the idea of central executive should be replaced by an emergent property of "how different brain networks interact and are deployed to meet task requirements" (p. 2104, Logie, 2016).

Although there is growing literature concerning the identification of executive processes, a range of different cognitive functions has been ascribed to this working memory component and the exact nature and relationships of these functions remain unclear (Collette & Linden, 2002; Logie, 2016). Moreover, the tasks used to interfere with central executive functioning have been criticised as being less pure than those used to interfere with the non-executive components, meaning that the roles of central executive cannot be clearly distinguished from those of phonological and visual working memory (Collette & Linden, 2002). For example, the random-digit generation task is widely used to measure attentional focus by the central executive (Baddeley, Emslie, Kolodny, & Duncan, 1998); however, in addition to imposing heavy loads on the central executive, it might also require phonological resources.

In the domain of mental arithmetic, the role of the central executive in carry operations has been investigated extensively (see DeStefano & LeFevre, 2004 for a review). However, rather than the temporary storage of carry information or intermediate solutions, its function is restricted to attention allocation in the problem-solving process, including attending to carry operations and monitoring the ongoing problem-solving process simultaneously (e.g., Imbo et al., 2007). The present research focuses on the relative involvement of working memory resources in the short-term storage of carry information or intermediate solutions, which is theorised to be the function of phonological or visual working memory. Any role of the central executive in carry operations for mental arithmetic was not investigated in the present research.

Episodic buffer

The tripartite model of working memory proposed by Baddeley and Hitch (1974) has been widely applied in research related to cognitive activities. Even so, it has failed to provide satisfactory explanations for the findings of a number of studies (Baddeley, 2000; Baddeley, Vallar, & Wilson, 1987; Baddeley & Levy, 1971). For example, in a study reported by Baddeley et al., (1987), better performance was observed when participants were required to recall words in the form of a sentence than when they recalled unrelated words. This effect could not be ascribed to the central executive, which theoretically has no storage capacity. It could not be addressed in terms of the phonological loop or the visuo-spatial sketchpad either, because they have been theorised to temporarily store and manipulate modality-specific information rather than semantic information. Baddeley (2000) suggested that the sentence advantage might be attributable to a process of grouping individual words into larger units based on semantic features stored in long-term memory.

Another empirical finding for which the tripartite model does not adequately account is a smaller-than-expected decrease in digit recall performance for sequences presented visually with a concurrent articulatory suppression task (Baddeley, 2000). As mentioned earlier, under the tripartite model, visually-presented information is postulated to be automatically transformed into phonological codes by the phonological loop (Baddeley & Hitch, 1974). If this is the case, then a significant performance decrement should be observed for digit recall with concurrent suppression. However, Baddeley (2000) showed that performance was only slightly impaired, which casts doubt on the tripartite model in terms of its lack of capacity to account for the way in which the information from the two subsystems is combined.

Based on these findings, Baddeley (2000) proposed an addition to the tripartite model, a component that holds information integrated from the phonological loop and the visuo-spatial sketchpad, and from long-term memory using a multi-modal code (see Figure 2). Baddeley termed this addition to his model the *episodic buffer*. The episodic buffer is distinct from the central executive: it is responsible for integrating rather than coordinating information. The coordination of information held separately by the two subsystems, and attentional control are still considered to be functions of the central executive. The episodic buffer is also postulated to serve as a mnemonic function, holding integrated information from the two subsystems, as well as its long-term associations (Baddeley, 2000).

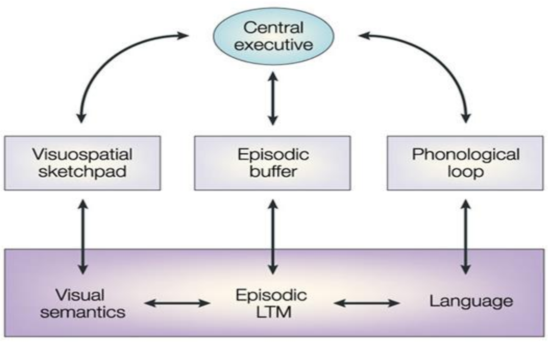


Figure 2. Baddeley's Revised Working Memory model. Adapted from "Working memory: looking back and looking forward" by A. Baddeley, 2003, Nature Review Neuroscience, 4, p. 835. Copyright 2003 by the American Psychological Association.

Empirical research investigating the episodic buffer component is limited, and there is still controversy regarding whether attentional control is required to retain associations between phonological and visual information (Cowan, Elliott, Saults, Morey, Mattox, Hismjatullina, & Conway, 2005). Any involvement of this component in mental arithmetic was not investigated in the present research.

In summary, Baddeley and Hitch's working memory model comprises an attentional control system (central executive) and two storage sub-systems (the phonological loop and the visuo-spatial sketchpad). All of these components of working memory have been demonstrated to play a crucial role in the execution of procedural calculation processes: The temporary storage of numerical information has been theorised to be associated with the phonological and visual working memory component, and the monitoring and manipulation of numerical information has been theorised to rely on central executive (Hubber, Gilmore & Cragg, 2014). In the present study, I focused on the ability to temporarily maintain digital carriers or intermediate solutions in visual and phonological components of working memory (see DeStefano & LeFevre, 2004 for a review).

The Triple Code Model

Arithmetic operations (e.g., addition, multiplication, subtraction and division) have been theorised to vary in the relevant involvement of phonological and visual working memory resources in mental arithmetic. Different routes through which individuals can perform different arithmetic operations have been proposed by the *Triple Code Model* (TCM, see Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene & Cohen, 1997), which is the predominant theoretical model of the cognitive processes underlying arithmetic.

The Triple Code Model is a multi-route model of numerical processing, which proposes that numbers are represented in three functionally independent codes: an analog magnitude code, an auditory verbal code and a visual Arabic code (Dehaene, 1992). *The analog magnitude code* represents numerical quantities on a mental number line (Dehaene, 1989; Dehaene & Cohen 1995), which facilitates magnitude comparison and approximation tasks (Dehaene, 1989; Dehaene et al., 1990; Moyer & Landauer, 1967). This code includes semantic content of number in terms of proximity (e.g., 2 is close to 3) and relative size (e.g., 2 is smaller than 4). It has been theorised that the analog magnitude code engages the bilateral inferior parietal regions (Chochon, Cohen, Moortele, & Dehaene, 1999; Dehaene et al., 1999), a brain structure that has previously been associated with the processing of numerical information. The auditory verbal code represents numbers in a verbal format (e.g., lexically, phonologically), which is used to retrieve familiarised arithmetic facts learned by rote learning, such as addition and multiplication tables (Gonzalez & Kolers, 1982). The auditory verbal code is predicted to rely on the left perisylvian network and the left basal ganglia and thalamic nuclei, which have been associated with memory and sequence execution (Dehaene, 1997; Houk & Wise, 1995). The visual Arabic code is involved in the representation and spatial manipulation of numbers in Arabic format (Ashcraft & Stazyk, 1981; Cohen & Dehaene, 1991; Dahmen et al., 1982). It is engaged in tasks that demand orientation of spatial attention (e.g., number comparison, approximation and counting). This code is predicted to engage bilateral inferior ventral occipitotemporal regions: the left used for visual identification of words and digits, while the right only for Arabic numbers (Dehaene, 1992, 1997). According to the Triple Code Model, these three distinct codes represent separate but connected calculation systems, which might be recruited in mathematical tasks (Dehaene, 2007).

Two major transcoding paths between the three representational codes have been proposed based on the Triple Code Model (Dehaene & Cohen, 1995; 1997): a *direct asemantic route* and an *indirect semantic route*. The former is theorised to convert written numerals into verbal representations to guide retrieval of arithmetic facts without semantic mediation. The latter is theorised to process the mental representation of quantity to compare operands and uses back-up strategies through manipulating visual Arabic representations when relevant arithmetic facts are not available, such as decomposing complex problems into simple single-digit problems for which facts *can* be retrieved (LeFevre Sadesky, & Bisanz, 1996; Schmithorst & Brown, 2004). A general workspace responsible for maintaining intermediate solutions in working memory and detecting errors has also been proposed, which is associated with prefrontal areas and the anterior cingulate (Dehaene, 1997; Dehaene & Naccache, 2001; Dehaene et al., 1996).

Basic Arithmetic Operations and The Tripe Code Model

The Triple Code Model predicts a double dissociation between the four basic arithmetic operations (Dehaene & Cohen, 1997): Addition is theorised to be either solved verbally (Dehaene & Cohen, 1992) or visually (Moll, Gobel, & Snowling, 2015) depending on the problem complexity. Simple addition problems (single digit and small numbers) have been theorised to predominantly rely on fact-retrieval, while solving complex addition problems have been theorised to require the analog magnitude system. Multiplication has been theorised to mainly rely on verbally-mediated retrieval of arithmetic facts (Dehaene & Cohen, 1997; De Visscher & Noel, 2016), and subtraction only to require the mental-magnitude representation system (Dehaene & Cohen, 1997;

Zamarian et al., 2009). There is still little consensus regarding the calculation process involved in division problems, hence this operation was not discussed in the present review.

According to The Triple Code Model, the distinction between four arithmetic operations are also reflected in neuroanatomical representations. Simple addition has been assumed to activate the left angular gyrus, which is theorised to be responsible for arithmetic fact retrieval that utilises the verbal code (Grabner, Ansari, Koschutnig, Reishofer, & Ebner, 2013), whereas complex addition activates bilateral intraparietal sulci that represent the “mental number line” that utilises the analog-magnitude code (visual-spatial, Schmithorst & Brown, 2004). Subtraction has been assumed to be similar to complex addition, and also to activate the bilateral intraparietal sulci (Piazza, Pinel, Le Bihan & Dehaene, 2007), whereas multiplication only activates the left angular gyrus, or middle temporal gyrus like simple addition (Delazer et al., 2003).

Previous lesion and neuroimaging studies have provided support to these predictions on the distinctions between arithmetic operations (e.g., Dehaene & Cohen, 1997; Fehr, Code, & Herrmann, 2007; Lee, 2000; Kong, Wang, Kong, Vengel, Chua & Gollub, 2005). For example, in the study of Dehaene and Cohen (1997), performance deficits in addition and multiplication were observed for a patient with impairments for rote verbal knowledge, whereas performance deficits in subtraction were observed in a patient with impaired quantitative knowledge. Similarly, Lee (2000) compared brain responses to single-digit multiplication and subtraction problems, and found greater activation in the left angular gyrus (fact-retrieval) for multiplication than for subtraction, whereas greater activation in the bilateral intraparietal sulci (mental number line) for subtraction than for multiplication. Based on these findings, it has been proposed that simple multiplication and addition predominantly require more phonological resources, whereas subtraction predominantly requires more visual resources (Dehaene, Piazza, Pinel & Cohen, 2003).

For complex mental arithmetic problems that cannot be solved through direct fact-retrieval, the Triple Code Model predicts the activation of bi-hemispheric parietal regions associated with visuo-spatial and numerical quantity processing (Dehaene & Cohen, 1995; Dehaene et al., 2003; Pinel et al., 1999). Moreover, the model predicts the lateral frontal regions would be more active for complex problems than for simple problems due to higher demands on working memory in the problem-solving process (Gruber et al., 2001; Dehaene et al., 2004). This notion has been supported by some studies (e.g., Fehr et al., 2007; Kong et al., 2005). For example, in the study of Kong et al., (2005), both behavioural and neuro-imaging data were compared between complex addition and subtraction problems. Specifically, problems with one carry operation (e.g., 35 + 7) and with no carry operations (e.g., 35 + 3) were used for both operations. A significant main effect of operations was revealed in the response time data, with subtraction problems solved slower than addition problems. Also, more activation in the right inferior parietal lobule, left precuneus and left superior parietal gyrus were observed for subtraction than for addition, indicating the differential involvement of neural networks in different arithmetic operations. The authors concluded that specific neural networks are associated with each arithmetic operation, with these networks serving as a common basis, with more regions becoming engaged with increasing problem difficulty.

However, this study has been criticised for its lack of comparison between carry and no-carry problems; carry and no-carry problems were collapsed into one factor in the analysis (Fehr et al., 2007). Moreover, only 11 participants were included in the final analysis, so the generalization of their results is questionable due to such a small sample size. In another study, Fehr et al., (2007) compared brain activation to visually-presented complex (double-digit) and simple (single-digit) problems involving each of the four arithmetic operations. Compared to single-digit problems, double-digit problems activated bilateral superior and middle frontal gyri and the right precuneus for all operations. All other regional activation was operation-specific: Compared to simple addition, complex addition showed more bilateral activation in frontal, temporal, occipital and left inferior parietal areas. Complex multiplication in contrast to simple multiplication showed activation in the bilateral frontal, left, central and right fusiform gyrus. Contrasting complex subtraction in contrast to simple subtraction resulted in bilateral frontal, bilateral inferior parietal, posterior cingulate activation. Thus, these results are consistent with the notion that different operations require different involvement of phonological and visual processes, while complex arithmetic problems activate the same brain structures (for domain general process associated with problem complexity) irrespective of the specific arithmetic operation, due to their increased demand on working memory (Dehaene & Cohen, 1995). However, it should be noted that brain activation between operations were not compared directly, thus the differences between operations were not clear (Rosenberg-Lee, Barth, & Menon, 2011).

The notion of operation-specific processing paths has later been challenged (e.g., Van Harskamp & Cipolotti, 2001; Kawashima et al., 2004; Rosenberg-Lee, et al., 2011; Tschentscher & Hauk, 2014). For example, three patients with selective impairments for addition, multiplication and subtraction have been reported in a study by Van Harskamp and Cipolotti (2001). This is inconsistent with the predictions of the Triple Code Model as it assumes that addition and multiplication both rely on an asemantic route. Therefore, both operations should be impaired for a patient with impairment to the asemantic route. Moreover, Kawashima et al., (2004) observed only minor differences in activation patterns of neural networks for simple addition, multiplication and subtraction, indicating overlap, rather than strict separation, of retrieval-based and magnitude-related processing in problem solving. Other studies have proposed that the operation-specific activation patterns might be a result of differing problem complexity (e.g., Rosenberg-Lee, Barth, & Menon, 2011; Tschenscher & Hauk, 2014). In the study of Rosenberg-Lee et al., (2011), the activation of a neural network involved in the verification of single-digit problems for all four operations were compared using functional magnetic resonance imaging (fMRI). Specifically, they compared neural activation patterns for both “small” problems (e.g., 2 + 3 =) and the “large” problems (e.g., 9 + 6 =) between addition, subtraction, division and multiplication. Regardless of problem size, similar activation in the intraparietal sulcus, superior parietal lobule and angular gyrus was found for all operations besides addition, challenging the view of operation-specific involvement of neural networks.

In conclusion, the operation-specific view of numerical processing paths (corresponding to working memory resources) for complex arithmetic problems is still under debate. Although there is some evidence supporting the differential involvement of visual and phonological processes for addition and multiplication, these studies have mainly employed single-digit problems (e.g., Lee, 2000). Although some studies employed multi-digit problems have nevertheless often mixed with single-digit problems in the final analysis (e.g., Kong et al., 2005). Moreover, a majority of the studies reviewed have not included a control condition (e.g., Kong et al, 2005), limiting the interpretability of their findings (Rosenberg-Lee et al., 2011). Finally, many of these previous studies often lacked the behavioural measures for the different operation types (e.g., Kawashima et al., 2004), which is necessary to assess compliance and brain-behaviour relationships.

The Role of Working Memory in Complex Mental Arithmetic

A number of behavioural studies have investigated the relative involvement of working memory resources in complex mental arithmetic (see DeStefano & LeFevre, 2004 for a review), apparent contradictory findings have emerged. Some of these studies have supported the involvement of working memory components in carry operations or the storage of intermediate solutions in multi-digit mental addition and multiplication (Fürst & Hitch, 2000; Heathcote, 1994; Imbo & LeFevre, 2010; Logie, Gilhooly, & Wynn, 1994; Trbovich & LeFevre, 2003), while others have not (Hubber et al., 2014; Imbo, Vandierendonck, & Vergauwe, 2007; Otsuka & Osaka, 2014; Seitz & Schumann-Hengsteler, 2000, 2002). Several factors related to experimental design and secondary interference tasks might be responsible for this apparent inconsistency.

Studies employing tasks intended to index the relative involvement of visual and phonological representations differ from one another in various ways; including whether the number of carriers was manipulated (see Heathcote, 1994; Seitz & Schumann-Hengsteler, 2000, 2002); whether arithmetic problems were presented vertically or horizontally (see Deslauiers Ouellette, Barnes, & LeFevre, 2008; Imbo & LeFevre, 2010; Trbovich & LeFevre, 2003), whether arithmetic operands were presented briefly or until a response was made (see Fürst & Hitch, 2003; Heathcote, 1994; Logie et al., 1994; Noël, Desert, Aubrun, & Seron, 2001; Seitz & Schumann-Hengsteler, 2000, 2002), and whether only dual-task trials with *both* arithmetic and working memory load tasks performed correctly were included in analyses (see Trbovich & LeFevre, 2003) as opposed to requiring only one task to be correctly performed were included in the analyses.

Lack of comparability between phonological and visual interference tasks within research might have led to inconsistent findings, such as the apparent involvement of phonological resources but not visual resources in the same study. For example, in some studies, participants were required to repeat a word throughout the calculation process (e.g., “bla, bla, bla”) as phonological interference, while hand movement (e.g., pressing buttons on a keyboard) has been used as visuo-spatial interference (Fürst & Hitch, 2002; Heathcote, 1994; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002). There is little comparability between repeating a word and pressing buttons: The former might require the employment of phonological resources but the latter might only require more attentional resources rather than visual resources.

In addition to the incomparability of phonological and visual interference tasks used *within* the same studies, variability of these tasks *across* studies might also have led to inconsistent results. For example, some studies have asked participants to repeat a word (either aloud or sub-vocally) as phonological interference (Fürst & Hitch, 2003; Heathcote, 1994; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002), while others have asked participants to recognise if a phonological nonword was one of a list of nonwords presented prior to the arithmetic task (Trbovich & LeFevre, 2003). Similarly, in some studies, participants were required to replicate patterns of asterisks or dots as visual interference tasks in some studies (see Hubber et al., 2014; Imbo & LeFever, 2010; Imbo & Vandierendonck, 2007; Trbovich & LeFevre, 2003), and in others, they were presented with irrelevant pictures (Heathcote, 1994), or pressed buttons on a keyboard (Logie et al., 1994; Otsuka & Osaka, 2014; Seitz & Schumann-Hengsteler, 2000).

Unlike studies investigating the role of phonological resources, variability in experimental paradigms used across studies might have only led to mixed results in the role of visual resources in complex mental arithmetic, as only experiment paradigms other than the dual-task paradigm were used in studies involving visual resources (e.g., Deslauriers et al., 2008). For example, in the study of Deslauriers et al., (2008) the visual similarity of digits in addends was manipulated as visual load, rather than using a concurrent visual load task. Other studies that have used a dual-task paradigm have required participants to perform visuo-spatial tasks simultaneously with arithmetic problems in the dual-task condition (e.g., Hubber et al., 2014; Imbo & LeFever, 2010; Imbo & Vandierendonck, 2007; Trbovich & LeFevre, 2003; Logie et al., 1994; Otsuka & Osaka, 2014; Seitz & Schumann-Hengsteler, 2000).

These differences noted above make it difficult to reconcile results, especially regarding their implications for the involvement of visual and phonological representations in complex mental arithmetic. In the following section, studies investigating the relative involvement of visual and phonological resources in complex mental arithmetic are reviewed in terms of the impact of their experimental design, selection of secondary interference tasks, and manipulation of number of carriers on their findings. It should be noted that the focus of the present study is the role of phonological and visual resources in *carry operations and/or intermediate solutions* rather than the entire problem-solving process (e.g., encoding, solution reporting) in complex mental arithmetic. To avoid repetition, this focus of the present research was referred as the role of phonological or visual resources in complex mental arithmetic in the following sections.

The Role of Phonological Loop in Complex Mental Addition and Multiplication

Phonological representations in complex mental addition and multiplication have been theorised to be associated with carry operations or the storage of intermediate solutions (Imbo & LeFevre, 2010, Trbovich & LeFevre, 2003). As noted previously, published studies have mainly assessed such a role using a dual-task interference paradigm, which comprises an arithmetic task and a secondary phonological interference task (see DeStefano & LeFevre, 2004 for a review).

The Role of Phonological Loop in Complex Mental Addition

The number of carry operations involved in mental addition problems is one of the most manipulated variables in dual-task studies. However, the nature of this manipulation has varied across them: In some studies (e.g., Heathcote, 1994; Logie et al., 1994), any performance decrement in arithmetic caused by a phonological load could not be compared between carry and no-carry problems because only problems with carry operations were used. As argued by DeStefano and LeFevre (2004), a load-by-carry interaction might be the most convincing evidence in dual-task studies examining the role of a working memory component in carry operations or the storage of intermediate solutions: Performance on carry problems should be affected differently from performance on no-carry problems by the same concurrent interference task, if the interference affects a working memory component that is implicated in the carry process (see Seiz & Schumann-Hengsteler, 2002 as an example).

Even when both carry and no-carry problems have been used, the number of carriers has not always been included as a factor in the analyses, leading to alternative explanation of results. For example, in the study of Trbovich and LeFevre (2003), addition problems with or without carry operations were visually presented, either horizontally or vertically. The involvement of phonological resources was implicated as a greater addition performance decrement was observed for horizontally-presented problems than for vertically-presented problems in the dual-task condition, compared to when addition task was performed alone. However, there was no direct comparison of the addition performance decrement between carry and no-carry problems because problem type (carry or no-carry) was collapsed in the analysis. Thus, their results cannot be interpreted from the perspective of temporary storage of carry operations or intermediate solutions; addition performance decrement was compared between horizontal and vertical presentation on the assumption that the involvement of phonological resources is more facilitated by horizontal presentation rather than by vertical presentation.

Variation in the methods by which participants have reported addition solutions might also have led to mixed evidence across studies. Many previous dual-task studies that failed to find a role of phonological resources in mental addition required final solutions to be written down (e.g., Fürst & Hitch, 2000; Heathcote, 1994) or typed using a keyboard (e.g., Hubber et al., 2014; Imbo et al., 2007). Compared to a *verbal report of solutions*, writing or typing solutions might have reduced the need to maintain intermediate solutions in working memory. For example, articulatory suppression did not interact with number of carriers in the study of Imbo et al., (2007), in which participants were asked to type their answers for the units, tens and hundreds respectively. Participants might have broken down the multi-digit addition problems to several single-digit problems (e.g., decomposed 123 + 456 = into 3 + 6 = for units, 2 + 5 = for tens, and 1 + 4 = for hundreds) and solved them through direct retrieval from long-term memory. In this case, no intermediate solutions would need to have been maintained in working memory, as participants only needed to perform one single-digit problem at a time. For problems involving carry operations, participants only need to maintain the digital carrier not the intermediate solutions, thus reducing the demand on working memory. It should be noted that compared to reporting final solutions verbally, this reduced requirement to maintain both carry information and intermediate solutions might not require enough phonological resources to evince a load-by-carry interaction. It should also be noted that typing or writing solutions is likely to affect the accuracy of response latency recording, leading to unreliable results.

Compared to typing or writing report methods, verbal report of solutions is more likely to achieve more accurate response times. For example, latencies of arithmetic problems can be recorded through the activation of a voice key when participants report a solution orally (e.g., Fürst & Hitch, 2000; Imbo & LeFevre, 2010; Logie et al., 1994; Noël et al., 2001; Otsuka & Osaka, 2014; Seitz & Schumann-Hengsteler, 2002; Trbovich & LeFevre, 2003). Nonetheless, this reporting method might affect the role of phonological resources in complex mental arithmetic when certain secondary phonological interference tasks are used, one of them being an *articulatory suppression* task. This task normally requires participants to repeatedly verbalise a word (e.g., "bla, bla, bla") throughout the calculation process (e.g., Fürst & Hitch, 2003; Heathcote, 1994; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002). Addition performance decrement has been observed in several studies using articulatory suppression (e.g., Heathcote, 1994; Seitz & Schumann-Hengsteler, 2002), however, this task doesn't require any phonological representation to be maintained, so there is no guarantee that there is any interference with phonological-represented carriers or intermediate solutions. Thus, it is more likely that the observed interference effect on addition performance was due to participants articulating both the word and solutions rather than maintaining carrying information or intermediate solutions. Moreover, when final solutions have been verbally reported, participants have had to switch between the word they were required to repeat and solution of a problem (Imbo et al., 2007), which might also have imposed a greater requirement for attentional resources in addition to, or instead of, a greater requirement for phonological resources.

Compared to the articulatory suppression task, the *phonological non-word recognition task* is unlikely to be affected by the verbal report method. In particular, in this task, participants are presented with a list of non-words (e.g., *mub*, *gad*) prior to the arithmetic task, and they were instructed to maintain these nonwords during the calculation process. After an answer to the calculation task has been provided, a test nonword is presented and participants indicate whether or not it is the same or not as one of the memory non-words by pressing the “yes” or the “no” button (see Trbovich & LeFevre, 2003). Using this task, an interference effect on addition performance was observed in the study of Trbovich and LeFevre (2003) when solutions were verbally reported, suggesting a role of phonological resources in complex mental addition. Phonological nonword recognition tasks are arguably more likely to interfere with verbally-represented carry information or intermediate solutions, as the nonwords must be maintained throughout the whole calculation process.

Presentation duration has also been demonstrated to affect the relative involvement of phonological resources in complex mental arithmetic. Two presentation durations have been used in previous studies: *brief presentation* and *continuous presentation*. Brief presentation involves arithmetic operands being presented sequentially, with each operand visible on the screen for a short period of time (e.g., 20ms to 4 seconds) before disappearing and the next operand being presented to participants (e.g., Heathcote, 1994; Logie et al., 1994; Otsuka & Osaka, 2014). Continuous presentation involves both operands remaining visible until a response is made (e.g., Seitz & Schumann-Hengsteler, 2002; Trbovich & LeFevre, 2003). More evidence supporting a role of phonological resources in complex addition has been observed in studies that have employed brief presentation than in those employing continuous presentation. For example, in the study of Logie et al. (1994), 3 to 6 two-digit numbers were presented sequentially for 20ms each, with participants required to sum them (e.g., 13 + 18 + 13 + 21, correct response: 65). Articulatory suppression was used as a concurrent secondary task, with participants saying “*the*” repeatedly. Performance of the running-total addition task was poorer in the articulatory suppression condition than in the addition-alone condition. The author thus concluded that intermediate solutions were phonologically represented, arguing that articulatory suppression interferes with the maintenance of intermediate results. Similar addition performance decrement was observed in the study of Heathcote (1994), which also employed a time-limited presentation of addends.

However, it should be noted that, such a role of phonological resources observed in the studies mentioned above might be actually associated with a requirement to maintain problem information (operands), rather than also being required in calculation itself. This is because brief presentation might have forced participants to retain the operands using verbal rehearsal (Trbovich & LeFevre, 2003). This possibly is supported by the findings of the study of Fürst and Hitch (Exp. 1, 2000), with an addition performance decrement caused by a concurrent phonological load only observed in a brief presentation condition but not in a continuous presentation condition. Specifically, addition problems were presented to participants visually, but the presentation duration was manipulated: Problems were either presented briefly for four seconds, or continuously until a response was made. Compared with a single-task condition in which no articulatory suppression task was performed, participants were less accurate in a dual-task condition when problems were briefly presented, but not when they were presented continuously.

The Role of Phonological Loop in Complex Mental Multiplication

Often regarded as a correlate of complex addition, mental multiplication has been theorised to share certain cognitive operations involved in addition (e.g., storage processes; see Seitz & Schumann-Hengsteler, 2000). It has been theorised that all of the elementary stages involved in complex mental addition are also necessary for complex mental multiplication (Geary, 1994; Geary & Widaman, 1987; Hope & Sherill, 1987; Seitz & Schumann-Hengsteler, 2000, 2002). Thus, if phonological resources are required in complex addition, for temporary storage of carry information or intermediate solutions, a similar role might be expected for complex multiplication (Imbo & LeFevre, 2010).

Contradictory findings in terms of the relative involvement of phonological resources in mental multiplication, have been reported in a few dual-task studies (e.g., Imbo & LeFevre, 2010; Imbo, Vandierendonk, & Vergauwe, 2007; Seitz & Schumann-Hengsteler, 2000, 2002). Similar to addition, one possible reason for these inconsistent findings might be that performance decrement have not been compared between carry and no-carry problems. For example, Imbo and LeFevre (2010) found a role of phonological resources in multiplication by comparing multiplication performance between easy problems (involving a carry value of zero or one) and difficult problems (involving a carry value of two or three). Conversely, the study of Seitz and Schumann-

Hengsteler (2000) reported no evidence supporting such a role when easy problems were defined as those comprising two single-digit operands both five or smaller, and difficult problems as those comprising a single-digit and a double-digit operand.

As noted previously, solution reporting methods might also have explained the absence of disruptive effect of phonological loads on multiplication performance. For example, Imbo, Vandierendonck and Vergauwe (2007) did not find a multiplication performance decrement under phonological load when they required participants to type the products starting from the unit column. Similar to mental addition, this might be because of only one digit, rather than all of the intermediate solutions or digital carriers involved, had to be maintained at a time (Imbo et al., 2007).

In conclusion, the role of phonological working memory component in mental arithmetic has received plenty of attention in previous research, yet inconsistent evidence has been reported in terms of whether phonological representations are required in the problem-solving process (e.g., Fürst & Hitch, 2000; Noël et al., 2001; Seitz & Shumann-Hengsteler, 2002), and specifically in carry operations (e.g., Heathcote, 1994; Logie et al., 1994; Noël et al., 2001). Similarly, mixed evidence supporting the involvement of phonological resources in carry operations or intermediates solutions has been reported for mental multiplication (e.g., Imbo & LeFevre, 2010; Seitz & Schumann-Hengsteler, 2000, 2002). Such inconsistency might be attributed to different choices in experiment design (e.g., presentation duration, presentation format and solution-reporting methods), in task-related factors (e.g., using articulatory suppression or nonword recognition task as phonological interference tasks), or to these factors in combination (e.g., using articulatory suppression and verbal report method in the same experiment).

The Role of Visual Working Memory in Complex Mental Arithmetic

Compared to phonological resources, considerably less attention has been paid to the role of visual resources in mental arithmetic. Like phonological resources, visual working memory resources have been theorised to be associated with the temporary storage of carrier information (Hay, 1973) or intermediate solutions (Heathcote, 1994). Only a few studies have investigated the contribution of visual resources in mental arithmetic, yet mixed evidence has been reported: Support for such a role of visual resources has been reported in some studies (e.g., Deslauries et al., 2008; Hay, 1973; Heathcote, 1994; Imbo & LeFevre, 2010; Trbovich & LeFevre, 2003), and not others (e.g., Hubber et al., 2014; Logie et al., 1994; Noël et al., 2001; Otsuka & Osaka, 2014).

The Role of Visual Working Memory in Complex Mental Addition

Unlike studies investigating phonological resources, experimental paradigms have varied across studies investigating the role of visual resources in complex mental addition. In some research, a dual-task interference paradigm involving a concurrently-performed visual interference task was used (e.g., Heathcote, 1994; Hubber et al., 2014; Imbo & LeFevre, 2010; Logie et al., 1994; Otsuka & Osaka, 2014; Trbovich & LeFevre, 2003). While in other research, rather than using a concurrent visual interference task, the visual similarity of operands of arithmetic problems was manipulated as visual load (e.g., Deslauriers et al., 2008; Noël et al., 2001). For example, in the research reported by Deslauriers et al., (2008), the involvement of visual procedures in complex mental addition was tested by manipulating the visual similarity of digits: The visual similarity was controlled using a seven-point scale developed by Campbell and Clark (1988), on which one indicates low visual similarity and seven indicates high visual similarity (e.g., 1 and 8 have a similarity index of one, while the similarity index for 3 and 8 is six). The visual similarity index for each arithmetic problem was determined by calculating the mean of the visual similarities between every pair of digits involved in a problem, and addition performance decrement was compared between problems with high and low similarity of digits in addends (Deslauriers et al., 2008). This is very likely only to have affected the encoding process rather than the calculation process when operands were presented continuously, as there was no need for participants to retain any visually-represented information. Although a visual interference effect was found in the research of Deslauriers et al., (2008), this might be due to interference with the visual encoding process, whereby encoding of addends with high similarity might be more time-consuming or error prone than the encoding of addends with lower similarity. Furthermore, if it was the digital carriers or intermediate solutions that were visualised, the manipulation of the visual similarity of addends would presumably have had little effect on this process. Compared to manipulating the visual similarity of digits in operands, a dual-task interference paradigm involving concurrent visual load tasks would be more likely to affect carry information or intermediate solutions, if they were visually represented in working memory.

For studies that have used a dual-task experimental paradigm, the selection of visual interference tasks might have contributed to the mixed results. Many dual-task studies that failed to find a role of visual resources in complex mental arithmetic have used *irrelevant pictures* (participants watch matrix patterns or pictures during calculation, see Heathcote, 1994; Logie et al., 1994), *hand movement* (participants repeatedly tapping a sequence of buttons, see Logie et al., 1994) and *visuo-spatial tapping* (participants are required to press the eight keys of a numeric keypad in clockwise order, once per second, see Otsuka & Osaka, 2014; Seitz & Schumann-Hengsteler, 2000, 2002) as secondary tasks. It is not obvious that any of these would interfere with visual representations of carry information or intermediate solutions. This is because there is no need to retain visual information whilst watching visual images or moving hands, as participants were not required to process visual stimuli for the purpose of any experimental task. Moreover, participants might have stored phonological, rather than visual representations of the irrelevant picture names if they encode them at all (e.g., Exp. 1, Logie et al., 1994) as they were of objects and animals, which can be easily named and encoded phonologically.

For studies that have found a disruptive effect of visual load on arithmetic performance using visual interference tasks mentioned above (e.g., Logie et al., 1994), such effect observed might be attributable to a more general demand for attentional resources rather than specifically for visual resources in the problem-solving process. For example, although Logie et al., (1994) observed disruptive effect caused by hand movement on addition performance, this might be because the hand movement process had involved attentional resources rather than visual resources since participants had to remember the sequence of the button to press.

Compared to studies using hand movement, irrelevant pictures or visuo-spatial tapping, a load-by-carry interaction is more likely to be observed in studies that employed a *visual recognition task* (e.g., Hubber et al., 2014; Imbo & Vandierendonck, 2007; Imbo & LeFever, 2010; Trbovich & LeFevre, 2003), in which visual stimuli need to be maintained throughout the calculation process. For example, the visual recognition task used in the study of Trbovich and LeFevre (2003) included patterns of asterisks presented on a grid. In each trial, participants were required to remember the memory pattern while solving an addition problem. A test pattern was presented after the addition task and participants indicated whether it was the same or not as the memory pattern. Thus, addition performance decrement observed can be addressed in terms of the competition for limited visual working memory resources between carry information and visual stimuli.

Inconsistency in the selection of trials included in data analyses might also, at least partially, explain discrepancies in findings regarding the involvement of visual resources in complex mental arithmetic. In some dual-task studies, only trials in which both tasks were correctly solved were included in the analyses (e.g., Trbovich & LeFevre, 2003), while in other studies, trials in which only one task had been correctly performed were selected (e.g., Logie et al., 1994; Imbo & LeFevre, 2010). Trials with only one correctly performed task (either the arithmetic task or the visual interference task) might effectively be single-task rather than dual-task trials, because participants may not have attended to the other task, which could lead to an apparent absence of a visual interference effect on mental calculation, even if visual representations are required. An example is the research of Logie et al. (1994). They found no interaction between visual load and problem complexity (defined by the value of the carry) through analysing data from dual-task trials with correctly performed addition problems, irrespective of whether or not the concurrent visual interference task was performed correctly.

Similar to studies that investigated phonological resources, presentation durations also mediate the relative involvement of visual resources in complex mental arithmetic: When all addends were presented briefly and sequentially, participants might have been constrained to use sub-vocal rehearsal to retain addends, forcing the involvement of a phonological procedure and consequently diminishing the role of a visual procedure (e.g., Noël et al., 2001).

The Role of Visual Working Memory in Complex Mental Multiplication

Even less attention has been paid to the role of visual resources in mental multiplication than in addition. Only two studies have investigated the role of visual resources in complex mental multiplication (Imbo & LeFevre, 2010, Seitz & Schumann-Hengsteler, 2002), and both suffered from some of the methodological deficits mentioned above. In the study of Imbo and LeFevre (2010), multiplication performance decrement was observed. However, the manipulation of visual load was analysed based on easy and difficult problems defined by the value of digital carriers (for ‘easy’ problems, the value of the digital carrier was 0 or 1 and for ‘difficult’ problems the value of the digital carrier involved was 2 or 3), rather than on whether carry operations were required or not. As noted in relation to the role of phonological resources in mental addition, only if a greater performance decrement between the dual-task and the single-task conditions was found for carry problems than for no-carry problems, could it convincingly be concluded that working memory resources are involved in carry operations. While a visuo-spatial tapping task (participants tapping the figure ‘8’ on a wooden board under the table) was used as the visual load task in the study of Seitz and Schumann-Hengsteler (2000), with no multiplication performance decrement observed as there is no guarantee that any visual representation had to be maintained in working memory to perform this tapping task.

In conclusion, there is mixed evidence concerning the role of visual resources in complex mental addition. As discussed in the role of phonological resources in mental arithmetic, the inconsistent results across studies could be attributable to the variability in experimental design, stimuli selection, trial selection and choice of secondary tasks.

Compared to addition, evidence supporting a role of visual resources in complex mental multiplication is even more sparse, comprising just two studies, which produced contradictory results. Although the number of carry operations involved were manipulated in both studies, only the study of Seitz and Schumann-Hengsteler (2000) attempted to address the role of visual resources in carry operations with a visuo-spatial tapping task, which might have imposed more attentional load than visual load. The methodological limitations and the sparseness of the evidence on any role of visual resources in complex multiplication is such that no conclusion can be drawn at present.

In summary, a majority of studies reviewed employ a dual-task interference paradigm to investigate the role of phonological and visual resources in complex addition and multiplication. Although there is some evidence that both phonological and visual representations are selectively involved in carry operations or intermediate solutions, the reliability of results of these studies might be affected by the choice of arithmetic problems, secondary tasks, solution reporting methods and presentation durations.

Levels of working memory load have only been manipulated beyond simple presence or absence (load vs. no load) in one study reviewed here (Trbovich & LeFevre, 2003). These researchers used two levels of load, easy and hard, defined by the number of visual or phonological stimuli participants had to maintain during the calculation process. For example, only one memory nonword was presented in the easy condition, and three memory nonwords were presented in the hard condition. However, it should be noted that addition performance decrement was measured by comparing performance between vertical and horizontal presentation conditions, rather than between no-carry and carry problems; the number of carry operations was not included as a factor in the analyses. Therefore, the extent to which a working memory load might affect the involvement of phonological or visual resources in carry operations was not convincingly clarified by this study. According to Imbo et al., (2006), more convincing evidence regarding the role of working memory in carry operations or intermediate solutions would be obtained by using different carry complexities as well as different working memory load complexity in the load condition (e.g., easy, medium, difficult). It is likely that not only levels of working memory load and numbers of carry operations, but also their interaction might affect the relative involvement of phonological or visual resources in complex mental arithmetic (e.g., lower levels of load might affect relative performance on no-, single- and double-carry problems differently than medium or higher levels of load). This is because working memory capacity varies across participants and it is likely that addition performance would be less affected in participants with higher capacity than in those with lower capacity by the same level of load. Thus, it is necessary to manipulate both working memory load levels in the load condition and numbers of carry operations to rigorously investigate the role of phonological or visual resources in complex mental arithmetic.

Moreover, the effect of arithmetic operations on the relative involvement of phonological and visual working memory resources has not been systematically investigated in complex arithmetic problems. Compared to simple problems that are assumed to be solved through direct retrieval of answers, complex problems require additional procedural manipulation, such as the temporary storage of carry operations or intermediate solutions. According to the Triple Code Model, this procedural number manipulation requires an analog-magnitude code that activates more visual processes regardless of the arithmetic operation, a process known as "*semantic elaboration*", (Dehaene & Cohen, 2003). For example, when solving the problem "39 + 48 =", participants might decompose the problem into a simpler one (e.g., 40 + 50 - 3). It has been theorised that this semantic elaboration process require a good understanding of the quantities presented in the original problem, so that the problem-solving process is expected to address the mental number line associated with visual processing (Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene et al., 2003). However, inconsistent findings regarding this notion were observed in previous behavioural studies. Also, few studies have included complex multiplication problems, either in behavioural or neuro-imaging studies reviewed. Therefore, the role of arithmetic operations in the relative involvement of phonological and visual resources in complex mental arithmetic needs further investigation.

Overview of the present project

Two main issues were investigated in the present research. The first was the relative involvement of visual and phonological representations in carry operations or intermediate solutions in mental addition and multiplication. As noted in the literature reviewed, not only the number of carry operations and the level of working memory load but also their interaction might have an impact on the relative involvement of phonological or visual working memory resources in complex mental arithmetic. In the present study, both the numbers of carry operations (0, 1 and 2) and the levels of working memory load (low, medium and high) were manipulated. A dual-task interference paradigm was used and performance differences between single-task and dual-task conditions were compared in light of the manipulation of numbers of phonological or visual stimuli and of digital carriers.

With respect to the experimental design, I decided to present arithmetic problems to participants visually and horizontally until a response is made, and they were required to verbally report the final solutions. This is because, first, as noted previously, continuous presentation can exclude the possibility that any evidence supporting a role of working memory resources in mental arithmetic could be addressed by participants being forced to use phonological resources to retain operands. Second, vertical presentation has been theorised to be associated with greater activation of visual resources than phonological resources (Trbovich & LeFevre, 2003). Thus, if evidence supporting a role of visual resources was observed, it is less likely they will be attributed to presentation format when a horizontal format is used. Finally, reporting the final solutions verbally requires participants to maintain carry operations and/or intermediate solutions in working memory and affords more accurate measurements of response times than typing or writing.

The second issue addressed by the present research was the role of arithmetic operations in the relative involvement of phonological and visual resources in complex mental addition and multiplication. According to the Triple Code Model, complex addition is supposed often to be solved using visual processing, while multiplication is theorised to most often be solved using verbal processing due to retrieving the facts verbally represented in a “multiplication fact table” (Dehaene, 1992, Dehaene et al., 1997). However, complex mental arithmetic problems, with solutions that cannot be retrieved directly from long-term memory, are also theorised to involve the mental manipulation of visual image with the aid of the analog-magnitude code (Dehaene, 1992; Dehaene & Cohen, 1995). Thus it is hypothesised on the basis of the prediction of the Triple Code Model, that visual processes will be required in both complex mental addition and mental multiplication, while phonological processes should only be observed for complex multiplication.

The present research differed from previous research in three major perspectives: the comparability of visual and phonological interference tasks, the choice of visual interference stimuli and the careful selection of trials for data analysis. As previously mentioned, equivalent visual and phonological interference tasks are required to rigorously compare the relative involvement of visual and phonological resources in carry operations. A change-detection paradigm (Luck & Vogel, 1997; Phillips, 1974) that can effectively interfere with both visual and phonological working memory was therefore used in the present research.

Visual stimuli used in previous research in the domain of mental arithmetic vary across studies. Furthermore, most previously-used visual stimuli have no obvious phonological analogue. In the present research, several types of stimuli typically used in visual cognition research were tested for their suitability as visual stimuli in a change-detection paradigm, and *Gaussian-windowed sinusoidal gratings* (Wright, Green & Baker, 2000) were selected as visual stimuli (see Chapter 2 Pilot Study for more details).

With respect to trial selection, only certain trials were included in the final analysis in the present research. In the accuracy analysis of dual-task performance, only trials in which the task not under analysis was performed correctly were included. For example, in analyses of accuracy data in the arithmetic tasks, a dual-task trial was included only if the concurrent change detection was correctly performed. This is because the dual-task experimental paradigm requires participants to manage the allocation of attentional resources. If only the calculation task or the visual change-detection task was correctly solved in a specific trial, then it is possible that the other (incorrectly performed) task was ignored by participants. In this case, the trial would be functionally equivalent to a single-task condition. Compared with a single-task condition in which only one task is presented and correctly solved, performance impairment in dual-task trials in which both tasks are correctly solved is taken to reflect the relative involvement of working memory resources when both tasks compete for the resources of the same working memory component.

Similarly, in the accuracy analysis of single-task performance, only trials that corresponded to the included dual-task trials were selected: Both tasks were performed in the same order in the dual- and single-task conditions; the only difference was that, in the dual-task condition, they were performed simultaneously rather separately. Thus, each arithmetic problem or change detection involved in single-task trials was the same as that in a corresponding dual-task trial. Compared with including single-task trials involving correctly-solved arithmetic problems or correct change detections, the impact of complexity discrepancies across arithmetic problems can be excluded when included dual-task and single-task trials comprise the same problem or change detection.

The same approach applied to the selection of trials in the response time analysis: For the dual-task condition, only trials in which both the arithmetic task and the visual change-detection task were correctly performed were included in the analysis. For the single-task condition, correct trials (either arithmetic or change detection) in which the other task was also performed correctly in the corresponding dual-task trials were selected.

Four experiments were conducted in the present research, with the same participants used for all experiments. A pilot study was conducted prior to the four main experiments to test the suitability of visual and phonological stimuli in change detection and estimating VWM and PWM capacities. The four main experiments were designed to investigate the relative involvement of visual and phonological representations in carry operations in mental addition and multiplication. A dual-task paradigm comprising a visual or phonological change-detection task and an arithmetic task was used in all experiments, with both numbers of carriers and levels of working memory load manipulated. For each experiment, a sub-analysis was conducted to investigate the potential role of individual differences in working memory capacity in the relative involvement of VWM and PWM resources in carry operations in mental addition and multiplication (Chapters 3, 4, 5, and 6 for more details). Findings of the present research were discussed in the seventh and final chapter, with reference to the implications for the relationship between arithmetic cognition and working memory, the theoretical structure of the multi-component working memory model and the impact of individual differences in working memory capacity on arithmetic performance.

Chapter 2

Pilot Study

Change detection is the ability to perceive changes in the sensory environment (Rensink, 2002). This ability varies across individuals in terms of how they encode items and allocate attention among items (Rensink, 2002). Controlled experiments investigating human change detection date back to 1950s and, at that time, patterns of dots in random locations were typically used as stimuli. Participants were instructed to determine whether one of the dots had changed its location in a pattern presented later (see Rensink, 2002). These studies contributed to the theoretical basis for the conceptualisation of visual short-term memory as a capacity-limited system and individual differences in visual perception.

Change-detection tasks normally comprise two steps: Study stimuli are presented to participants (e.g., a picture of a car park with some cars), followed by probe stimuli that may be identical or modified (e.g., one of the cars may have been removed). The task is to determine whether or not there was a change (Rensink, 2002).

In the present research, change detection means the noticing of a change through the use of vision, including the abilities to identify the change in object (what) and the location (where). One of the change-detection paradigms is referred as “Gap-contingent Techniques” (Rensink, 2002). In this paradigm, study stimuli are presented for a short period of time to participants (e.g., 500ms). After a brief interstimulus interval, either a same or a modified stimulus is presented. The task is to determine whether or not the probe stimulus was the same as it was in the study stimuli.

This change-detection paradigm was used for all pilot experiments not only because it has been widely used to estimate working memory capacity across individuals, but also because both phonological and visual stimuli can be effectively used under this paradigm.

As shown in Figure 3 (from Rouder, Morey, Morey & Cowan, 2011, p. 325), there are two common versions of the Gap-contingent Techniques: a *single-probe recognition* *paradigm*, and a *whole-display recognition paradigm*. In a single-probe recognition task, participants are required to make an unspeeded same/different response at the end of each trial, to indicate whether a probe item was the same as, or different from, a previous item presented at the same location as part of a study array. In a whole-display recognition paradigm, all memory items are presented as probe items in two conditions: either the same as the memory items, or, alternatively, one of the items is novel (Rouder et al., 2011). Participants must assess all items before making a recognition judgment (Wheeler & Treisman, 2002).

The single-probe recognition paradigm has been widely used in studies involving individual differences across visual tasks. This paradigm has been shown to yield more accurate performance than an equivalent whole-display recognition paradigm. This is because only one probe item needs to be attended to and evaluate in the single-probe recognition paradigm, thus there is less interference with the vulnerable visual representation of memory items compared to the whole-display paradigm (Wheeler & Treisman, 2002). The single-probe paradigm was therefore used throughout the experiments of this study.

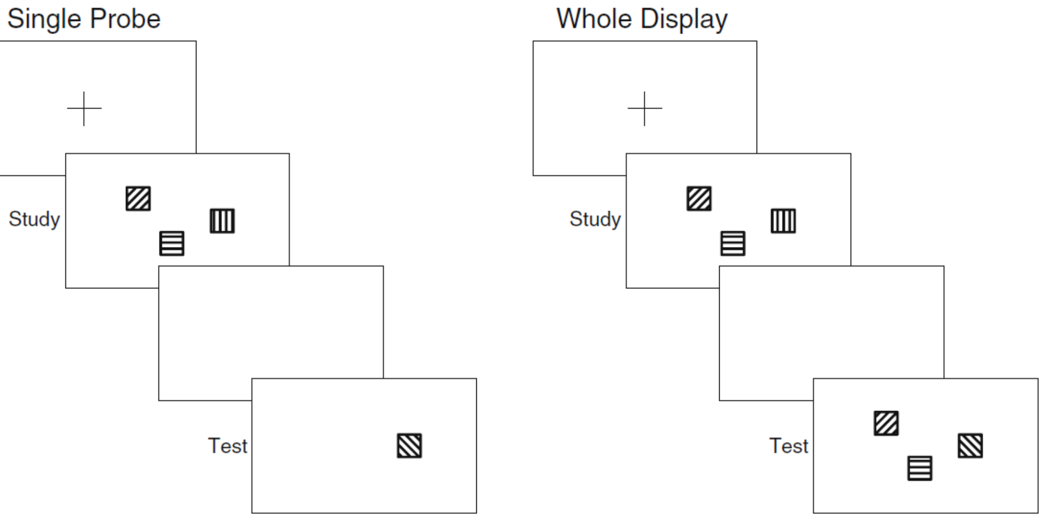


Figure 3. Single-probe and whole display change-detection paradigms. Adapted from "How to measure working memory capacity in the change detection paradigm" by J. N., Rouder, R. D., Morey, C. C., Morey, & N., Cowan, 2011, Psychonomic Bulletin & Review, 18, p.325. Copyright 2003 by the American Psychological Association.

The aim of the pilot experiments was to test the suitability of different types of stimuli as visual and phonological stimuli, in terms of change detection accuracy and the estimates of visual and phonological working memory capacity they yield.

Accuracy in visual single-probe recognition tasks is theorised to be affected by many factors, including the number of visual stimuli presented, selected features of visual stimuli and time limits for encoding study arrays (Luck & Vogel, 1997; Phillips, 1974). Change-detection accuracy has been found to be insensitive to relatively large changes in the presentation duration of study arrays within a certain range of encoding time (see Alvarez & Cavanagh, 2004; Awh et al., 2007; Luck & Vogel, 1997). In the study of Alvarez and Cavanagh (2004), change detection was conducted using shaded cubes with presentation durations varying from 50ms to 850ms. The maximised accuracy of change detection was found when duration was 450ms, and remained the same even when exposure time was longer. Although these results do not necessarily mean that all items in study arrays were encoded, they do suggest that the number of items that can be maintained in working memory, rather than presentation duration, is the main limiting factor in change detection when the presentation duration is at least 450ms (Awh et al., 2007).

Estimation of working memory capacity is theorised to be affected by the stimuli used, such as certain stimulus features (Alvarez & Cavanagh, 2004). This is because processing (encoding) speed varies with feature information to be encoded, which consequently affects change detection performance (Alvarez & Cavanagh, 2004). For example, estimates of visual working memory capacity might be different for shape and colour, because shape might be more difficult to encode visually than colour.

However, the present study has employed a simple model of WM to estimate capacity from change detection tasks, based on the assumption that participants have a fixed WM capacity that is not affected by presentation duration, number of items presented. This model assumes that participants working memory capacity is fully utilized on every trial. Errors only arise when more items are presented than can be stored and participants happened to be tested on one of the excess items. Errors associated with encoding and retrieval process are not considered in this model. Based on this notion, the working memory capacity can be directly estimated from the observed accuracy (Cowan, 2001; Magen, Emmanouil, McMains, Kastner, & Treisman, 2009).

According to this assumption, Cowan (2001) proposed a formula to estimate working memory capacity in the single-probe task (see Figure 4).

$$\mathbf{K = N\ (H - F)}$$

Figure 4. Cowan's formula for working memory capacity measurement in the single-probe paradigm: N refers to number of items presented, H refers to the correct detection rate, F refers to the false alarm rate. Adapted from "How to measure working memory capacity in the change detection paradigm" by J. N., Rouder, R. D., Morey, C. C., Morey, & N., Cowan, 2011, Psychonomic Bulletin & Review, 18, p.325. Copyright 2003 by the American Psychological Association.

In this formula, estimates of working memory capacities are subject to the assumption that K is less than or equal to N (Rouder et al., 2001). The rationale of this approach is that, when number of presented items (represents by N) equals or is smaller than number of items can be stored in working memory (represents by K), participants will have perfect performance, otherwise, they guess. Since only one probe item is presented, so only two conditions need to be considered: the probe item is in the working memory or it is not. In the first condition, participants' hit rate will always be 1, and their false alarm rate will always be 0. In the second condition, participants guess (the guessing probability is represented by g), and their hit and false alarm rates is as follows:

$$\mathbf{H = K/N + g\ (1 – K/N)}$$

$$\mathbf{F = g\ (1 – K/N)}$$

This formula was used to estimate visual and phonological working memory capacity in the analysis of all data in pilot experiments in the present study.

A key assumption of the present pilot study is that the only factor that affects change detection is the number of items to be maintained in working memory. However, notwithstanding the argument of Awh et al., (2007) that, within a certain range of encoding time, accuracy of change detection should not be affected by presentation durations within each set size, larger set sizes could nonetheless lead to more incorrect responses, even in the absence of any capacity limitations. This is because more comparisons between the probe item and memory items must be made when more memory items need to be maintained in working memory (Green, 1961; Palmer, 1990). For example, Awh et al., (2007) compared change detection performance between coloured squares, Chinese characters, random polygons and shaded cubes as stimuli using two set sizes (four and eight). Significantly higher accuracy was found for set size four than for set size eight for each type of stimuli.

The pilot experiments that follow were designed to test the suitability of different stimuli, in terms of the impact of set size, feature information and presentation duration on estimates of working memory capacity. Within a certain range of encoding time, the suitability of a particular type of visual stimulus is determined by the stability of capacity estimated across presentation durations for each set size.

It is expected that of each presentation duration, lower percentages of correct responses will be observed for larger set sizes than for smaller set sizes. If ceiling performance is observed for a set size, then that set size might be below the actual capacity of working memory of a participant, leading to underestimated capacity. For set sizes beyond the actual working memory capacity of a participant, instead of decreasing with set size, similar estimates of working memory capacities should be observed regardless of the number of items in the study array (Zhang, Johnson, Woodman, & Luck, 2012).

If a visual stimulus is suitable for VWM capacity estimation, similar accuracy of change detection should be observed across presentation durations at any given set size. This is because if accuracy improves with increasing presentation durations, then the resulting estimates of memory capacity might be influenced by encoding process rather than that of the working memory capacity. If accuracy of change detection is not affected by variation in presentation durations, then errors at set sizes beyond the actual working memory capacity of a participant will reflect “limitations in storage capacity rather than limitations in perceiving or encoding the stimuli” (p. 279, Luck & Vogel, 1997).

Pilot Experiments 1 to 4 were designed to test the suitability of bars with different orientations, Gabor patches with different orientations and spatial frequencies as visual stimuli in estimating visual working memory capacity. Pilot Experiment 5 was designed to test the suitability of aurally-presented non-words as stimuli for the estimation of phonological working memory capacity.

Visual Stimuli Selection

To prevent visual stimuli from being phonologically recoded, features of visual stimuli that can be easily labelled verbally, such as colour and shape, were excluded from consideration. Recall that visual information with a *verbal label*, such as a picture of a cat, has been theorised to be automatically converted into phonological representation and stored in the phonological loop through a phonological recoding process, which is theorised to be a function of the articulatory rehearsal system (Baddeley & Hitch, 1974). Therefore, only visual stimuli comprising features that are difficult to verbally encode, such as orientations and spatial frequencies, were considered in the pilot study.

Pilot Experiment 1

Orientation is a target feature commonly used in visual change-detection tasks, and items with different orientations have been found to be effective in procedures estimating VWM capacity (e.g., Keogh & Pearson, 2014; Luck & Vogel, 1997).

Luck and Vogel (1997) used a study array comprising small bars varying in orientation as stimuli in a change-detection task. After a 900ms retention interval, a probe array that was either identical to the study array, or in which a single probe bar was changed in orientation, compared with the original one in the same location, was presented. Change detection performance was the same as in experiments using different stimuli (such as coloured squares with an articulatory suppression task to prevent verbal encoding), suggesting the suitability of orientation as a feature of visual stimuli in change detection, without articulatory suppression (Luck & Vogel, 1997).

The first pilot experiment was designed to investigate the suitability of orientations as the target feature in visual change detection, with both presentation durations (500ms, 1,000ms, and 1,500ms) and set sizes (four and six) manipulated. In previous research, smaller set sizes (e.g., less than 3 stimuli) have resulted in ceiling performance in change-detection tasks than larger set sizes (see Zhang et al., 2012), hence larger set sizes (4 or 6 stimuli) were used here. For each set size, estimates of VWM capacity and percentage of correct responses should be similar across presentation durations. As previously noted, for each set size, if the percentage of correct responses of the change-detection task improves with increasing presentation duration, then observed estimates of memory capacity might be the result of encoding rather than indexing working memory storage. For each presentation duration, the percentage of correct responses should decrease with increasing set size.

Method

Participants

Two participants who were staff members at Victoria University of Wellington completed all pilot experiments. They had normal colour vision and normal or corrected-to-normal visual acuity.

Equipment

Testing took place individually in a dimly lit room throughout the pilot study. Eprime 2.0 Professional was used to programme all experimental tasks. The same equipment was used for all pilot experiments and formal experiments, including a 400MHz Intel PC fitted with a 19-in CRT monitor, resolution 1090×780 pixels; 60Hz, a PST serial response box (Model 200A) with five keys, a foot pedal that connected to the serial response box, and two microphones with frequency response ranges of 80-12,000 Hz.

Materials

Black bars with different orientations were generated with paint affiliated to the Windows 7 operation system, with a visual angle of 0.06° × 1.15°. In each condition, all bars were presented on an imaginary circle with radius of 2cm on a grey background with RGB (128, 128, 128) to reduce visual aftereffects. The probe bars were always rotated 90° from the stimulus in the target position in the study array in the different condition. Each study array comprised bars randomly chosen from a pool including orientations ranging from 15° to 170° without replacement. The difference of orientations between bars in a study array was at least 20°.

There were 12 patterns of bars for each set size (4 or 6), and each pattern was presented twice in each presentation duration (500ms, 1000ms, and 1,500ms) resulting 144 test trials in total. There were four practice trials prior to test trials.

Procedure

Each participant was tested individually in a quiet and dimly lit room. The procedure lasted about an hour with frequent breaks. Participants sat with their eyes 40 cm from the computer screen. They were provided with an explanation of the experiment.

The experimental trials were divided randomly and evenly into two set-size conditions (four and six) and two correct-response conditions (same/different). Participants initiated each trial using a foot pedal. All participants were instructed to respond without hesitating for too long. After providing a response, the words “Please step on the foot pedal to continue” appeared on the screen. This allowed participants to regulate their pace from trial-to-trial throughout the experiment.

Each trial began with a black fixation cross (0.5° × 0.5° of visual angel) in the centre of the screen presented against a grey background. After 500ms the study set was presented for 500ms, followed by a 1,000ms blank interval. Then a single probe square appeared randomly at one of the study set locations, and participants were asked to indicate whether the probe square was the same or different than the one at that location in the study set, by pressing one of the two response keys. Only accuracy was measured. Figure 5 illustrates the trial sequence of the same condition.

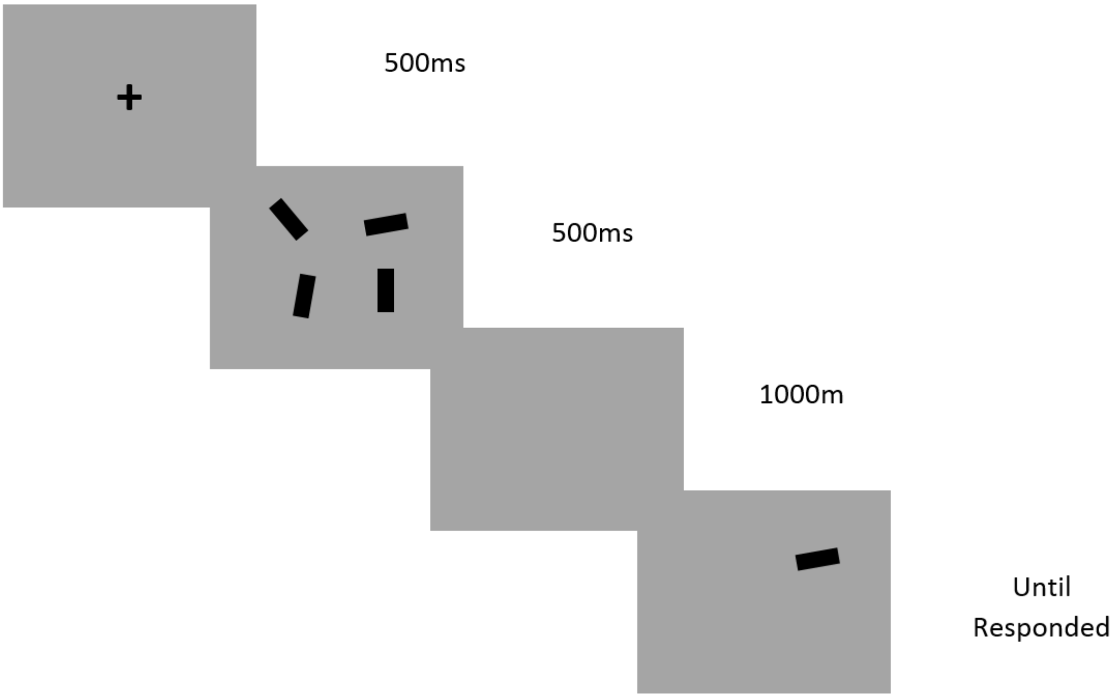


Figure 5. The trial sequence in the 500ms presentation duration condition (‘same’ condition)

Results and Discussion

Table 1 shows estimates of VWM capacity and proportions of correct responses as a function of presentation duration and set size for Participant 1. Higher accuracy was observed for set size four than for set size six, and while similar estimates of VWM capacity were observed for all presentation durations for set size four, as predicted, performance was close to ceiling, suggesting that VWM capacity might have been underestimated.

Moreover, estimates of VWM capacity for set size six increased with presentation duration, suggesting that these estimates reflect the encoding processes rather than the capacity limits of visual working memory.

Table 1 VWM capacity estimates for different presentation durations and set sizes (percentage of correct responses) for Participant 1

Set size	Presentation Duration		
	500ms	1,000ms	1,500ms
4	3 (87.5%)	3.33 (91.7%)	3.67 (93.8%)
6	2 (75%)	2.75 (72.9%)	5 (83.3%)

Table 2 shows estimates of VWM capacity and proportion of correct responses as a function of presentation durations and set sizes for Participant 2. Change detection accuracy was higher for set size four than for set size six, and estimates of VWM capacity tended to increase with presentation duration for both set sizes. Again, these results suggest that performance was not limited by the number of items maintained in working memory, but reflected the encoding process.

Substantially lower accuracy of change detection (66.7%) was observed at 1,000ms duration for set size six than the same duration for set size four. It could be argued that this is attributable to insufficient time to encode all six bars in study arrays. However, if this were the case, then even lower accuracy should have been observed at 500ms. Another possibility is that the unexpectedly low accuracy was attributable to representations of bars interfering with each other. However, this is also unlikely because high accuracy was observed for the same set size at 1,500ms duration. Rather, it is probably attributable to statistical noise.

Table 2 VWM capacity estimates for different presentation durations and set sizes (percentage of correct responses) for Participant 2

Set size	Presentation Duration		
	500ms	1,000ms	1,500ms
4	2.83 (85.4%)	2.5 (81.3%)	3.5 (95.8%)
6	2.75 (72.9%)	2 (66.7%)	4 (83.3%)

In summary, the results of pilot experiment 1 were inconsistent with those of previous studies, in which stable performance of change detection was obtained across presentation durations and set sizes, using bars with different orientations. A possible explanation could be visual after-effects: Although a grey background was used to reduce after-effects, it might not have been sufficient to eliminate the strong after-effects associated with the boundaries of the black bars.

Pilot Experiment 2

To avoid visual after-effects, the suitability of orientations as a target feature in change detection was tested using *Gaussian-windowed sinusoidal gratings*, commonly called *Gabor patches*. These are generated by "multiplying a sine-wave luminance grating with a circular Gaussian function" (p. 240, Wright et al., 2000). Gabor patches have been theorised to have the best quantum efficiency for detection (Watson, Barlow & Robson, 1983). Spatial limits are applied by imposing a Gaussian intensity envelope on the sinusoids (Westheimer, 1988).

Gabor patches have been regarded as ideal probes for the visual system (Westheimer, 1998). Compared to graphs with sharp edges, Gabor patches are distributed patterns devoid of apparent contours, which makes them more efficient in mitigating after-effects in tasks requiring precise spatial localisation (Westheimer, 1998). In particular, Gabor patches have been theorised to minimise uncertainty in stimulus localisation and spatial frequency simultaneously (Brillouin, 1962).

Another advantageous properties of Gabor patches is that they are unlikely to be coded verbally, which minimises the possibility of using phonological storage rather than visual storage. Two commonly used features of Gabor patches were tested: orientation (e.g., Carrasco, Penpeci-Talgar, & Eckstein, 2000; Keogh & Pearson, 2011; Laurent, Hall, Andewrson & Yantis, 2015; Westheimer, 1997) and spatial frequency (e.g., Carrasco, Talgar & Cameron, 2001; Laurent et al., 2015).

This experiment was designed to examine the suitability of Gabor patches as stimuli in visual change-detection tasks. Gabor patches with varied orientations were used as stimuli and VWM capacity was estimated at a range of set sizes and presentation durations.

Method

Material

Twenty-eight Gabor patches with orientation ranging from 0° to 135° were generated as visual stimuli using an online Gabor patch generator (see http://www.cogsci.nl/software/online-gabor-patch-generator). All Gabor patches had a standard deviation of the isotropic Gaussian envelope of 0.45˚, phase of 0; and mean luminance of 60cd/m^2, contrast 90/170, size 72 × 72 pixel presented on a grey background with RGB (128, 128, 128).

Twelve study arrays were generated comprising either four or six Gabor patches randomly selected from the pool without replacement. Gabor patches in each study array were either 30°, 45°, or 60° different from each other. Each probe Gabor patch was identical to one of the Gabor patches in the study array to prevent participants from judging on the basis of familiarity.

All memory Gabor patch patterns were presented on a 1920 × 1080 pixel (50.8˚ × 35.4˚) screen. Each study array was repeated for four times, resulting in 96 trials in total. There were four practice trials prior to the start of the test trials.

One Gabor patch was presented at one of the four or six locations used in the study array, selected at random. On half of the trials, the orientation of the probe Gabor patch was the same as that of the study patch at that location. On the other half of the trials, the probe patch was 90° different from the study patch that had been in the same position.

Procedure

Figure 6 shows the trial sequence. The procedure was the same as for Pilot Experiment 1, except that Gabor patches with different orientations rather than bars were used.

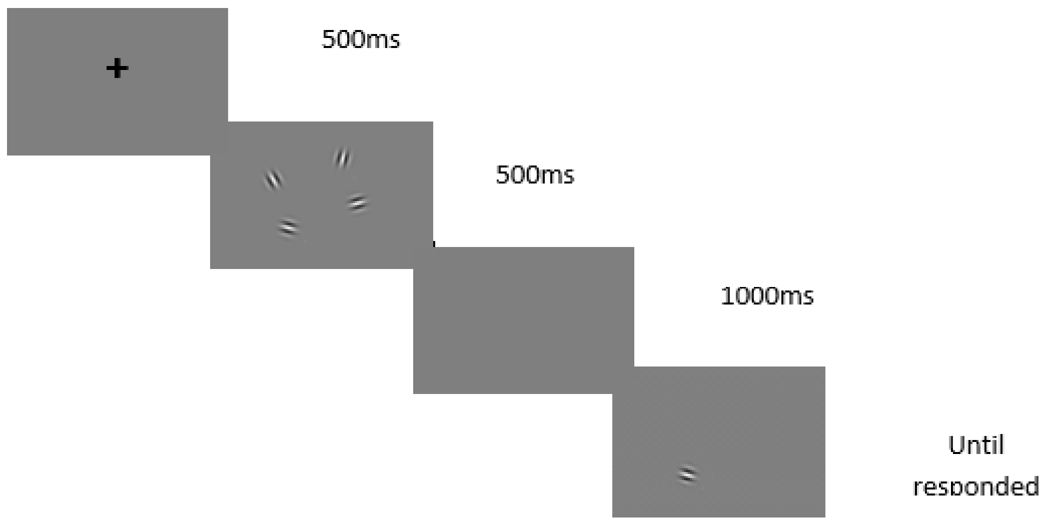


Figure 6. The trial sequence for the change-detection task for set size four (same response)

Results and Discussion

Table 3 shows estimates of VWM capacity and change detection accuracy as a function of set size and presentation duration for Participant 1. Although lower accuracy was observed for set size six than for set size four at durations of 500ms and 1,000ms, when the presentation duration was 1,500ms, the same accuracy was observed for both set sizes, perhaps because 1,500ms is enough time to encode all items in the study arrays. This interpretation is further supported by the estimate of VWM capacity (4.25) at 1,500ms presentation duration, for set size six, which was substantially greater than estimates for this set size under other presentation durations.

Compared to 500ms duration, lower accuracy of change detection was observed at 1,000ms duration in set size six. It is very likely that noise contributed to the inconsistent results, as each participant performed each condition only one time.

Table 3 VWM capacity estimated for different presentation durations and set sizes (percentage of correct responses) for participant 1

Set size	Presentation Duration		
	500ms	1,000ms	1,500ms
4	2.67 (83%)	3.16 (90%)	2.83 (85%)
6	3.75 (81%)	3.59 (79%)	4.25 (85%)

Table 4 shows estimates of VWM capacity and change detection accuracy as a function of set size and presentation duration for Participant 2. Similar to Participant 1, although greater accuracy was observed for set size four than for set size six at 500ms and 1,000ms durations, the accuracy difference disappeared at 1,500ms duration. Moreover, greater estimate of VWM capacity (3. 67) was associated with set size six at the 1,500ms duration than at shorter durations. These results replicate the pattern observed for Participant 1, suggesting that change detection was affected by presentation duration rather than by number of items that could be maintained in visual working memory.

Table 4 VWM capacity as the result of different luminance of squares and set sizes (percentage of correct responses) for Participant 2

Set size	Presentation Duration		
	500ms	1,000ms	1,500ms
4	1.83 (77.1%)	2.25 (85%)	2.67 (87.5%)
6	2.75 (71%)	2.83 (73.3%)	3.67 (85.4%)

In summary, similar to Pilot Experiment 1, present results based on orientations as the target feature suggest that change detection is affected by the encoding process. This might be attributable to an encoding strategy: representations of orientations might be chunked, or participants could have verbally encoded the orientations (such as “up left, down right”) when there is sufficient time to do so.

To avoid the potential for this kind of strategy, spatial frequency was used as the target feature in the following experiment.

Pilot Experiment 3

Pilot Experiment 3 was designed to test the suitability of spatial frequency as a target feature for visual change detection. If the manipulation of spatial frequency is suitable for VWM capacity measurement, then presentation durations and set sizes should not have an effect on estimates of VWM capacity.

Materials

Twenty-eight Gabor patches were generated for this experiment using the same online Gabor generator as that in Pilot Experiment 2; the only difference was that all Gabor patches were oriented vertically but differed in spatial frequency, ranging from 0.03c/pixel to 0.2c/pixel, with 0.01c/pixel increments. Twelve study arrays were generated with Gabor patches selected randomly from the pool without replacement, each containing four or six Gabor patches with different spatial frequencies, placed at equal distances on an imaginary circle (radius = 4.28˚) centred at fixation. Figure 7 shows a sample pattern for set size four.

Figure 7. Sample pattern of set size four

One probe patch was presented at one of the four or six locations used in the study array, selected in random. On half of the trials, the spatial frequency of the probe Gabor patch was the same as that of the study patch at the same location. On the other half of the trials, the probe patch was 0.05c/pixel higher or lower in spatial frequency than the memory patch that had been in the same position.

Procedure

The procedure was the same as that of Pilot Experiment 2.

Results and Discussion

Table 5 shows estimates of VWM capacity and percentages of correct responses as a function of presentation duration and set size for Participant 1. Greater accuracy was observed for set size six than for set size four at 1,000ms duration. Moreover, the accuracy of change detection decreased between 1,000ms and 1,500ms durations for both set sizes. These results suggest that change detection was not affected by either presentation durations or the number of items maintained in working memory.

However, this was not the case. For both set sizes, accuracy maximised at 1,000ms duration. Even with sufficient time to encode all Gabor patches in study arrays, change detection accuracy was similar between 1,000ms and 1,500ms duration in set size four, but accuracy decreased in set size six.

Table 5 VWM capacity estimated for different presentation durations and set sizes (percentage of correct responses) for Participant 1

Set size	Presentation Duration		
	500ms	1,000ms	1,500ms
4	0.75 (64.6%)	2 (75%)	2.16 (71%)
6	1.16 (56%)	2.75 (83.3%)	1.75 (64.5%)

Table 6 shows estimates of VWM capacity and percentages of correct responses as a function of presentation duration and set size for Participant 2. Higher accuracy was observed for set size six than for set size four except at 1,500ms duration, which is against the prediction. Moreover, although similar estimates of VWM capacity across presentation durations were observed for both set sizes, accuracy increased between 500ms and 1,000ms for both set sizes, suggesting that presentation duration affected change detection performance. However, similar accuracy was observed for both set sizes between 1,000ms and 1,500ms presentation duration.

Table 6 VWM capacity estimated for different presentation durations and set sizes (percentage of correct responses) for Participant 2

Set size	duration		
	500ms	1,000ms	1,500ms
4	0.5 (54%)	0.67 (62.5%)	1 (66.7%)
6	1 (58.3%)	2 (66.7%)	2 (62.5%)

The results of the present experiment are inconsistent with the prediction that stable percentages of correct responses and estimates of VWM capacity would be observed across presentation duration for each set size. A possible explanation might be that performance was affected by errors during encoding, or during the comparison of the probe patch and the study array. A premise of the notion that the number of items maintained in working memory is the only factor affecting change detection is that stimuli in study arrays are sufficiently discriminable from each other (Rouder et al., 2011). The increment in spatial frequency of the Gabor patches used in the present experiment was the same for all frequencies, which might have led to poor discriminability for Gabor patches with high spatial frequencies; sensitivity to changes in spatial frequency decreases with higher frequencies (Brady & Field, 1994). This is supported by the rates of correct change detection of just above 50%, suggesting that participants have trouble differentiating Gabor patches with similar high spatial frequencies; change detection trials involving higher-frequency patches were performed at chance levels, leading to underestimates of VWM capacity.

This possibility is further supported by the substantially increased estimates of VWM capacities and accuracy observed between 500ms and 1,000ms duration for both participants and both set sizes. Participants might find it difficult to differentiate Gabor patches with high spatial frequencies in the same study array, which might therefore take longer to encode correctly. It is likely that 1,000ms is sufficient for differentiating and encoding Gabor patches with high similarity in spatial frequency, so that performance did not further improve with durations longer than this (see Table 6).

It should also be noted that each participant only performed the experiment once, so it is likely that noises might have contributed to the apparently confusing results observed for Participant 1 at set size six.

To eliminate the impact of insufficiently discriminable visual stimuli on change detection, a similar experiment was conducted with modified increments for Gabor patches with a different range of spatial frequencies.

Pilot Experiment 4

This experiment was designed to further examine the suitability of Gabor patches with different spatial frequencies as visual stimuli in change detection using adjusted increments for different ranges of spatial frequencies.

Material

The same materials as those used in Pilot Experiment 3 were used in this experiment, expect that the Gabor stimuli had different increments of spatial frequency. For Gabor patches with spatial frequencies from 0.03-0.09c/pixel, the increment was 0.02c/pixel. For Gabor patches with spatial frequencies from 0.10-0.15c/pixel, the increment was 0.05c/pixel, while for Gabor patches with spatial frequencies from 0.16-0.20c/pixel, the increment was 0.1c/pixel.

The increment between target and probe Gabor patches in the study set was 0.05c/pixel for Gabor patches with spatial frequencies ranging from 0.03-0.15c/pixel, and 0.1c/pixel for Gabor patches with spatial frequencies ranging from 0.16-0.2c/pixel.

In total, 12 study arrays with different spatial frequencies were generated, half with probes 0.05c/pixel or 0.1c/pixel lower than the corresponding study patch, the other half with probes 0.05 or 0.1c/pixel higher than the corresponding study patch. The locations of probe patches were counterbalanced among study sets, and the location of the patches with the highest and lowest spatial frequencies were similarly counterbalanced. Figure 8 shows a sample pattern of study arrays for set size four.

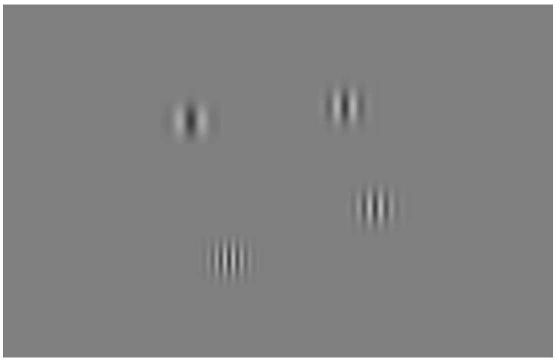


Figure 8. Sample pattern for set size four

Procedure

The procedure was the same as that of Pilot Experiment 3.

Results and Discussion

Tables 7 shows estimates of VWM capacity and percentages of correct responses as a function of set size and presentation duration for Participants 1 and 2. For both participants, the predicted higher accuracy was observed for set size four than for set size six for all presentation durations. Moreover, similar estimates of VWM capacity were observed for all set sizes and presentation durations. Finally, for each set size, similar accuracy of change detection was observed across presentation durations.

These results are consistent with presentation duration being not a limiting factor on change detection performance, and performance yielding a valid estimate of the number of Gabor patches maintained in working memory: For each participant, although less time was available to encode each Gabor patch at each presentation duration for set size six than for set size four, the number of Gabor patches that could be maintained in working memory is the same regardless of set size. Thus, lower accuracy for larger set sizes nonetheless generated similar estimates of VWM capacity to those generated on the basis of higher accuracy for smaller set sizes.

Relatively larger estimates of VWM capacity and higher accuracy of change detection were observed at both set sizes for Participant 1 than for Participant 2, suggesting that differences in the numbers of items could be maintained in working memory is the only reason for performance differences between the participants.

The substantially higher estimates of VWM capacity observed in Pilot Experiments 1 and 2 for Participant 2 than in the present experiment might be due to different encoding processes for orientations and spatial frequencies. Moreover, verbal encoding strategies might have been employed by participants to encode orientations; both estimates of VWM capacity and accuracy improved with presentation duration in Pilot Experiments 1 and 2. The results of the present experiment also provide further support for the possibility that Pilot Experiment 3 underestimated VWM capacities because the Gabor patches were insufficiently discriminable.

Table 7 VWM capacity estimated for different presentation durations and set sizes (percentage of correct responses) for Participants 1 and 2

Participants	Set size	Presentation Duration		
		500ms	1,000ms	1,500ms
Participant 1	4	2.17 (83%)	2.67 (83.3%)	2.25 (83%)
	6	2.92 (77.1%)	2.75 (72.9%)	2.38 (76%)
Participant 2	4	2.17 (77%)	1.67 (71%)	2.16 (77.1%)
	6	1.75 (64.5%)	2 (66.7%)	1.75 (64.6 %)

Phonological Stimulus Selection

Articulatory suppression tasks have been used widely to provide phonological interference in experiments investigating the role of phonological processing in mental arithmetic (see DeStefano & LeFevre, 2004 for a review). However, this approach was not employed in the present work because it is difficult to find a comparable visual interference task. As mentioned in General Introduction, comparability in interference tasks might be crucial in the comparison of mechanisms of the relative involvement between visual and phonological working memory resources in carry operations in mental arithmetic. Moreover, there is no way to guarantee that participants carry out a sub-vocal articulatory suppression task rigorously when focusing on a primary task. To address this concern, in some published experiments participants have been required to articulate aloud rather than sub-vocally (e.g., LeFevre, 2003). However, in the dual-task paradigm used in the present work, under which a mental arithmetic task is performed simultaneously with a change-detection task, the precision of response time measurement would be impaired by this approach, especially considering the requirement to report the solution of mental arithmetic problems verbally. Finally, the rationale of articulatory suppression is inconsistent with the rationale of the dual-task interference paradigm used in this study; articulatory suppression is intended to prevent information stored in the phonological loop from being reactivated and maintained through a rehearsal process (Baddeley & Hitch, 1974; Baddeley, 1992). However, the rationale of the dual-task interference paradigm used in the present research is capacity-sharing: dual-task costs are caused by competing for resources within capacity-limited working memory components, when the resources of that component are required by both primary and secondary tasks (Wickens, 1991). If rehearsal is prevented, the consequences of this competition cannot be observed.

An experiment was therefore designed to test the suitability of *pronounceable consonant-vowel-consonant non-words* as stimuli in a phonological change detection task (see Imbo & LeFevre, 2010; Trbovich & LeFevre, 2003 for previous usage of this approach).

Pilot Experiment 5

Method

Material

Twenty-six phonological non-words (e.g., gul, mek) were read and recorded by a female native speaker of English who has a clear and gentle voice. Phonological non-words that might be spelled as names or that corresponded to common abbreviations were avoided (Trbovich & LeFevre, 2003).

In total, 12 study nonwords lists, each comprising seven different non-words were constructed. All study nonwords were presented to participants through headphones. A single probe non-word was presented after a 1,000ms interval. On half of the trials, the probe non-word was identical to one of the non-words in the study string. Its location in the study string was equally allocated across study nonwords; for example, it was the third non-word in the study string as often as it was the sixth non-word in the study string. On the other half of the trials, the probe was different from all of the non-words in the study string. Each study string was repeated four times, resulting in a total of 48 trials.

Procedure

Each participant was tested individually in a quiet room. A pair of headphones was provided, and they adjusted the volume to suit themselves. A brief explanation of the experiment was provided and participants were instructed to make a response to each trial without hesitating too long. Two practice trials were administered to make sure participants were familiar with the procedure.

Each trial began with a 500ms tone, which was the cue to be ready for the following non-words. After a 1,000ms interval, participants heard the nonwords, with 500ms inter-stimulus interval between non-words. Following another 1,000ms inter-stimulus interval the probe non-word was presented. Participants were asked to indicate whether or not the probe non-word was the same as one of the study non-words by pressing one of the two response keys. After a response was given, the words "Please step on the foot pedal to continue" appeared on the screen. This allowed participants to regulate their pace from trial-to-trial throughout the experiment.

Results and Discussion

Table 9 shows the estimated PWM capacities and percentages of correct responses for each participant.

A high percentage of correct responses was observed for both participants. Estimated PWM capacities differed between the two participants, although their percentages of correct responses were similar to each other. This is because estimates of PWM capacity relies on both correct-positive and false-positive rates, which might differ across participants even if the overall accuracy is similar. Estimates of PWM capacity for both participants were between ceiling and floor performance, suggesting the suitability of nonwords as phonological stimuli in change detection.

It might be argued that participants could have visualised the non-words rather than using sub-vocal rehearsal to maintain them in PWM. This is unlikely due to the theorised limitations of the capacity of working memory components. As mentioned above, in general, VWM capacity has been theorised to be about three to four items (Alvarez & Cavanagh, 2004; Luck & Vogel, 1997; Olsson & Poom, 2005; Vogel, Woodman, & Luck, 2001), but the number of non-words in each string used in this experiment was seven, which substantially exceeds that capacity limit.

Table 8 Mean PWM capacity estimated using phonological non-words (percentage of correct responses)

	Estimated PWM Capacity (percentage of correct responses)
Participant1	4.29 (80.4%)
Participant2	6.13 (84%)

In summary, five pilot experiments were conducted using a single-probe change-detection paradigm. Findings indicated that Gabor patches with spatial frequencies as the target feature are suitable as visual stimuli for visual change detection, while phonological non-words are suitable as phonological stimuli for phonological change detection.

When Gabor patches with different spatial frequencies were used as visual stimuli in the visual change-detection task (see Pilot Experiment 4), steady performance was observed for both participants across presentation durations and set sizes. Estimates of PWM capacity consistent with previous research were observed for both participants.

Chapter 3

Experiment 1

Few studies have investigated the effect of arithmetic operations on the relative involvement of working memory resources in complex mental arithmetic. As reviewed in the General Introduction, complex addition has been theorised to be solved with the aid of an analog magnitude code, which activates visual processing (Dehaene, 1992; Dehaene & Cohen, 1997). Therefore, it is hypothesised that visual working memory resources are involved in complex addition for maintaining carry operations or intermediate solutions if strategies other than adding units and tens are used (e.g., decomposing the problem into easier problems).

Although a few studies have reported experiments in which numbers of digital carriers have been manipulated (e.g., Fürst & Hitch, 2000; Trbovich & LeFevre, 2010), no studies have simultaneously manipulated digital carriers and the level of working memory load, as is the case in the present experiment. In line with previous research, it was expected that problems involving carry operations would be solved less rapidly and accurately than no-carry problems. A role for visual working memory resources in carry operations would be supported by interactions between single/dual-task conditions and the number of carry operations required in the arithmetic task, with the influence of visual interference growing larger with increasing numbers of carry operations. As reviewed in General Introduction, this is because, compared to no-carry problems, more working memory resources are likely to be required for temporarily retaining digital carriers or interim-solution information (Imbo et al., 2007).

For both arithmetic and secondary interference tasks, it was further expected that if visual resources are involved, such effects will grow larger with both the level of VWM load and the number of carry operations. This would be evidenced by a three-way interaction between task conditions, number of carriers and level of visual interference. Specifically, it is hypothesised that a greater effect of increasing VWM load on addition performance should be observed for carry problems than for no-carry problems. This is because, compared to no-carry problems, solving carry problems requires more working memory resources to maintain digital carriers or intermediate solutions. If VWM resources are used to maintain representations of digital carriers or intermediate solutions, the competition for limited VWM resources under the same level of visual load would be greater during solving carry problems than during solving no-carry problems. Thus, greater performance decrement should be observed with increasing level of VWM load for carry problems than for no-carry problems.

A dual-task paradigm involving an addition task and a visual change-detection task was used in the present experiment, with task conditions and carry conditions manipulated. In the dual-task condition, participants were required to maintain calculation-irrelevant visual stimuli throughout the calculation process. In the single-task conditions, the addition and visual change-detection tasks were performed alone. All arithmetic and change detection tasks were presented in exactly the same order in both the dual- and single-task conditions to guarantee that only matched trials were selected for dual-task cost analysis.

Method

Participants

Sixty participants, all native speakers of English, received gift vouchers as a token of appreciation for participating in the experiment. A majority of the participants were undergraduate students of Victoria University of Wellington. Participants came from various educational backgrounds, including social/humanity (e.g., philosophy, education, art) and natural science (e.g., chemistry, physics, biology). Since arithmetic performance could be affected negatively by high level of mathematics anxiety, all participants were required to complete the short Mathematics Anxiety Rating Scale (sMARS; Alexander & Martray, 1989). This survey is a 25-item version of the 98-item MARS, which is the most widely used survey for assessing mathematics anxiety (Richardson & Suinn, 1972). The sMARS assesses anxiety about mathematics using a five point Likert scale (1 not at all anxious to 5 very anxious). Participants make responses about how anxious they would be in various settings and experiences related to mathematics (e.g., "How anxious you will be when looking through pages on a math test?"). A high reliability of the sMARS was observed in the present study (Cronbach's

$\alpha = 0.917$). Four participants showed high levels of math anxiety and did not proceeded to the experiments.

The data of eight participants were removed from the analyses either because of too many failures of the sound-activated relay, or because they withdrew before completing all of the experimental sessions. The remaining 48 participant showed acceptable math anxiety levels (M = 2.75, SD = 1.23). The mean age of the remaining respondents was 23 years. There were 37 females (77%) and 11 males (23%).

Materials

Addition task

One-hundred and eight double-digit problems were used in the experiment, with nine additional problems used for practice trials. Double-digit operands and their sums ranged between 12 and 95, and 35 and 156, respectively. Problems involving digits with zeros in the unit positions were avoided because they could straightforwardly be solved using fact-retrieval rather than calculation (Ashcraft, 1992, 1995). The 108 problems were divided into three sets of 36 problems. In each set, there were 4 to 8 no-carry problems, 20 to 23 single-carry problems and 4 or 5 double-carry problems. In the no-carry problems, the sums of both the unit and decade columns were less than ten (e.g., 16 + 23). Single-carry problems required participants either to carry one from the units and add it to the decades (e.g., 14 + 58), or to carry from the sum of the decades to the hundreds (e.g., 85 + 42). Double-carry problems required both of these carry operations. Appendix A shows all of the problems used in the study.

Visual Change-Detection Task

The stimuli for the change-detection task used in this experiment were the same as those for Pilot Experiment 4, except that three sets of Gabor patches were constructed, with nine study arrays in each set. The first set comprised two Gabor patches in each study array, the second comprised three Gabor patches in each study array, and the third, four Gabor patches in each study array. Each study array was repeated four times in the experiment, twice with a probe Gabor patch identical to one of the Gabor patches in the study array, and twice with a probe Gabor patch different from all Gabor patches in the study array, giving a total of one hundred and eight trials in each of the dual-task and single-task conditions.

Versions of Arithmetic Problem/Memory Load Combinations

Six versions of problem/study-array combinations were generated to control for variations in difficulty across arithmetic problems. Each problem was presented in both the dual- and single-task conditions in every version, but the change-detection study array presented with each problem varied across versions. For example, if an addition problem, 12 + 45, was presented with two-patch arrays in Version 1, it would be presented with three-patch arrays in Version 2.

Participants were randomly allocated to versions of the experiment, with eight participants completing each version. Each participant completed every arithmetic problem and every study array across versions, the only difference was that problem/study-array *combinations* varied across versions.

For each version of the experiment, the same arithmetic problems or study arrays were presented in two single-task conditions, in the same order as those of the dual-task condition.

Procedure

Each participant was tested individually in a quiet and dimly-lit room. Participants sat with their eyes 40 cm from the computer screen. They were provided with an explanation of the experiment and instructed to make a response to each trial as quickly and as accurately as possible. Each participant performed the task in three conditions: a dual-task condition in which the addition task and the visual change-detection tasks were performed simultaneously, and two single-task conditions in which the addition task and the visual change-detection task were performed alone. Four practice (dual-task) trials with addition problems, randomly selected without replacement from the stimulus list, were administered, to make sure participants were familiar with the procedure. Of the four practice problems, two involved a carry operation from units to tens, and the other two were no-carry problems. In the dual-task condition, participants were instructed to perform both tasks as accurately and quickly as possible. No feedback was given during the experiment.

Figure 9 shows the display sequence for the dual-task condition. Each trial began with a black fixation point in the centre of the screen presented against a grey background. This was a cue to be ready for the study array of the change-detection task. After

500ms, a study array comprising four (or two or three) Gabor patches with different spatial frequencies was presented for 1,000ms, followed by a mental addition problem presented in the centre of the screen in visual Arabic font for 4,000ms. Participants were instructed to speak their responses into the microphone loudly and clearly. Response times were recorded automatically through the triggering of a voice key. The accuracy of response times was to 1 millisecond (ms). After responding to the problem, a single probe patch appeared randomly at one of the four locations where the study Gabor patches had been presented. Participants were asked to indicate whether or not the probe patch was exactly the same as the Gabor patch that had been in that position previously, by pressing one of the two response keys. After responding, the words "Please step on the foot pedal to continue" appeared on the screen. The use of the foot pedal to initiate each trial allowed participants to regulate their pace from trial-to-trial throughout the experiment.

In the single-task condition in which the visual change-detection task was performed alone, arithmetic problems were replaced by a 4,000ms inter-stimulus interval.

In both the single- and the dual-task conditions, participants took a short break after 36 trials before proceeding with the next 36 trials. Another short break occurs before participants performed the last 36 trials.

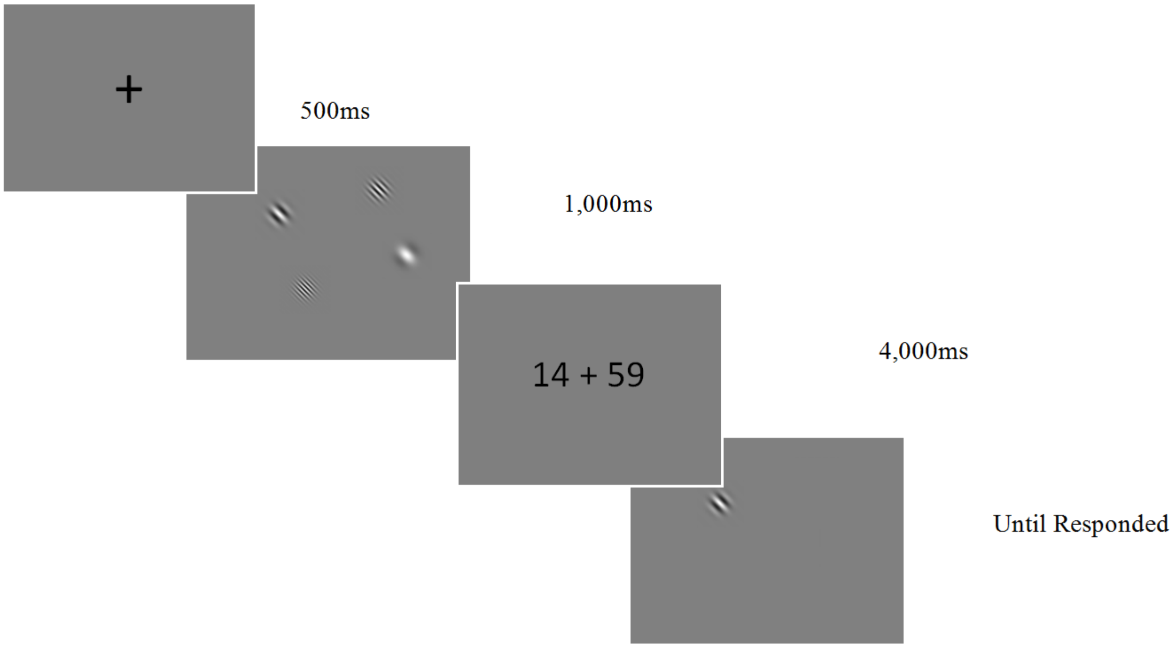


Figure 9. The sequence of a dual-task trial ('same' condition)

Design

Response time and accuracy data from each of the addition task and the visual change-detection task were analysed in a 2 (task conditions: dual-task, single-task) × 3 (number of digital carriers: 0, 1, 2) × 3 (number of Gabor patches: 2, 3, 4) × 6 (version: 1, 2, 3, 4, 5, 6) mixed-measures analysis of variance (ANOVAs), with repeated measures on the first three factors and the last factor as a between-group factor.

Results involving version are not reported because they were simply used to remove variance associated with any variations in task difficulty across versions.

Any response time data for arithmetic tasks with a value more than 4000ms were excluded from the analysis for all experiments. This is because the interval between the arithmetic task and the change-detection task was 4,000ms, and participants needed to make responses within this time range, otherwise it is likely that they were still calculating the arithmetic problems rather than focusing on the change-detection task.

Counterbalance of Trials Between and Within Sessions

To avoid fatigue and practice effects, each participant attended several sessions, with each lasting for about one hour; only one session was performed per day. The number of sessions and interval between days in which sessions were performed varied across participants due to different availability. For example, some participants might have been only available on Mondays, so their intervals between sessions were a week. However, no participants performed sessions on consecutive days. The minimum interval between sessions was one day (e.g., participants performed one session on Monday, then they will perform the following session on Wednesday).

In each session, participants performed one or two arithmetic or change-detection blocks alone, as well as dual-task blocks involving both the arithmetic and the change-detection tasks. If two dual-task conditions were performed, then the arithmetic task and the change-detection task performed in each dual-task condition were never identical in the same session. For example, if a participant performed the addition task under visual load first, and then he or she would perform the multiplication task under phonological load in the same session. A short rest was allowed during the experiment and was compulsory between conditions. Following rest periods, the experiment continued when the participant was ready. The order of the tasks was the same for all participants: Single-tasks conditions were always performed before dual-task conditions to guarantee familiarity with the separate task before approaching them in combination. In single-task sessions, addition tasks were always performed prior to multiplication tasks. This is because multiplication has been theorised to be more difficult than addition, so the performance of the addition tasks might be affected if more complicated multiplication tasks had been performed earlier. Visual change-detection tasks were always performed before the phonological change-detection tasks as they are faster to complete because nonwords can only be presented one at a time. The single-task sessions were then followed by sessions including four dual-task conditions. All participants first solved addition problems under visual load, then solved multiplication problems under phonological load. In the following session, all participants solved addition problems under phonological load, followed by multiplication problems solved under visual load.

Results

Addition task

Accuracy Rates- In total, one participant was excluded from the analysis due to having no correctly responded double-carry problems. For each condition (single-task, dual-task condition), 67.6% (3,502 out of 5,184) of trials were included in the analysis.

Table 9 shows the mean accuracy rates and response times for the addition task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Table 9. Mean accuracy rates and standard errors for the addition task as functions of the number of Gabor patches and digital carriers in each of the dual- and single-task conditions.

Question types	Visual loads	Single-task condition *M (SD)*	Dual-task condition *M (SD)*	N
No-carry problems	2 Gabor patches	0.94 (0.01)	0.93 (0.01)	47
	3 Gabor patches	0.97 (0.01)	0.92 (0.02)	47
	4 Gabor patches	0.96 (0.01)	0.89 (0.02)	47
Single-carry problems	2 Gabor patches	0.80 (0.02)	0.76 (0.03)	47
	3 Gabor patches	0.79 (0.02)	0.75 (0.02)	47
	4 Gabor patches	0.81 (0.02)	0.74 (0.03)	47
Double-carry problems	2 Gabor patches	0.68 (0.04)	0.52 (0.06)	47
	3 Gabor patches	0.73 (0.04)	0.58 (0.06)	47
	4 Gabor patches	0.73 (0.04)	0.53 (0.05)	47

As predicted, the presence of carry operations in complex addition was associated with greater performance decrement under visual load [F (2, 82) = 4.017, MS_{error} = 0.07, p = 0.022, η^2 = 0.1], with a greater dual-task cost observed for double-carry problems (0.17) than for no-carry or single-carry problems (both 0.05). Such significant load-by-carry interaction supported a role of visual resources in carry operations or intermediate solutions in mental addition, and is also consistent with the assumption that visual processing is required during solving complex addition. However, this effect of number of carriers on addition performance was not further enlarged by increasing visual load as predicted, with the load-by-carry interaction mentioned above did not interact with visual load level, reflected by an insignificant three-way interaction between task conditions, number of Gabor patches and number of carriers [F <1]. This is inconsistent with the hypothesis that increasing load would affect the dual-task performance decrement differently for no-carry, single-carry and double-carry problems.

As is typically found, participants made more errors on problems with more carry operations, evinced by a significant main effect of number of carriers [F (2, 82) = 99.524, $MS_{error} = 0.066$, $p < 0.01$, $\eta^2 = 0.7$]. Specifically, no-carry problems (0.94) were solved more accurately than single-carry problems (0.78), which were solved more accurately than double-carry problems (0.63). Also, addition problems were solved less accurately when under a visual load (0.74) than when presented alone (0.82), evinced by a significant main effect of task conditions [F (1, 41) = 15.139, $MS_{error} = 0.111$, $p < 0.01$, $\eta^2 = 0.27$]. These result patterns replicate the findings of Imbo et al., (2007).

Contrary to the prediction, error rates did not increase with level of load, reflected by a non-significant main effect of number-of-Gabor patches [$F < 1$]: Accuracy did not differ between addition problems solved with two-patch arrays (0.77), three-patch arrays (0.79) or four-patch arrays (0.77).

Solution Latencies- In total, 23 participants were excluded from the analysis due to neither, or only one task being correctly performed in trials involving double-carry problems. Dual-task trials with response time longer than 4,000ms were excluded from the analysis. For the single-task condition, 45.4% (2,355/5,184) of trials were included in the analysis. For the dual-task condition, 34.5% (1,786/5,184) of trials were included in the analysis.

Table 10 shows mean accuracy rates and response times for the addition task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Table 10. Mean solution times and standard errors for the addition task as functions of the number of Gabor patches and digital carriers in each of the dual- and single-task conditions.

Question types	Visual loads	Single-task condition *M (SD)*	Dual-task condition *M(SD)*	N
No carry problems	2 Gabor patches	1974 (63)	2094 (69)	25
	3 Gabor patches	1948 (66)	2126 (69)	25
	4 Gabor patches	1992 (76)	2078 (79)	25
Single-carry problems	2 Gabor patches	2548 (99)	2430 (80)	25
	3 Gabor patches	2452 (75)	2476 (84)	25
	4 Gabor patches	2558 (72)	2580 (83)	25
Double-carry problems	2 Gabor patches	2941 (124)	2799 (140)	25
	3 Gabor patches	2696 (123)	2663 (141)	25
	4 Gabor patches	2572 (134)	2743 (158)	25

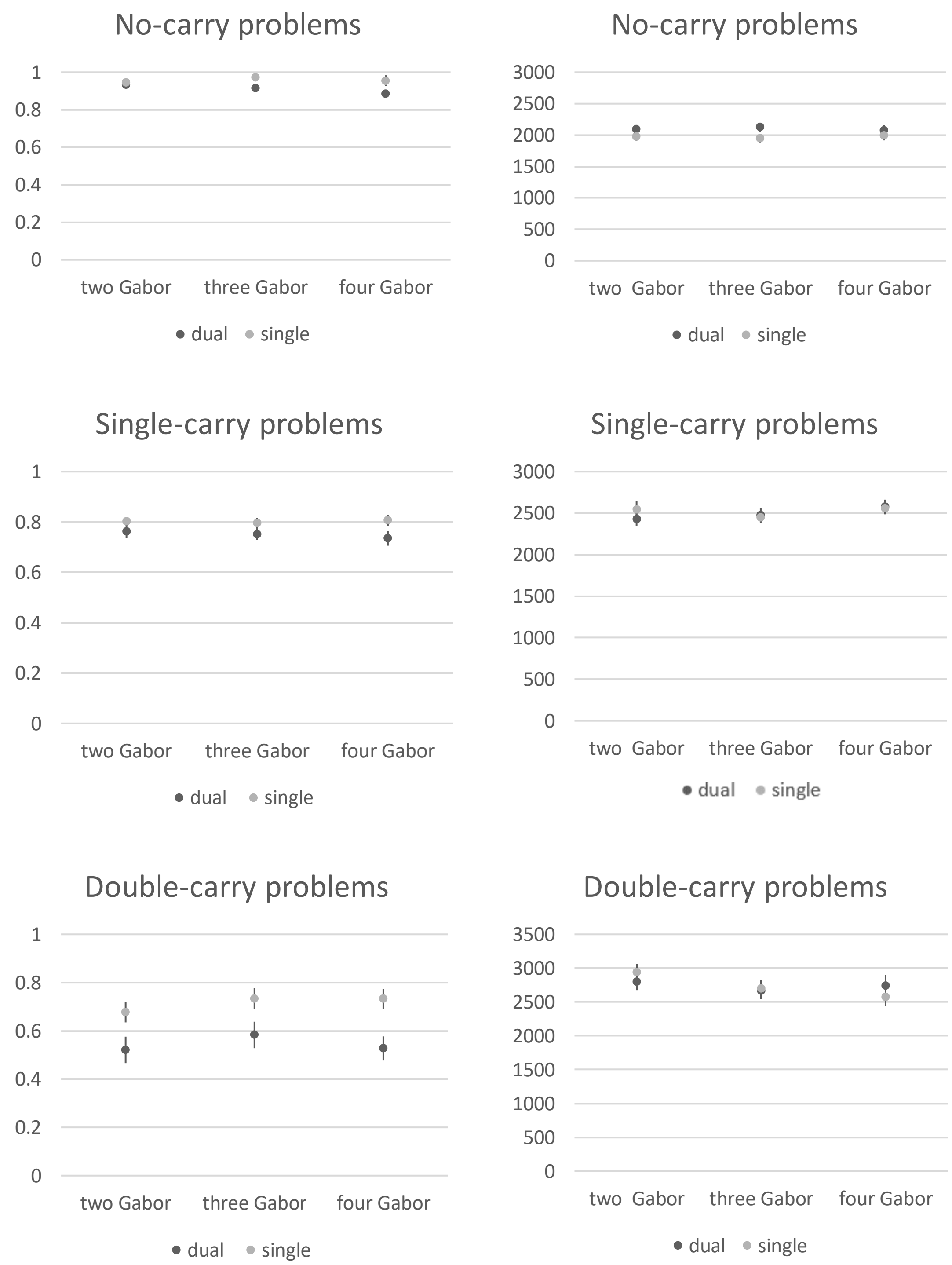
Unlike the accuracy data, the response time data did not show evidence for a role of visual representations in carry operations of complex addition: The task-by-carry interaction showed no significant difference in performance decrement observed between carry problems and no-carry problems [F (2, 38) = 1.479, MS_{error} = 115,964, p = 0.24, $\eta^2 = 0.07$]. Similar to the accuracy analysis, there was no predicted significant three-way interaction between task conditions, the number of carriers and the number of Gabor patches [F <1], which contradicted the prediction that increasing visual load will further enlarge the effect of number of carriers on addition performance.

As is typically found, participants took longer to answer problems with more carry operations [F (2, 38) = 125.443, MS_{error} = 103,285, p < 0.01, $\eta^2 = 0.9$], with no-carry problems (2,035ms) solved faster than single-carry problems (2,507ms) and double-carry problems (2,735ms).

However, participants were not slower when problems were solved under a visual load (2,443ms) than when they were solved alone (2,409ms) [F <1]. Moreover, consistent with the results of accuracy analysis, mean latencies on addition problems did not increase with visual load as predicted [F <1]. Conversely, similar mean latencies were observed for addition problems presented with two-patch arrays (2,465ms), with three-patch arrays (2,393ms) and with four-patch arrays (2,421ms). This is inconsistent with the prediction that addition performance decrement will increase with load level.

Figure 10 shows mean accuracy rates (proportion of trials answered correctly, left column) and response times (milliseconds, right column) for the addition task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Figure 10. Accuracy rates (left column) and response times (right column) for the addition task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions. Error bars represent standard errors.



Visual Change-detection Task

Accuracy Rates- In total, 11 participants were excluded from the analysis due to no correctly responded double-carry problems. In each condition (dual- and single-task condition), 61.7% (3,198 out of 5,184) trials were included in the analysis.

Table 11 shows the mean accuracy rates for the visual change-detection task as functions of the number of Gabor patches and digital carriers, in each of the dual-task and single-task conditions.

Table 11. The mean accuracy rates for the visual change-detection task as functions of the number of Gabor patches and digital carriers, in each of the dual-task and single-task conditions.

Question types	Visual loads	Single-task condition *M(SD)*	Dual-task condition *M (SD)*	N
No carry problems	2 Gabor patches	0.86 (0.02)	0.73 (0.03)	37
	3 Gabor patches	0.80 (0.02)	0.76 (0.03)	37
	4 Gabor patches	0.77 (0.03)	0.65 (0.03)	37
Single-carry problems	2 Gabor patches	0.87 (0.02)	0.66 (0.03)	37
	3 Gabor patches	0.84 (0.02)	0.68 (0.03)	37
	4 Gabor patches	0.76 (0.03)	0.67 (0.05)	37
Double-carry problems	2 Gabor patches	0.84 (0.02)	0.74 (0.05)	37
	3 Gabor patches	0.77 (0.04)	0.67 (0.05)	37
	4 Gabor patches	0.76 (0.04)	0.60 (0.06)	37

As is typically found, participants were more accurate when change detection was performed alone (0.81) than when performed with a concurrent addition task (0.68), reflected by a significant main effect of task condition [F (1, 31) = 35.565, MS_{error} = 0.039, p < 0.01, d = 0.5]. Moreover, accuracy of change detection decreased with increasing number of Gabor patches involved [F (2, 62) = 7.354, MS_{error} = 0.039, p = 0.001, η^2 = 0.19], with participants more accurate in change-detection involving two-patch arrays (0.78) than with three-patch arrays (0.75) or four-patch arrays (0.70).

However, a similar performance decrement caused by a concurrent addition task was observed for visual change detection regardless the number of Gabor patches involved [F < 1]. Furthermore, similar to the addition task, there was no predicted three-way interaction between task conditions, the number of carriers and the number of Gabor patches observed [F <1]. These results were inconsistent with the hypothesis that an effect of the number of Gabor patches on change detection performance would be further mediated by the calculation demand imposed by the concurrent addition task.

Also, accuracy of change detection did not decrease with increasing addition requirements (defined by the number of carriers involved in the concurrent addition problems) [F (2, 62) = 1.242, MS_{error} = 0.039, p = 0.296, η^2 = 0.04]. In particular, similar accuracy was observed for visual change-detection solved with no-carry problems (0.76), with single-carry problems (0.74) and with double-carry problems (0.73).

No other effects were significant.

Solution Latencies- In total, 22 participants were excluded from the analysis due to no correctly responded double-carry problems or no correct change detection. For the dual-task condition, 40.3% (2088/5184) of trials were included in the analysis. For single-task condition, 36.6% (1895/5184) of trials were included in the analysis.

Table 12 shows the mean solution times for the visual change-detection task as functions of the number of Gabor patches and digital carriers, in each of the dual-task and single-task conditions.

Table 12. The mean solution times for the visual change-detection task as functions of the number of Gabor patches and digital carriers, in each of the dual-task and single-task conditions.

Question types	Visual loads	Single-task condition *M (SD)*	Dual-task condition *M(SD)*	N
No carry problems	2 Gabor patches	1403 (86)	1391 (102)	26
	3 Gabor patches	1418 (89)	1461 (100)	26
	4 Gabor patches	1596 (97)	1415 (113)	26
Single-carry problems	2 Gabor patches	1429 (99)	1587 (119)	26
	3 Gabor patches	1490 (87)	1530 (100)	26
	4 Gabor patches	1620 (89)	1603 (103)	26
Double-carry problems	2 Gabor patches	1461 (94)	1762 (145)	26
	3 Gabor patches	1420 (95)	1856 (147)	26
	4 Gabor patches	1577 (120)	1549 (114)	26

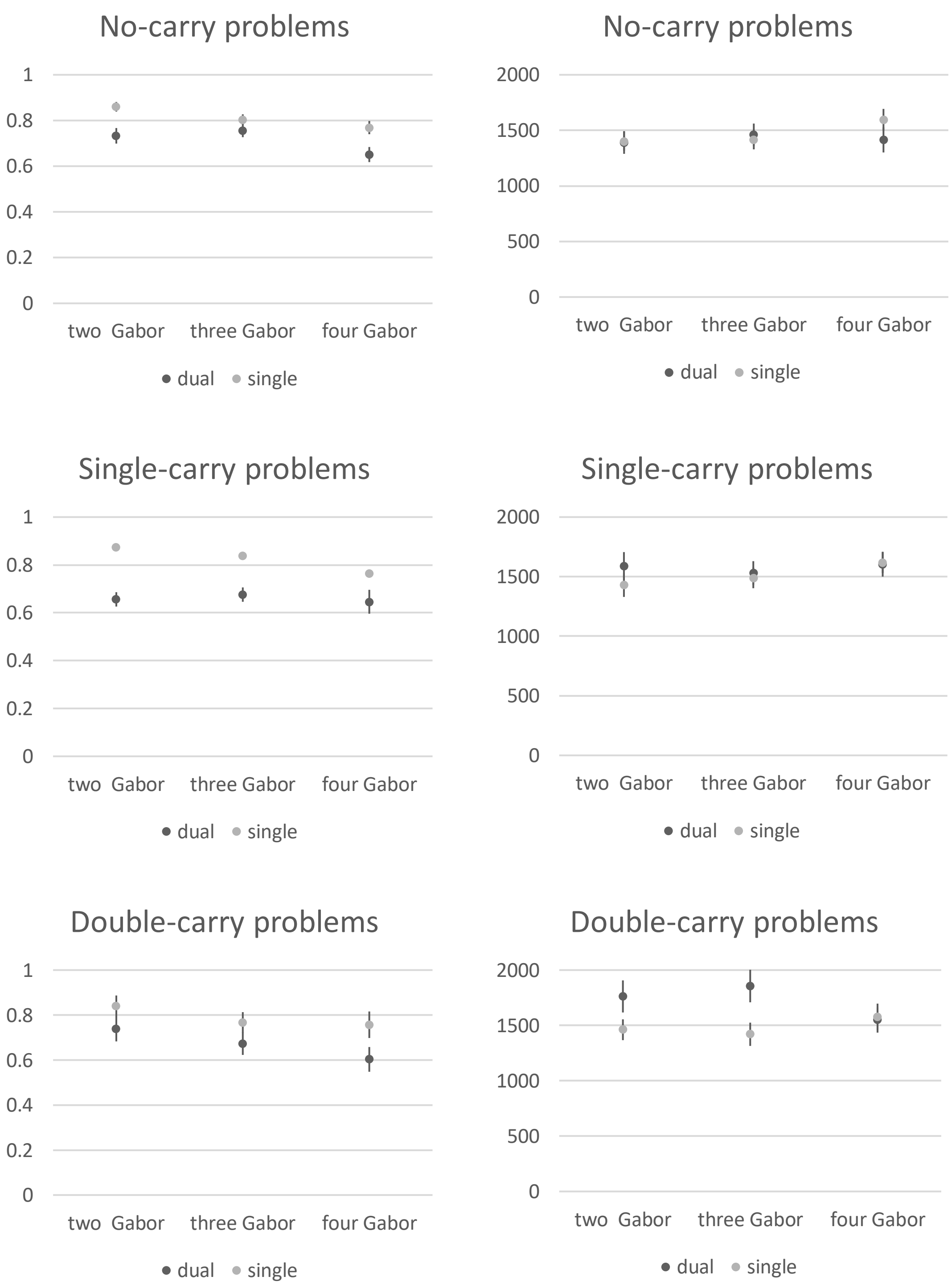
Inconsistent with the accuracy analysis, participants did not respond more rapidly when change detection was performed alone (1,490ms) than when it was performed with concurrent addition problems (1,573ms), reflected by a non-significant main effect of task condition [F (1, 20) = 1.185, MS_{error} = 584,063, p = 0.289, η^2 = 0.06]. Moreover, latencies on visual change detection did not increase with the number of Gabor patches involved [F <1]: Mean latencies did not significantly differ between correct change detections comprising two-patch arrays (1,506ms), three-patch arrays (1,529ms) and four-patch arrays (1,559ms).

However, rather than increasing with number of Gabor patches as expected, the change detection performance decrement *decreased* with increasing size of Gabor array [F (2, 40) = 7.888, MS_{error} = 78,599, p = 0.001, η^2 = 0.28], with a dual-task cost 149ms for correct change-detection with two-patch arrays, 173ms for three-patch arrays and -75ms for four-patch arrays. Furthermore, this unexpected effect of number of Gabor patches on visual change detection performance did not further interact with number of carriers as predicted, with no significant three-way interaction observed [F <1]. These results were inconsistent with the prediction.

As expected, latencies on correct change detection increase with the number of carriers involved in the concurrent addition task [F (2, 40) = 8.819, MS_{error} = 96,474, p = 0.01, η^2 = 0.31], with participants being slower with change detection performed with single-carry problem (1,543ms) or with double-carry problems (1,604ms) than performed with no-carry problems (1,448ms). Moreover, the change-detection performance decrement increased with number of carriers [F (2, 40) = 8.605, MS_{error} = 82,640, p = 0.01, η^2 = 0.4]. Specifically, a greater dual-task cost was observed for correct change detection performed with double-carry problems (236ms), than with single-carry problems (60ms), than with no-carry problems (-50ms). These result patterns are consistent with the dual-task prediction that change detection performance would decrease with the increasing cognitive demand imposed by the carry operations in the concurrent addition task.

Figure 11 shows the mean accuracy rates (proportion of trials answered correctly) and response times (milliseconds) for the visual change-detection task as functions of the number of Gabor patches and digital carriers, in each of the dual-task and single-task conditions.

Figure 11. Accuracy rates (left column) and response times (right column) for the visual change-detection task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions. Error bars represent standard errors.



Discussion

Consistent with the previous research in which the number of carriers was manipulated (e.g., Fürst & Hitch, 2000; Noël et al., 2001), the results of the present experiment showed that the difficulty of addition problems increases with the number of carry operations involved, evidenced by the significant main effects of the number of carriers in both the accuracy and response time data of the addition task. This effect can be addressed in terms of the number of calculation steps involved during solving carry problems: Addition problems involving carry operations require an extra step in the problem-solving process compared with no-carry problems. Thus, manipulation of the number of carry operations increases the time required for execution of this extra step, which results in slower performance and, when there are more processing steps to be performed, the probability of errors also increases (Imbo et al., 2007).

The results of this experiment also provide evidence of the main issue addressed, which is a role for visual representations in carry operations in mental addition. Task conditions (single or dual) interacted in the predicted way with the number of carriers in the accuracy data of the addition task. Specifically, there was a greater performance decrement associated with dual-task performance compared with single-task performance observed for double-carry problems than for single-carry, or no-carry problems. These findings were consistent with previous research (e.g., Hayes, 1973; Heathcote, 1994; Hitch, 1978; Hubber et al., 2014; Trbovich & LeFevre, 2003), in which the presence of visual loads impaired performance of carry problems more than performance of no-carry problems. That a significant task-by-carry interaction was observed in the accuracy data but not the response time data of the addition task could be addressed in terms of competition for the same working memory resources: When the task requires Gabor patches and carriers to be maintained simultaneously, visual representations of each are likely to have competed for limited VWM resources. The values of the digital carriers or intermediate solutions might therefore not have been maintained correctly in VWM due to insufficient capacity. This has been theorised to affect the accuracy rather than the speed of the calculation (Imbo & Vandierendonck, 2007).

These results are also consistent with finding of the previous study (e.g., Deschuyteneer, De rammaelaere & Fias, 2005), indicating that complex addition with carry operations is solved via the semantic elaboration on the number line activating visual processing (Schmithorst & Brown, 2004). This can be addressed in terms of strategies used to solve complex addition problems involving carry operations: participants calculated the solution through the combination of the elementary addition facts or through decomposing the original problem into easier problems. The former strategy requires the temporary storage of digital carriers (from adding the units and/or the tens), which might have utilised visual working memory resources in complex addition. The latter strategy requires the temporary storage of intermediate solutions (e.g., decomposing 39 + 48 into 40 + 50 - 1 - 2, with the intermediate solutions 90 and 3 needing to be maintained temporarily in working memory). As introduced in General Introduction, this process requires a good understanding of the quantities presented in the original problem that engages the mental number line that activates visual processing (Dehaene, 1992; Dehaene & Cohen, 1997), which corresponds to the visual working memory component in Baddeley's model. Thus, the present finding lend support to the operation-dependent involvement of cognitive resources proposed by the Triple Code Model, which predicts the involvement of visual processing for complex mental addition.

The task-by-carry interaction was not mediated by the level of visual load in the addition task as predicted by the mechanism of manipulation of load level: The performance decrement should have been increased when fewer visual resources were available with higher levels of load. Although a significant main effect of the number of Gabor patches was observed in the response time data of the addition task, it did not further interact with the task-by-carry interaction, suggesting that the greater performance impairment for carry problems than for no-carry problems associated with the presence of visual loads was not exacerbated by increasing load.

There was less evidence supporting the role of VWM in carry operations in mental addition in the visual change-detection task. Although a significant interaction between task conditions and load level was observed in the response time data of the visual change-detection task, the dual-task cost *decreased* with increasing number of Gabor patches.

It could be argued that this performance improvement in the visual change-detection task with increasing load was due to a trade-off between tasks, with participants prioritising the maintenance of the Gabor patches at the price of carry information involved in the concurrent addition problems. However, the trial selection method used in the present experiment excluded this possibility: For dual-task conditions, only trials in which the concurrent addition problem was correctly solved was included in the analysis. Thus, the maintenance of the carry information could not have been compromised.

A possible explanation might be adult participants switching between representational modalities in working memory. According to Hubber et al., (2014), "*When participants are prevented from using visuospatial storage, due to the dual task, they fall back onto using verbal storage*" (p. 67, Hubber et al., 2014). Thus, it is possible that participants switched to PWM to maintain the representations of the carriers or intermediate solutions, rather than continuing to use VWM when it was heavily loaded. This possibility will be further elaborated in the General Discussion.

Chapter 4

Experiment 2

Multiplication can be regarded as repeated addition (Imbo & Vandierendonck, 2007). It has been theorised that complex multiplication includes carry operations similar to those in complex addition (Geary et al., 1986). Multiplication is assumed to heavily rely on retrieving arithmetic facts stored in long-term memory. However, for complex multiplication problems involving carry operations, the problem is likely to be decomposed into several sub-problems, and solutions of each sub-problem will be summed to achieve a final solution. For example, when solving question 24 × 3, the problem might first be broken down into components (20 × 3, 4 × 3), with solutions achieved through fact-retrieval (60, 12) and then temporarily stored. The final solution might then be achieved by adding the two partial solutions (60 + 12 = 72). This procedural process has been theorised under the Triple Code Model to utilise the magnitude number line, which activates visual processing (Dehaene, 1992; Dehaene & Cohen, 1997).

Few neuro-imaging studies have investigated complex multiplication, and little behavioural research has investigated the role of visual working memory in carry operations in multiplication. The present experiment was designed to investigate the role of visual resources in carry operations in complex mental multiplication.

It is hypothesised that visual processing is required for complex multiplication. Moreover, evidence supporting the role of VWM in mental multiplication is likely to be stronger than in addition (Experiment 1): Compared to addition problems used in Experiment 1, greater values of digital carriers and intermediate solutions are involved in multiplication problems used in the present experiment. Both numbers of carry operations and values of digital carriers have been found to affect calculation performance (Imbo & Vandierendonck, 2007). Specifically, the influence of working memory load has been found to be greater when the numbers of carry operations and values of the carriers are greater (Exp. 4, Imbo et al., 2007). A plausible interpretation of this finding is that more working memory resources are required for maintaining carrier information and intermediate solutions with greater values.

Method

Participants

Participants were the same as those that completed Experiment 1.

Materials

Multiplication task

One hundred and eight double-digit multiplication problems, each involving one double-digit operand and one single-digit operand were used for this experiment and nine similar problems were used as practice trials. Double digit operands and their products ranged from 12 and 98, and 26 and 315, respectively. Problems with values of zero in the unit positions were avoided because they are likely to be solved using fact-retrieval rather than being calculated (see Ashcraft, 1992; 1995). The 108 problems were divided into three groups of 36 problems. In each group, there were 6 or 7 no-carry problems, 20-to-25 single-carry problems and 5-to-9 multi-carry problems. Single-carry problems required participants to either carry a digital carrier from the product in the units column and add it to a digit in the decades column (e.g., 15 × 3), or from the product in the decade unit to the hundred column (e.g., 54 × 2). The values of digital carriers ranged from one to five. Appendix B shows all of the problems used in this experiment.

Visual change-detection task

The visual stimuli for the change-detection task were the same as those used in Experiment 1.

Procedure

The procedure used in the present experiment was the same as the procedure used in Experiment 1.

Design

The design used in the present study was the same as the design used in Experiment 1.

Results

Multiplication Task

Accuracy Rates- In total, three participants were excluded from the analysis due to no correctly responded double-carry problems. For each condition (single-task, dual-task condition), 67.3% (3,489 out of 5,184) of trials were included in the analysis.

Table 13 shows the mean accuracy rates for the multiplication task as functions of numbers of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Table 13. Mean accuracy rates for the multiplication task as functions of numbers of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Question types	Visual loads	Single-task condition *M(SD)*	Dual-task condition *M (SD)*	N
No carry problems	2 Gabor patches	0.94 (0.02)	0.94 (0.02)	45
	3 Gabor patches	0.91 (0.02)	0.96 (0.02)	45
	4 Gabor patches	0.96 (0.02)	0.95 (0.02)	45
Single-carry problems	2 Gabor patches	0.83 (0.02)	0.79 (0.02)	45
	3 Gabor patches	0.82 (0.02)	0.79 (0.02)	45
	4 Gabor patches	0.83 (0.02)	0.80 (0.02)	45
Double-carry problems	2 Gabor patches	0.77 (0.04)	0.71 (0.03)	45
	3 Gabor patches	0.79 (0.04)	0.73 (0.04)	45
	4 Gabor patches	0.82 (0.04)	0.76 (0.03)	45

Contrary to the prediction that complex arithmetic is solved with the aid of a magnitude number line activating visual processing, the presence of carry operations in complex multiplication was not associated with greater performance decrement under visual load [F (2, 78) = 2.357, MS_{error} = 0.034, p = 0.101, η^2 = 0.06]. This non-significant load-by-task interaction suggests that VWM resources are not required by carry operations or intermediate solutions in mental multiplication. This is also reflected by the absence of a significant predicted three-way interaction between task conditions, number of carriers and number of Gabor patches [F <1].

As is typically found, participants made more errors on problems involving greater number of carry operations [F (2, 78) = 59.199, MS_{error} = 0.039, p < 0.01, η^2 = 0.6]. Specifically, no-carry problems (0.94) were solved more accurately than single-carry problems (0.81) and double-carry problems (0.76).

However, participants were not more accurate when multiplication problems were solved alone then when under a visual load as expected [F (1, 39) = 2.987, MS_{error} = 0.046, p = 0.092, η^2 = 0.07], with multiplication problems solved almost as accurately in the single-task condition (0.81) as in the dual-task condition (0.83). Also, contrary to the prediction, error rates did not increase with visual load [F (2, 78) = 1.471, MS_{error} = 0.031, p = 0.236, η^2 = 0.04]: Multiplication accuracy did not differ between multiplication problems solved with two-patch arrays (0.83), three-patch arrays (0.83) or four-patch arrays (0.85).

Other two-way interactions were not statistically significant, all Fs <1.

Solution Latencies- In total, 13 participants were excluded from analysis due to no correctly responded questions. Dual-task trials with response time longer than 4,000ms were excluded from the analysis. For the single-task condition, 43.4% (2,247/5,184) of trials were included in the analysis. For the dual-task condition, 50% (2,593/5,184) of trials were included in the analysis.

Table 14 shows the mean solution time for the multiplication task for each combination of visual load (numbers of Gabor patches), digital carriers and task (single, dual) condition.

Table 14. Mean solution times for the multiplication task as of numbers of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Question types	Visual loads	Single-task condition *M (SD)*	Dual-task condition *M(SD)*	N
No carry problems	2 Gabor patches	1962 (73)	1994 (76)	35
	3 Gabor patches	1868 (70)	1940 (71)	35
	4 Gabor patches	1973 (97)	1927 (69)	35
Single-carry problems	2 Gabor patches	2332 (94)	2408 (80)	35
	3 Gabor patches	2311 (91)	2576 (97)	35
	4 Gabor patches	2295 (80)	2596 (110)	35
Double-carry problems	2 Gabor patches	2488 (98)	2730 (115)	35
	3 Gabor patches	2582 (100)	2620 (111)	35
	4 Gabor patches	2619 (98)	2846 (111)	35

Unlike the accuracy data, the predicted load-by-carry interaction supporting a role of visual resources in carry operations or intermediate solutions in mental multiplication was observed in the latency analysis [F (2, 58) = 3.787, MS_{error} = 139,572, p = 0.028, η^2 = 0.12], with greater dual-task costs observed for single-carry problems (213ms) and double-carry problems (168ms) than for no-carry problems (19ms). However, this effect of number of carry operations on multiplication performance was not further enlarged by increasing visual load, in spite of a significant three-way interaction between task conditions, the number of carriers and the number of Gabor patches [F (4, 116) = 2.523, MS_{error} = 99,652, p = 0.045, η^2 = 0.08]. Specifically, performance decrement did not differ significantly between no-carry problems solved with two-, three- patch arrays (32ms and 72ms respectively), and rather than increasing with the level of load, the dual-task cost turned into a benefit when solved with four-patch arrays (-46ms). For single-carry problems, as predicted, greater dual-task costs were observed for single-carry problems solved with three-patch arrays (265ms) and four-patch arrays (301ms) than for those solved with two-patch arrays (75ms). However, for double-carry problems, dual-task costs decreased and then increased with level of load (242ms for two Gabor patches, 38ms for three Gabor patches, and 227ms for four Gabor patches), which is inconsistent with the prediction.

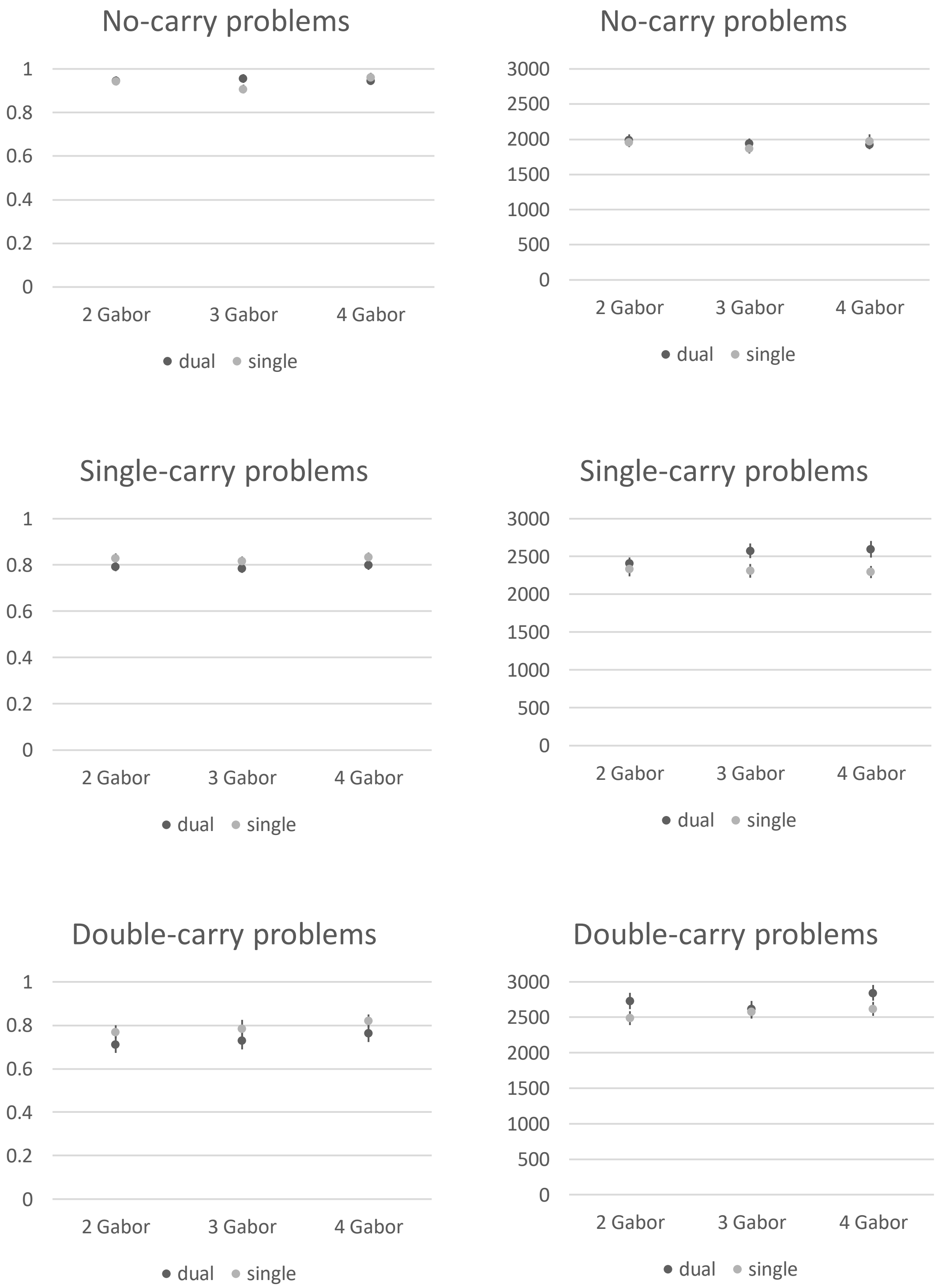
As is typically found, participants took longer to solve problems presented with a visual load (2,404ms) than problems presented alone (2,270ms) [F (1, 29) = 5.024, MS_{error} = 547,547, p = 0.033, η^2 = 0.15]. Moreover, participants were slower when solving problems with more number of carry operations [F (2, 58) = 152.631, MS_{error} = 172,254, p < 0.01, η^2 = 0.84], with double-carry problems (2,647ms) solved more slowly than single-carry problems (2,419ms) and no-carry problems (1,944ms).

Similar to the accuracy data, mean latencies on correct multiplication did not increase with visual load [F (2, 58) = 1.668, MS_{error} = 139,880, p = 0.198, η^2 = 0.05], with similar mean response times observed between multiplication problems solved with two-patch (2,319ms), three-patch (2,316ms) and four-patch arrays (2,376ms).

There were no other significant two-way interactions.

Figure 12 shows the mean accuracy rates and response times in the multiplication task as functions of numbers of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Figure 12. Mean accuracy rates (left column) and response times (right column) of the multiplication task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions. Error bars represent standard errors.



Visual Change-detection Task

Accuracy Rates- In total, 1 participant was excluded from the analysis due to no correctly responded double-carry problems. For each condition (single-task, dual-task condition), 78.4% (4,065 out of 5,184) of trials were included in the analysis.

Table 15 shows the mean accuracy rates for the visual change-detection task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Table15. Mean accuracy rates for the visual change-detection task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Question types	Visual loads	Single-task condition *M (SD)*	Dual-task condition *M(SD)*	N
No carry problems	2 Gabor patches	0.86 (0.02)	0.68 (0.03)	47
	3 Gabor patches	0.85 (0.02)	0.70 (0.03)	47
	4 Gabor patches	0.78 (0.02)	0.70 (0.03)	47
Single-carry problems	2 Gabor patches	0.86 (0.02)	0.68 (0.02)	47
	3 Gabor patches	0.80 (0.02)	0.64 (0.02)	47
	4 Gabor patches	0.76 (0.02)	0.67 (0.02)	47
Double-carry problems	2 Gabor patches	0.87 (0.02)	0.69 (0.03)	47
	3 Gabor patches	0.83 (0.03)	0.64 (0.04)	47
	4 Gabor patches	0.68 (0.04)	0.67 (0.03)	47

As is typically found, participants were more accurate when change detection was performed alone (0.81) than when performed with a concurrent multiplication task (0.67), reflected by a significant main effect of task condition [F (1, 41) = 76.7, MS_{error} = 0.058, p < 0.01, η^2 = 0.65]. Moreover, change detection accuracy decreased with increasing complexity (defined by number of Gabor patches involved) as expected [F (2, 82) = 11.207, MS_{error} = 0.026, p < 0.01, η^2 = 0.22]. Specifically, participants were more accurate in change detection involving two-patch arrays (0.77) than three-patch arrays (0.74) or four-patch arrays (0.71). Change detection accuracy also decreased as the number of carriers involved in the concurrent multiplication task increased as expected [F (2, 82) = 4.625, MS_{error} = 0.019, p = 0.012, η^2 = 0.1], with change detection being performed more accurately with no-carry problems (0.76) than with single-carry problems (0.73) or with double-carry problems (0.73).

However, contrary to the prediction, the change-detection performance decrement in dual-task conditions *decreased* with increasing number of Gabor patches. Although there was a significant interaction between task conditions and number of Gabor patches [F (2, 82) = 5.334, MS_{error} = 0.038, p = 0.007, η^2 = 0.12], a greater dual-task cost was observed for two-patch arrays (0.18) and three-patch arrays (0.17) than for four-patch arrays (0.09). Furthermore, this unexpected effect was not mediated by increasing calculation demands, with no predicted significant three-way interaction of task conditions, the number of carriers and the number of Gabor patches observed [F <1].

There were no other significant two-way interactions.

Solution Latencies- In total, 12 participants were excluded from analysis due to no correctly responded questions. For the single-task condition, 47.2% (2,540/5,184) of trials were included in the analysis. For the dual-task condition, 52.6% (2,725/5,184) of trials were included in the analysis.

Table 16 shows mean solution times for the visual change-detection task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Table16. Mean solution times for the visual change-detection task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Question types	Visual loads	Single-task condition *M (SD)*	Dual-task condition *M (SD)*	N
No carry problems	2 Gabor patches	1357 (73)	1376 (154)	36
	3 Gabor patches	1488 (64)	1341 (148)	36
	4 Gabor patches	1583 (90)	1450 (122)	36
Single-carry problems	2 Gabor patches	1437 (73)	1513 (118)	36
	3 Gabor patches	1522 (93)	1407 (112)	36
	4 Gabor patches	1570 (85)	1536 (109)	36
Double-carry problems	2 Gabor patches	1347 (79)	1561 (118)	36
	3 Gabor patches	1461 (99)	1579 (108)	36
	4 Gabor patches	1747 (107)	1518 (117)	36

Unlike the accuracy data, participants were not significantly faster when performing the change detection alone (1,512ms) than when performing them with a concurrent multiplication task (1,466ms) [F <1].

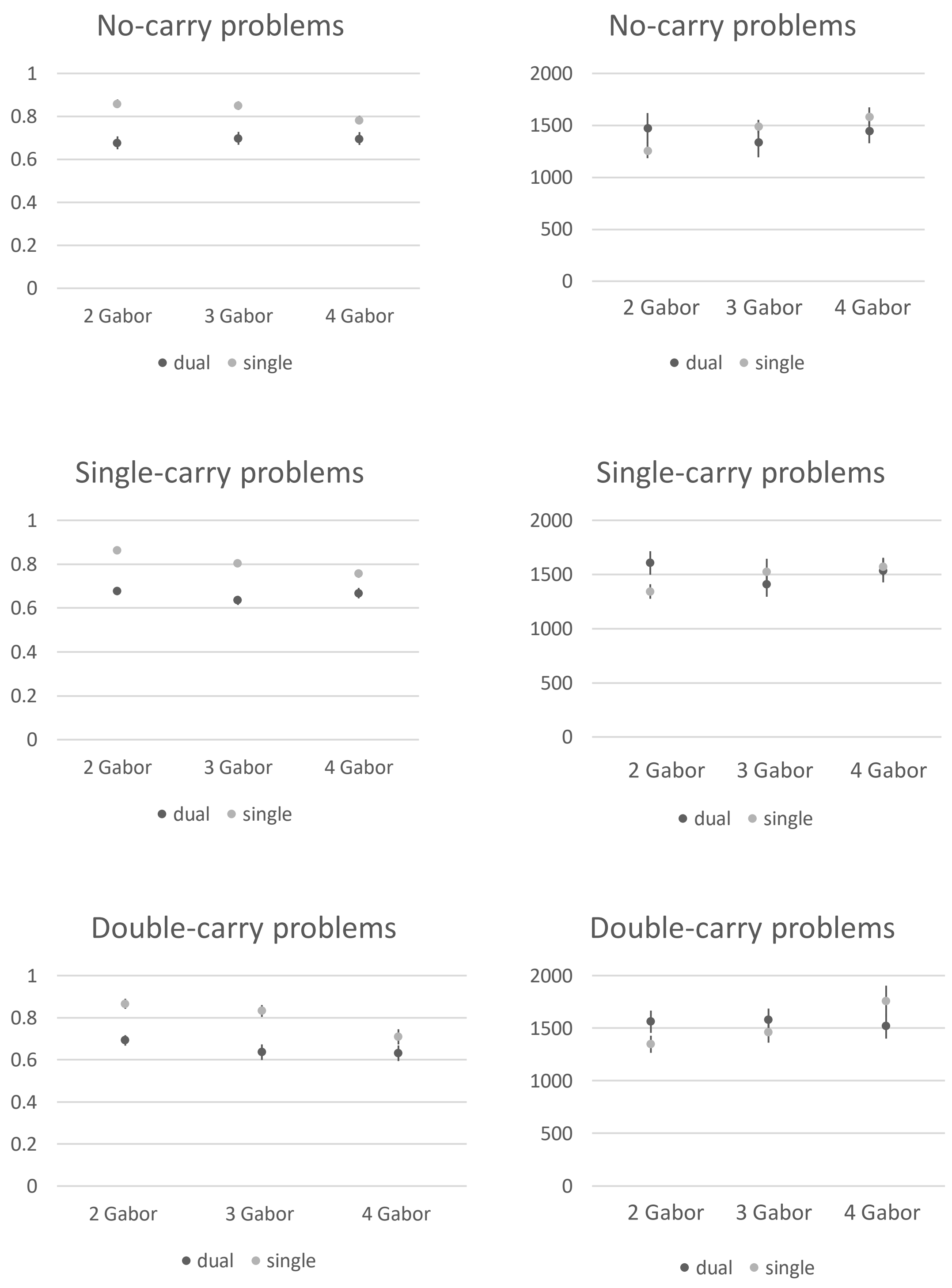
Consistent with previous findings, participants took longer to perform more complex change detection [F (2, 60) = 7.417, MS_{error} = 134,918, p = 0.001, η^2 = 0.2]. In particular, correct change detection with four-patch arrays (1569ms) was performed more slowly than those with three-patch arrays (1466ms) or with two-patch arrays (1,432ms). Also, longer latencies were associated with increasing calculation demand imposed by the concurrent multiplication task [F (2, 60) = 4.004, MS_{error} = 136,280, p = 0.023, η^2 = 0.12], with correct change detection performed faster with no-carry problems (1,433ms) than with single-carry problems (1,497ms), than with double-carry problems (1,537ms).

However, rather than increasing with number of Gabor patches as expected, the performance decrement caused by a concurrent arithmetic task *decreased* with increasing change detection complexity [F (2, 60) = 4.815, MS_{error} = 201,323, p = 0.011, η^2 = 0.06]. Specifically, a dual task cost was only observed for mean correct response times with two-patch arrays (231ms), whereas a dual-task advantage was observed for three-patch arrays (-48ms) and four-patch arrays (-136ms). Furthermore, this negative effect of the number of Gabor patches on change-detection performance was further enhanced by the increasing calculation demand [F (4, 120) = 2.627, MS_{error} = 82,490, p = 0.038, η^2 = 0.09], which is opposite from the prediction. Further comparison showed that the dual-task cost for mean response times for correct change detection performed with no-carry problems *reduced* with increasing visual load (216ms for two-patch arrays, -148ms for three-patch arrays and -134ms for four-patch arrays). Similarly, a *reduction* in the dual-task cost with increasing number of Gabor patches was observed for change detection performed with single-carry problems (264ms for two-patch arrays, -115ms for three-patch arrays and -34ms for four-patch arrays). While presented with double-carry problems, although a similar reduction in the dual-task cost was observed, the degree of such a reduction was not as substantial as it was for no-carry or single-carry problems (214ms for two-patch arrays, 117ms for three-patch arrays and -239ms for four-patch arrays).

There were no other significant two-way interactions, all F <1.

Figure 13 shows the mean accuracy rates and response times for the visual change-detection task as functions of the number of Gabor patches and digital carriers in each of the dual-task and single-task conditions.

Figure 13. Mean accuracy rates (left column) and response times (right column) for the visual change-detection task as functions of the numbers of digital carriers and Gabor patches in each of the dual-task and single-task conditions. Error bars represent standard errors.



Discussion

The results of this experiment resemble the findings in Experiment 1, with multiplication problems involving carry operations solved less accurately and rapidly than no-carry problems. This was evidenced by the significant main effects of the number of carriers in both the accuracy data and the response time data of the addition task, indicating an influence of the number of carry operations in mental multiplication similar to that shown for mental addition; in both cases, the results suggest that more working memory resources are required to solve problems involving carry operations than to solve problems without such operations (Imbo et al., 2005).

The present findings provide more evidence supporting the role of VWM in carry operations in mental multiplication than was found in support of a similar role in addition (Experiment 1). Specifically, the influence of number of carry operations on arithmetic performance was affected by the presence or absence of a concurrent visual load task. Statistically this finding was supported by a significant task-by-carry interaction in the response time data of the multiplication task, with a greater performance decrement observed for carry-problems than for no-carry problems. Moreover, a three-way interaction was also observed in the response time data of the multiplication task, suggesting that the effect of number of carry operations on arithmetic performance was mediated by the level of visual load.

However, unlike Experiment 1, a similar task-by-carry interaction was observed more clearly in the latency analyses than in accuracy analyses in the present experiment. Visual load only slowed arithmetic performance but did not impair the accuracy of the calculation compared to the control condition. These findings suggest a limited role of visual working memory in carry operations in mental multiplication, which are inconsistent with the previous research conducted by Imbo and LeFevre (Exp. 2, 2010) in which a role of visual resources in mental multiplication was observed.

It could be argued that the absence of a significant task-by-carry interaction in the accuracy data of the multiplication task might be attributable to fewer working memory resources being required to solve multiplication problems in the present experiment: Multiplication problems comprising a two-digit number and a one digit-number could be decomposed into easier tasks (Imbo et al., 2007). For example, 28 × 7 can be broken into two simple multiplication problems relying on fact retrieval rather than on working memory resources: 2 × 7 and 8 × 7. However, if this was the case, then no significant task-by-carry interaction should be observed in the response time data of the multiplication task.

A possible explanation for the task-by-carry interaction appearing in the response time data rather than the accuracy data as it did in Experiment 1, might arise in relation to differences between addition and multiplication. Multiplication problems are typically more complicated than addition problems due to greater values of digital carriers and larger magnitudes of solutions (Ashcraft, 1992; Ashcraft & Battaglia, 1978; Campbell, 1995; Geary, 1996; Groen & Parkman, 1972). Therefore, in the present experiment, participants might have focused on achieving the correct products to multiplication problems, so that the effect of visual load was shown in the response time data. Whereas in Experiment 1 in which (relatively less-complicated) addition problems were used, participants could focus on speed. This notion is also supported by different patterns of the main effects of task conditions in the present experiment and in Experiment 1: A significant main effect of task condition was observed in the response time data of the present experiment but not in the accuracy data, whereas the converse was true in Experiment 1.

These results did not support the hypothesis that complex multiplication is solved through semantic elaboration on a mental number line, which is assumed to activate visual processing. This might be due to differences between multiplication and addition: Compared to addition, solving multiplication is probably more heavily rely on fact-retrieval, which is theorised to require verbal processing. Campbell and Xue (2001) found retrieval rates of 76% for addition and 96% for multiplication in university students. However, the practice effect might also have contributed to the absence of visual processing in complex mental multiplication, since the same problems were solved in both single-task and dual-task conditions. Although tasks involving the same problems were never performed in the same session, it is likely that participants still subconsciously familiarised themselves with some problems, so that products were remembered rather than calculated.

The role of visual resources in carry operations in mental multiplication was not further supported by the visual change-detection data. Although significant interactions between task conditions and level of visual load were observed in both the accuracy and response time data of the visual change-detection task, the dual-task cost between the dual-task and single-task condition *decreased* with increasing load level.

Moreover, inconsistent with the prediction, the visual change-detection task solved with no-carry problems was performed faster in the dual-task condition than in the single-task condition, and this performance improvement grew larger with increasing numbers of carriers. Thus, the present experiment provided inconsistent findings regarding the role of working memory in carry operations in mental multiplication.

A possible explanation for the apparent anomalous results in the change detection data could be provided by the switching hypothesis, as discussed in relation to Experiment 1; carrier information initially represented visually could be recoded phonologically when the capacity of the visual storage is approached. As reviewed in the General Introduction, empirical evidence supporting the relative involvement of phonological representations in complex mental multiplication has been found in previous studies (e.g., Imbo & LeFevre, 2010; Imbo & Vandierendonck, 2007; Seitz & Schumann-Hengsteler; 2000; 2002), with performance of carry-problems more affected by a concurrent phonological task than performance of no-carry problems. Therefore, when less VWM resources than PWM resources are available, it is possible that some participants might switch to using phonological representations for maintaining the (originally visually-represented) intermediate solutions or digital carriers.

Chapter 5

Experiment 3

According to the Triple Code Model, complex addition is solved using semantic elaboration on a mental magnitude number line that utilises visual resources (Dehaene, 1992; Dehaene & Cohen, 1997). The Triple Code Model predicts that no phonological processing is required in complex addition. However, inconsistent results have been found regarding this issue in previous behavioural studies (e.g., Imbo & LeFevre, 2010; Logie et al., 1994). Moreover, little previous research has directly compared the involvement of phonological and visual resources in carry operations in mental arithmetic. This is because the interference tasks used for tapping PWM normally have little comparability with those used for tapping VWM. For example, it is hard to conceive of a visual interference task that is comparable to the articulatory suppression task.

The present experiment was designed to address these issues. The role of phonological representations in carry operations in mental addition was investigated using a change-detection task that was comparable to the visual change-detection task used in Experiments 1 and 2. The only difference was that pronounceable consonant-vowel-consonant non-words were used as stimuli rather than Gabor patches.

Evidence supporting phonological representations of carrier information during calculation would be the same as described in Experiment 1; a task-by-carry interaction in the addition task, with a greater dual-task cost for problems with more carriers. For the phonological change-detection task, supporting evidence would be interactions between single/dual-task conditions and the number of nonwords. Further supportive evidence would be three-way interactions between task conditions, number of carriers and nonwords in both tasks, with the dual-task cost increased by increasing load levels and numbers of carry operations.

Method

Participants

The same participants as those in Experiment 1 completed the present experiment.

Materials

Addition task

The addition problems used in the present experiment were the same as those used in Experiment 1.

Phonological Change-Detection Task

Twenty-eight non-words were recorded, all read by the same person as those used in Pilot Experiment 5. Three sets of study nonword lists were constructed, with nine nonword lists in each. The first set comprised four non-words in each study list. The second set comprised five non-words in each study list, and the third set comprised six non-words in each study list. No non-word was used more than once in any study list.

Each study nonword list was repeated four times in the experiment, twice with a probe non-word the same as one of the non-words in the study nonwords, and twice with a probe non-word different from all the non-words in the study nonwords, resulting in 108 trials in total in each of the single and dual-task conditions.

Procedure

Each participant was tested individually in a quiet room. A pair of headphones was provided and each participant adjusted the volume to suit themselves. A brief explanation of the experiment was provided and participants were instructed to make a response to each trial as quickly and as accurately as possible. Six dual-task practice trials were administered to make sure participants were familiar with the procedure.

Each trial began with a 500ms tone, which was the cue to be ready for the following nonword list. After 500ms, a study string comprising four, five or six non-words was presented, with 500ms intervals between each non-word, followed by a mental addition problem presented in the centre of the screen in the visual Arabic font for 4,000ms. Participants were instructed to speak their responses into the microphone loudly and clearly. After responding to the problem, a probe non-word was presented. Participants were asked to indicate whether or not the probe was the same as one of study non-words by pressing one of the two response keys. After a response was given, the words "Please step on the foot pedal to continue" appeared on the screen.

In the single-task condition in which the phonological change-detection task was performed alone, arithmetic problems were replaced by a 4,000ms inter-stimulus interval.

Design

The mean response times and the accuracy rates for both the arithmetic task and the phonological change-detection tasks were analysed separately in a 2 (task conditions: dual, single) × 3 (numbers of digital carriers: 0, 1, 2) × 3 (number of nonwords: 4 non-words, 5 non-words, 6 non-words) × 6 (version: 1, 2, 3, 4, 5, 6) mixed measures analysis of variance (ANOVAs), with repeated measures on the first three factors and the last factor as a between-group factor.

Results

Addition Task

Accuracy Rates - No participants were excluded from the analysis. For each condition (single-task, dual-task condition), 73.9 % (3,831 out of 5,184) of trials were included in the analysis.

Table 17 shows mean accuracy rates for the addition task as functions of the number of non-words and digital carriers in each of the dual-task and single-task conditions.

Table17. Mean accuracy rates for the addition task as functions of the number of non-words and digital carriers in each of the dual-task and single-task conditions.

Question types	Phonological loads	Single-task condition *M (SD)*	Dual-task condition *M (SD)*	N
No-carry problems	4 nonwords	0.91 (0.01)	0.97 (0.01)	48
	5 nonwords	0.96 (0.01)	0.96 (0.01)	48
	6 nonwords	0.93 (0.01)	0.95 (0.01)	48
Single-carry problems	4 nonwords	0.81 (0.02)	0.81 (0.02)	48
	5 nonwords	0.82 (0.02)	0.75 (0.03)	48
	6 nonwords	0.80 (0.02)	0.76 (0.03)	48
Double-carry problems	4 nonwords	0.65 (0.04)	0.51 (0.06)	48
	5 nonwords	0.75 (0.04)	0.56 (0.06)	48
	6 nonwords	0.68 (0.04)	0.55 (0.05)	48

Contrary to the hypothesis that phonological processing is not required during solving complex mental addition, the predicted load-by-carry interaction supporting a role of phonological working memory in carry operations or intermediate solution in mental addition was observed [F (2, 84) = 11.609, MS_{error} = 0.053, p = 0.001, η^2 = 0.22]. Further comparison revealed a greater dual-task cost for double-carry problems (0.15) than for no-carry problems (-0.03) or for single-carry problems (0.03). However, unlike predicted, the increasing level of phonological load did not differently impact on the dual-task performance decrement of no-, single- and double-carry problems, with no significant three-way interaction between task, number of carrier and level of load [F <1].

As is typically found, participants were less accurate on problems with more carry operations [F (2, 84) = 95.172, MS_{error} = 0.083, p < 0.01, η^2 = 0.69]. Further comparison showed that double-carry problems were solved less accurately (0.95) than single- (0.79) or no-carry problems (0.62). Also, participants were more accurate when solving addition problems alone (0.81) than when solving them under a phonological load (0.76) as expected [F (1, 42) = 6.216, MS_{error} = 0.104, p < 0.01, η^2 = 0.13].

However, contrary to the prediction, participants were not less accurate under increasing level of phonological load [F (2, 84) = 1.395, MS_{error} = 0.03, p = 0.254, η^2 = 0.03], with similar accuracy of addition problems solved with four (0.78), five (0.80) and six nonwords (0.78) observed.

Solution Latencies- In total, 19 participants were excluded from the analysis due to no correctly responded questions. Dual-task trials with response time longer than 4,000ms were excluded from the analysis. For the single-task condition, 53.3% (2,768/5,184) of trials were included in the analysis. For the dual-task condition, 45.3% (2,349/5,184) of trials were included in the analysis.

Table 18 shows mean solution times for the addition task as functions of the number of non-words and digital carriers in each of the dual-task and single-task conditions.

Table18. Mean solution times the addition task as functions of the number of non-words and digital carriers in each of the dual-task and single-task conditions.

Question types	Phonological loads	Single-task condition *M (SD)*	Dual-task condition *M (SD)*	N
No carry problems	4 non-words	1933 (69)	2175 (75)	29
	5 non-words	1930 (52)	2357 (82)	29
	6 non-words	1989 (76)	2253 (77)	29
Single-carry problems	4 non-words	2457 (77)	2614 (60)	29
	5 non-words	2451 (75)	2672 (70)	29
	6 non-words	2488 (70)	2688 (75)	29
Double-carry problems	4 non-words	2710 (103)	2906 (134)	29
	5 non-words	2842 (128)	2977 (152)	29
	6 non-words	2812 (135)	2994 (138)	29

Inconsistent with the accuracy analysis, no load-by-carry interaction was observed [F <1], a finding that does not support a role of phonological resources in carry operations or intermediate solutions in mental addition. Furthermore, the dual-task performance decrement on solving addition problems involving carry operations were not more impaired than no-carry problems with increasing phonological load [F <1]. These results are consistent with that for the prediction of the Triple Code Model that phonological processing is not required in complex mental addition.

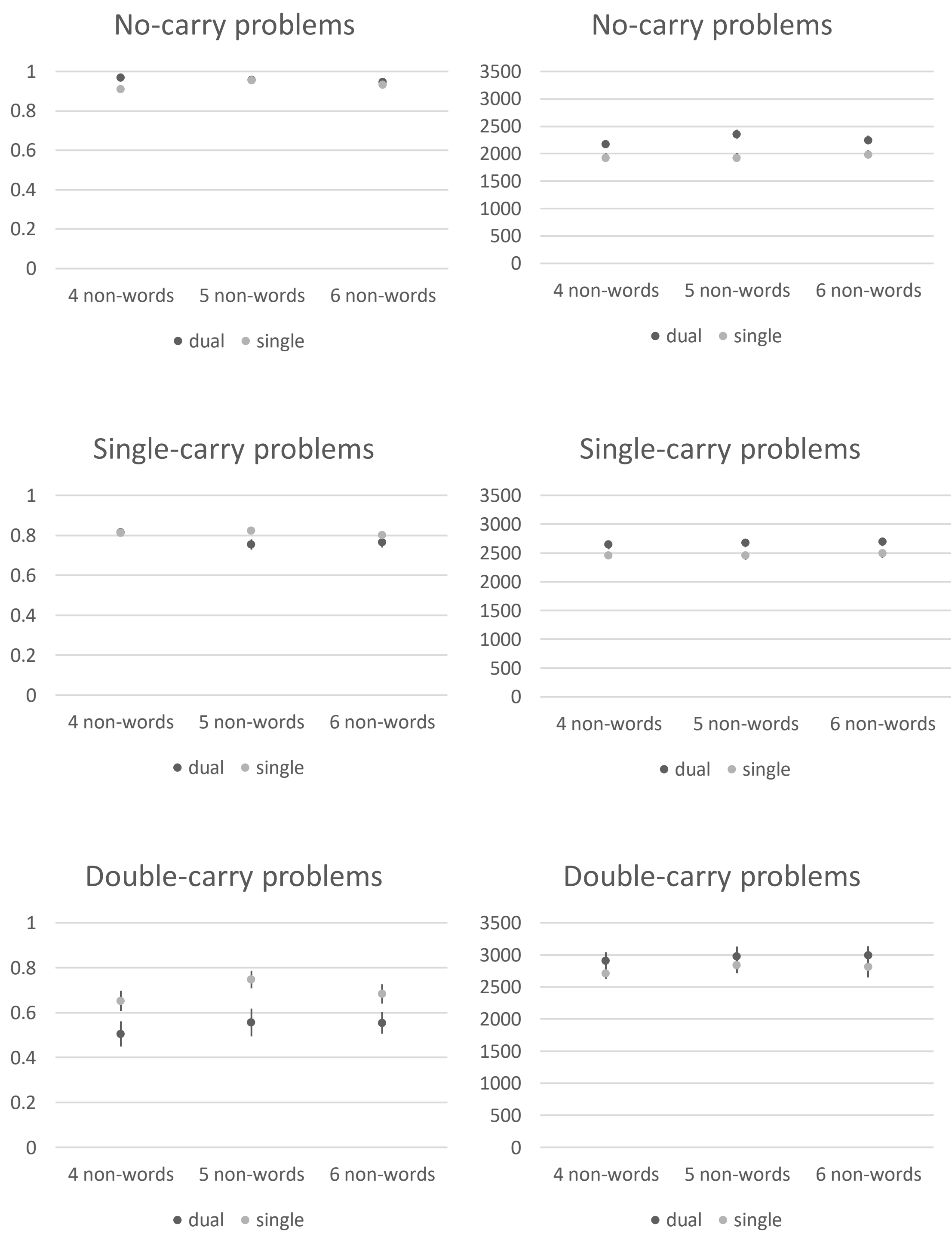
Consistent with previous findings, participants took longer to answer problems with more carry operations [F (2, 46) = 94.636, MS_{error} = 202,740, p < 0.01, η^2 = 0.80]. Further analysis showed no-carry problems (2,140ms) were solved faster than single-carry problems (2,578ms) and double-carry problems (2,873ms). Moreover, participants were faster when solving addition problems alone (2,435ms) than when solving them under a phonological load (2,626ms) as expected [F (1, 23) = 11.238, MS_{error} = 343,226, p < 0.01, η^2 = 0.33].

Similar to the accuracy analysis, mean latencies on correct solved addition problems did not increase with phonological load level as expected [F (2, 46) = 1.213, MS_{error} =95,013, p = 0.307, η^2 = 0.05], with similar mean latencies observed for correct addition problems solved with four (2,499ms), five (2,555ms) and six nonwords (2,538ms).

There were no other significant two-way interactions, all Fs < 1.

Figure 14 shows the mean accuracy rates and response times for the addition task as functions of number of nonwords and numbers of digital carriers in each of the dual-task and single-task conditions.

Figure 14. Mean accuracy rates (left column) and response times (right column) for the addition task as a function of numbers of nonwords and numbers of digital carriers in each of the dual-task and single-task conditions. Error bars represent standard errors.



Phonological Change-detection Task

Accuracy Rates- In total, 18 participants were excluded from the analysis due to no correctly responded double-carry problems in the dual-task condition. For each condition (single-task, dual-task condition), 58.4% (3,027 out of 5,184) of trials were included in the analysis.

Table 19 shows the mean accuracy rates of the phonological change-detection task as functions of the number of non-words and digital carriers, in each of the dual-task and single-task conditions.

Table 19. Mean accuracy rates of the phonological change-detection task as functions of the number of non-words and digital carriers, in each of the dual-task and single-task conditions.

Question types	Phonological loads	Single-task condition *M (SD)*	Dual-task condition *M (SD)*	N
No carry problems	4 nonwords	0.75 (0.02)	0.64 (0.02)	30
	5 nonwords	0.85 (0.02)	0.86 (0.02)	30
	6 nonwords	0.87 (0.02)	0.75 (0.02)	30
Single-carry problems	4 nonwords	0.75 (0.02)	0.69 (0.02)	30
	5 nonwords	0.84 (0.02)	0.83 (0.02)	30
	6 nonwords	0.84 (0.03)	0.79 (0.03)	30
Double-carry problems	4 nonwords	0.76 (0.03)	0.70 (0.05)	30
	5 nonwords	0.85 (0.02)	0.81 (0.04)	30
	6 nonwords	0.80 (0.04)	0.74 (0.05)	30

As is typically found, participants were more accurate when change detection was performed alone (0.81) than performed with a concurrent addition task (0.76) [F (1, 24) = 7.129, MS_{error} = 0.053, p = 0.013, η^2 = 0.23]. Moreover, participants were less accurate with more complicated change detection as expected [F (2, 48) = 19.826, MS_{error} = 0.033, p < 0.01, η^2 = 0.45], with change detection comprising four nonwords (0.72) was performed less accurately than with six (0.80) or five nonwords (0.84).

However, contrary to the dual-task prediction, change detection accuracy did not decrease with increasing cognitive demand imposed by the concurrent addition task [F <1], with similar change detection accuracy observed when performed with no-carry (0.79), single-carry (0.79) and double-carry problems (0.77). Moreover, the change-detection performance decrement did not increase with the number of nonwords involved [F (2, 48) = 1.904, MS_{error} = 0.027, p = 0.16, η^2 = 0.07]. Furthermore, no predicted three-way interaction between task condition, number of carriers and load level was observed [F <1], suggesting that change detection performance decrement caused by increasing number of nonwords is not mediated by the complexity of concurrent addition problems.

There were no significant two-way interactions, all *F*s < 1.

Solution Latencies- In total, 22 participants were excluded from the analysis due to no correctly responded double-carry problems or phonological change detection. For the dual-task condition, 41.7% (2,161/5184) of trials were included in the analysis. For the single-task condition, 39.2% (2,034/5184) of trials were included in the analysis.

Table 20 shows the mean solution time of the phonological change-detection task as functions of the number of non-words and digital carriers, in each of the dual-task and single-task conditions.

Table 20. Mean solution times the phonological change-detection task as functions of the number of non-words and digital carriers, in each of the dual-task and single-task conditions.

Question types	Phonological loads	Single-task condition *M (SD)*	Dual-task condition *M (SD)*	N
No carry problems	4 nonwords	1570 (76)	1829 (128)	26
	5 nonwords	1845 (88)	1763 (81)	26
	6 nonwords	1981 (85)	1753 (72)	26
Single-carry problems	4 nonwords	1629 (78)	1949 (125)	26
	5 nonwords	1799 (73)	1792 (106)	26
	6 nonwords	2009 (86)	1806 (77)	26
Double-carry problems	4 nonwords	1586 (81)	2386 (237)	26
	5 nonwords	1938 (96)	1912 (103)	26
	6 nonwords	2004 (99)	1869 (131)	26

Contrary to the accuracy analysis, participants performed change detection with a concurrent addition task (1,895ms) at a similar speed to change detection presented alone (1,818ms) [F (1, 20) = 2.51, MS_{error} = 234,181, p = 0.366, η^2 = 0.11]. Moreover, mean latencies of correct change detection did not increase with complexity of change detection as expected [F (2, 40) = 1.031, MS_{error} = 218,023, p = 0.366, η^2 = 0.05], with similar mean latencies observed for correct phonological change detection involving four (1,825ms), five (1,841ms) and six nonwords (1,903ms).

Unlike the accuracy analysis, mean latencies of correct change detection increased with the number of carriers involved in the concurrent addition task as expected [F (2, 40) = 5.677, MS_{error} = 156,998, p = 0.007, η^2 = 0.22], with correct change detection presented with no-carry problems (1,790ms) performed significantly faster than that presented with single-carry problems (1,831ms) or double-carry problems

(1,949ms). However, the change-detection performance decrement *decreased* (rather than increasing) number of nonwords involved as expected [F (2, 40) = 21.534, MS_{error} = 174,089, $p < 0.01$, $\eta^2 = 0.52$], turning in to a small dual-task benefit (460ms for change detection comprising four nonwords, -36ms for five nonwords and -188ms for six nonwords). Furthermore, this unexpected effect was mediated by the increasing complexity of the concurrent addition task [F (4, 80) = 2.205, MS_{error} = 118,555, $p = 0.076$, $\eta^2 = 0.1$]: When performed with no-carry problems, the dual-task cost first decreased and then reversed with increasing number of nonwords (260ms for four nonwords, -82ms for five nonwords and -227ms for six nonwords). A similar pattern was observed for single-carry problems (320ms for four nonwords, -7ms for five nonwords and -203ms for six nonwords). Although the same pattern was also observed for change detection performed with double-carry problems (799ms for four nonwords, -27ms for five nonwords and -135ms for six nonwords), a significantly greater dual-task cost was observed for change detection comprising four nonwords performed with double-carry problems (799ms) than with no-carry (260ms) or with single-carry problems (320ms). These results suggest that low level of load was affected by increasing calculation demand most, which is opposite to the prediction.

Figure 15 shows the mean accuracy rates and response times to the phonological change-detection task as functions of number of nonwords and digital carriers, in each of the dual-task and single-task conditions.

Figure 15. Mean accuracy rates (left column) and response times (right column) of the phonological change-detection task as a function of numbers of digital carriers and number of nonwords in each of the dual-task and single-task conditions. Error bars represent standard errors.

Discussion

The present study replicated the findings of some previous research (e.g., Imbo et al., 2007), showing that the difficulty of addition problems is increased by the number of carry operations involved. This was evident by the significant main effect of the number of carriers observed in both the accuracy and the latency analyses in the addition task, with addition problems comprising more carry operations solved less accurately and slower. As discussed in relation to Experiment 1, the number of calculation steps increases with the number of carry operations involved, resulting in slower performance and greater probability of error (Imbo et al., 2007).

The present experiment also suggests a role of PWM resources in carry operations in mental addition. The predicted task-by-carry interaction was observed in the accuracy data of the addition task, suggesting that the complexity of addition problems, defined by the number of carry operations involved, was increased by the presence of concurrent phonological loads. These findings were consistent with previous research showing evidence for a role of the phonological loop in maintaining intermediate solutions (e.g., Ashcraft & Kirk, 2001; Heathcote, 1994; Noël et al., 2001; Seitz & Schumann-Hengsterler, 2002; Trbovich & LeFevre, 2003), while contradicting the predictions of the Triple Code Model that addition mainly relies on visual processing. However, this could be attributable to a practice effect: The same addition problems were solved three times in the present experimental series (the previous two times are in the single-task condition and under visual load in Experiment 1), which could have activated more answer retrieval than calculation. Moreover, the number of double-carry problems was substantially less than the number of no-carry and single-carry problems. This could have affected the activation of working memory resources, since no-carry problems and some single-carry problems might be solved by separate fact-retrieval for the tens and the units rather than procedural strategies utilising the mental number line.

A similar pattern of results to those of Experiment 1 (in which the same addition problems were used) was also observed in the present experiment, except that the predicted task-by-carry interaction appeared in the accuracy analysis rather than in the latency analysis of the addition task. As discussed in relation to Experiment 1, this could be addressed by the view of capacity sharing. Representations of carry information and phonological nonwords compete for limited PWM resources, resulting in insufficient capacity for retaining carry information or intermediate solutions correctly, which affects accuracy rather than the response time of the calculation. With the interfering task comparable to the visual interference task used in this experiment, these results also suggest that phonological and visual resources might be required in carry operations in mental addition in the same way: to temporarily maintain the carry information or intermediate solution.

Also similar to the findings of Experiment 1, the level of phonological load did not interact with the task-by-carry interaction in either the accuracy or latency data of the addition task, suggesting that the performance cost of carriers was not further increased by increasing phonological load. Thus, the role of phonological loop in carry operations in mental addition was not further supported due to the absence of the predicted three-way interaction. These patterns of results are exactly the same as in the addition task described in Experiment 1, suggesting a similar role for phonological and visual resources in carry operations in mental addition: to temporarily maintain intermediate solutions and carry information.

No further evidence in favour of the phonological-involvement hypothesis was provided by the analysis of phonological change-detection data. Specifically, the presence of a concurrent addition task did not negatively affect the performance of the phonological change-detection task, with the dual-task cost decreasing with increasing number of nonwords. As discussed in Experiment 1, a possible explanation could be switching between PWM and VWM. It has been theorised that “horizontal … presentation creates a heavy phonological load that, for these complex problems, quickly overwhelms the capacity of the phonological loop” (p. 245, Deslauriers et al., 2008). Thus, besides the phonological load, it is likely that the horizontally-presented problems used in the present study have already imposed a heavy demand on phonological resources, resulting in an increased possibility of switching.

There is also a possibility that switching might occur for non-words as well as (or instead of) carrier information. This is because, unlike Gabor patches, phonological non-words can be relatively easily recoded. For example, individuals might visualise a non-word *fud* by visualising the letter f, u, and d. Therefore, it is possible that, when

PWM resources are insufficient to maintain representations of both calculation-related information and non-words, participants might visually recode them.

Chapter 6

Experiment 4

As reviewed in the General Introduction, the importance of phonological working memory in carry operations in complex mental multiplication is predicted by the Triple Code Model: Multiplication is often a rote verbal-memory task with facts stored in an internal phonological code, which utilizes phonological working memory resources (Cohen & Dehaene, 1996; 2000; Dehaene, 1992; Dehaene & Cohen, 1995; 1997; Lee & Kang, 2002). Such a role of phonological resources has also been found to be similar as to its role in complex mental addition. However, although the number of carry operations has been manipulated in a fairly detailed way in some previous research (e.g., Imbo & LeFevre, 2010), no study has simultaneously manipulated the level of phonological load. The present experiment was designed to replicate the involvement of phonological processing in complex multiplication, and to further examine the apparent similarity of the role of phonological resources in carry operations in mental multiplication and in mental addition. Both the number of carry operations involved in the calculating problems and level of phonological load were manipulated the same way as they were in Experiment 3.

Compared to Experiment 3, it is expected that more supportive evidence for a role of PWM in carry operations in multiplication might be observed in the present experiment. This is because a greater range of values of digital carriers and intermediate solutions are involved in multiplication problems than in addition problems, and might therefore be more resource-demanding to maintain (Imbo et al., 2007).

Method

Participants

The participants were the same as those who participated in Experiment 1.

Materials

Multiplication task

The same multiplication problems as those used in Experiment 2 were used in the present experiment.

Phonological change-detection task

The same phonological stimuli as those used in Experiment 3 were used in the present experiment.

Procedure

The same procedure as that used in Experiment 3 was used in the present experiment.

Design

The same design as that used in Experiment 3 was used in the present experiment.

Results

Multiplication Task

Accuracy Rates- No participants were excluded from the analysis. For each condition (single-task, dual-task condition), 71.6% (3,712 out of 5,184) of trials were included in the analysis.

Table 21 shows the mean accuracy rates for the multiplication task as functions of the number of non-words and digital carriers, in each of the dual-task and single-task conditions.

Table 21. Mean accuracy rates for the multiplication task as functions of the number of non-words and digital carriers, in each of the dual-task and single-task conditions.

Question types	Phonological loads	Single-task condition *M (SD)*	Dual-task condition *M (SD)*	N
No carry problems	4 non-words	0.94 (0.02)	0.94 (0.02)	48
	5 non-words	0.92 (0.03)	0.92 (0.03)	48
	6 non-words	0.90 (0.02)	0.91 (0.02)	48
Single-carry problems	4 non-words	0.76 (0.02)	0.74 (0.03)	48
	5 non-words	0.79 (0.02)	0.75 (0.03)	48
	6 non-words	0.76 (0.02)	0.75 (0.03)	48
Double-carry problems	4 non-words	0.75 (0.04)	0.69 (0.05)	48
	5 non-words	0.75 (0.04)	0.66 (0.05)	48
	6 non-words	0.73 (0.04)	0.63 (0.04)	48

As predicted, a role of phonological working memory resources in carry operations or storage of intermediate solutions of mental multiplication, supported by a load-by-carry interaction, was found [F (2, 42) = 2.706, MS_{error} = 0.05, p = 0.073, η^2 = 0.06], with a greater performance decrement observed for double-carry problems (0.08) than for single- (0.03) or no-carry problems (-0.06). These results also supported the notion that solving multiplication problems mainly relies on phonological processing. However, unlike predicted, this effect did not further interact with the level of phonological load [F <1], which is inconsistent with the prediction that a greater multiplication performance decrement caused by more number of carriers would further enlarged by increasing phonological load.

As previously found, participants were more accurate when multiplication problems were solved alone (0.81) than when they were solved under phonological load [F (1, 42) = 3.074, MS_{error} = 0.08, p = 0.087, η^2 = 0.07]. Moreover, participants were less accurate when solving more complicated multiplication problems [F (2, 84) = 90.917, MS_{error} = 0.04, p < 0.01, η^2 = 0.68], with double-carry problems (0.70) solved less accurately than single-carry problems (0.76), which were solved less accurately than no-carry problems (0.92).

However, contrary to prediction, mean accuracies of multiplication problems did not decrease with increasing phonological load level [F (2, 84) = 1.37, MS_{error} = 0.03, p = 0.26, η^2 = 0.03]. Specifically, similar mean accuracies were observed for multiplication problems solved with four nonwords (0.80), with five nonwords (0.80) and with six nonwords (0.78).

There were no other significant two-way interactions [*F*s <1].

Solution Latencies- In total, 17 participants were excluded from analysis due to no correctly responded double-carry problems. Dual-task trials with response time longer than 4,000ms were excluded from the analysis. For the dual-task condition, 43.2% (2,238/5,184) of trials were included in the analysis. For the single-task condition, 46.7% (2,422/5,184) of trials were included in the analysis.

Table 22 shows mean solution times for the multiplication task as functions of the number of non-words and digital carriers, in each of the dual-task and single-task conditions.

Table 22. Mean solution times for the multiplication task as functions of the number of non-words and digital carriers, in each of the dual-task and single-task conditions.

Question types	Phonological loads	Single-task condition *M (SD)*	Dual-task condition *M (SD)*	N
No carry problems	4 non-words	1978 (92)	2194 (111)	31
	5 non-words	1902 (86)	2335 (104)	31
	6 non-words	1936 (100)	2150 (104)	31
Single-carry problems	4 non-words	2434 (96)	2602 (112)	31
	5 non-words	2478 (99)	2599 (116)	31
	6 non-words	2531 (94)	2486 (113)	31
Double-carry problems	4 non-words	2679 (113)	2765 (142)	31
	5 non-words	2451 (121)	2646 (161)	31
	6 non-words	2699 (107)	2723 (159)	31

Conversely to the accuracy analysis, although a marginally significant load-by-carry interaction was observed [F (2, 50) = 2.936, MS_{error} = 209,880, p = 0.062, η^2 = 0.11], the performance decrement *decreased* as the number of carry operations increased, with a greater performance decrement observed for *no-carry* problems (287ms) than for double- (101ms) than for single- carry problems (74ms). This might be addressed by a speed-accuracy trade-off effect: Participants might have focused on the accuracy at the cost of response times, which is evident by a dual-task benefit (-0.06) observed for no-carry problems in the accuracy analysis. Therefore, these results support the relative involvement of phonological processing in complex mental multiplication, which is consistent with the assumption under the Triple Code Model. However, this effect was not further mediated by the increasing phonological load as predicted [F <1], which was not consistent with the prediction that performance decrement would be further enlarged by increasing load level.

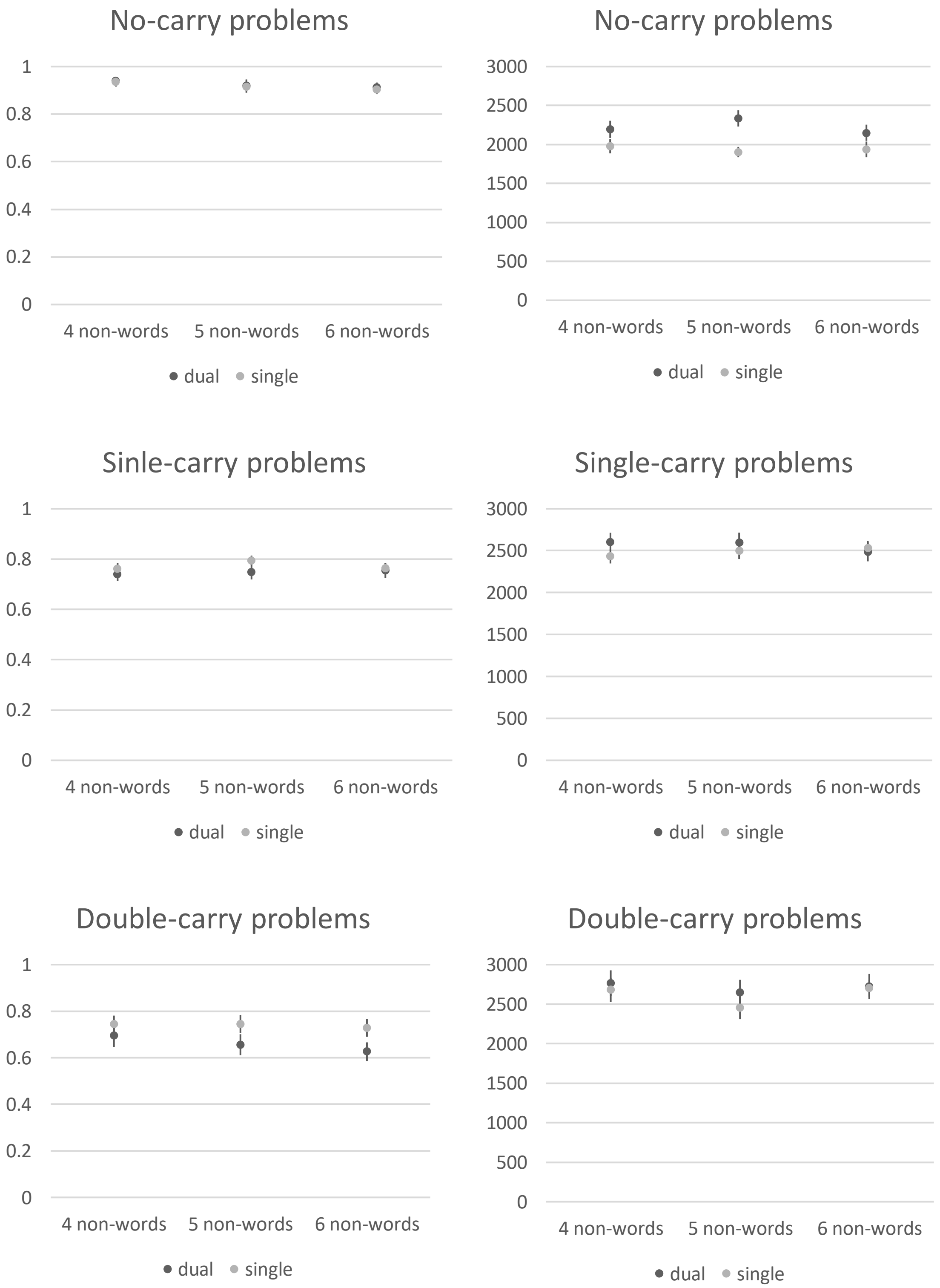
Similar to the accuracy analysis, participants took longer to solve problems with more carry operations as expected [F (2, 50) = 55.553, MS_{error} = 300,096, p < 0.01, η^2 = 0.69], with double-carry problems (2,660ms) solved slower than single-carry problems (2,526ms), which were solved slower than no-carry problems (2,083ms).

However, participants were not faster when solving multiplication problems alone (2,500ms) than when solving them under a phonological load (2,346ms) [F (1, 25) = 1.834, MS_{error} = 1776,205, p = 0.188, η^2 = 0.07]. Moreover, mean latencies of correct multiplication problems did not increase with phonological load level [F <1], with similar mean response times observed between multiplication problems solved with four nonwords (2,442ms), five nonwords (2,406ms) and six nonwords (2,421ms).

There were no other significant two-way interactions, all Fs <1.

Figure 16 shows the mean accuracy rates and response times for the multiplication task as functions of number of nonwords and numbers of digital carriers, in each of the dual-task and single-task conditions.

Figure 16. Mean accuracy rates (left column) and response times (right column) of the multiplication task as functions of number of nonwords and numbers of digital carriers in each of the dual-task and single-task conditions. Error bars represent standard errors.



Phonological Change-detection Task

Accuracy Rates- In total, four participants were excluded from the analysis due to no accurate double-carry problems. For each condition (single-task, dual-task condition), 71.1% (3688 out of 5184) of trials were included in the analysis.

Table 23 shows the mean accuracy rates of the phonological change-detection task as functions of the number of non-words and digital carriers in each of the dual-task and single-task conditions.

Table 23. Mean accuracy rates of the phonological change-detection task as functions of the number of non-words and digital carriers in each of the dual-task and single-task conditions.

Question types	Phonological loads	Single-task condition *M (SD)*	Dual-task condition *M (SD)*	N
No carry problems	4 nonwords	0.82 (0.02)	0.60 (0.02)	44
	5 nonwords	0.84 (0.02)	0.83 (0.03)	44
	6 nonwords	0.84 (0.02)	0.75 (0.02)	44
Single-carry problems	4 nonwords	0.81 (0.02)	0.66 (0.02)	44
	5 nonwords	0.83 (0.02)	0.75 (0.02)	44
	6 nonwords	0.83 (0.02)	0.77 (0.02)	44
Double-carry problems	4 nonwords	0.79 (0.03)	0.59 (0.03)	44
	5 nonwords	0.85 (0.02)	0.72 (0.04)	44
	6 nonwords	0.86 (0.02)	0.70 (0.03)	44

As expected, participants were more accurate when change detection was performed alone (0.83) than when it was performed with a concurrent multiplication task (0.71) [F (1, 38) = 62.010, MS_{error} = 0.05, p < 0.01, η^2 = 0.62].

However, contrary to the prediction, change detection accuracy *increased* with the number of nonwords involved [F (2, 76) = 33.780, MS_{error} = 0.02, p < 0.01, η^2 = 0.47], with change detection comprising four nonwords (0.71) performed less accurately than those comprising five nonwords or six nonwords (both 0.80). Moreover, this effect was enlarged by the presence of a concurrent multiplication task [F (2, 76) = 6.692, MS_{error} = 0.02, p = 0.002, η^2 = 0.15]. Further analysis found a *greater* dual-task cost for change detection with four nonwords (0.17) than with five nonwords (0.08) or six nonwords (0.11). This unexpected effect was further mediated by increasing number of carriers involved in the concurrent multiplication task [F (4, 152) = 3.012, MS_{error} = 0.02, p = 0.02, η^2 = 0.07]. Further analysis showed that when performed with no-carry problems, a greater dual-task cost was observed for change detection with four nonwords (0.22) than five (0.02) or six nonwords (0.09). A similar pattern was observed for change detection performed with single-carry problems (0.14 for four nonwords, 0.08 for five nonwords and 0.07 for six nonwords). When performed with double-carry problems, the dual-task effect did not differ significantly with increasing numbers of non-words (0.16 for four nonwords, 0.15 for five nonwords and 0.18 for six nonwords). These results were opposite to the prediction that increasing phonological load would affect the dual-task performance decrement differently for no-carry, single-carry and double-carry problems.

Consistent with the prediction, participants were more accurate on change detection performed with easier multiplication problems [F (2, 76) = 3.055, MS_{error} = 0.03, p = 0.053, η^2 = 0.07]. Specifically, change detection was performed more accurately with no-carry problems and single-carry problems (both 0.78) than with double-carry problems (0.75). Moreover, change detection performance decrement also increased with the complexity of the concurrent multiplication task as expected [F (2, 76) = 2.94, MS_{error} = 0.03, p = 0.059, η^2 = 0.07], with a greater dual-task cost observed for change detection performed with double-carry problems (0.16) than with single-carry problems (0.10) or no-carry problems (0.11).

Solution Latencies- In total, 12 participants were excluded from the analysis due to no correctly responded double-carry problems or change detection in the dual-task condition. For the single-task condition, 47.7% (2,474/5,184) of trials were included in the analysis. For the dual-task condition, 50.6% (2,621/5,184) of trials were included in the analysis.

Table 24 shows mean solution times rates of the phonological change-detection task as functions of the number of non-words and digital carriers in each of the dual-task and single-task conditions.

Table 24. Mean solution times rates of the phonological change-detection task as functions of the number of non-words and digital carriers in each of the dual-task and single-task conditions.

Question types	Phonological loads	Single-task condition *M (SD)*	Dual-task condition *M (SD)*	N
No carry problems	4 non-words	1734 (77)	1887 (128)	36
	5 non-words	1848 (76)	1813 (105)	36
	6 non-words	2059 (74)	1889 (92)	36
Single-carry problems	4 non-words	1689 (67)	1888 (92)	36
	5 non-words	1865 (72)	1812 (83)	36
	6 non-words	2091 (78)	1868 (83)	36
Double-carry problems	4 non-words	1739 (82)	1858 (112)	36
	5 non-words	1777 (87)	1778 (113)	36
	6 non-words	2058 (78)	1967 (115)	36

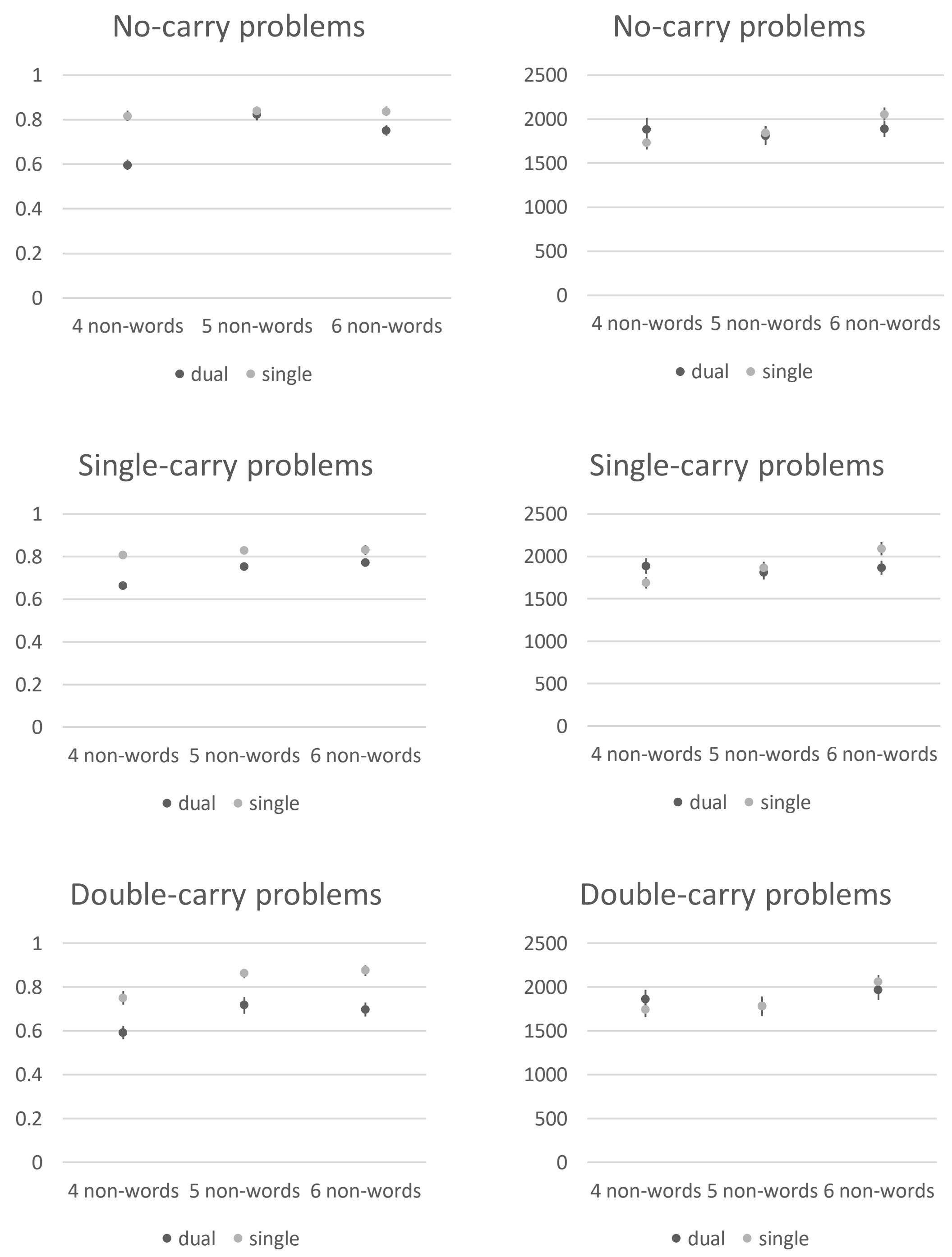
Unlike the accuracy analysis, participants did not take longer to perform change detection presented alone (1,873ms) than with a concurrent multiplication task (1,851ms) [F <1]. Although latencies on correct change detection increased with the number of nonwords involved as expected [F (2, 60) = 8.202, MS_{error} = 233,640, p = 0.01, η^2 = 0.22] (1,799ms for four nonwords, 1,816ms for five nonwords and 1,972ms for six nonwords), this effect of number of nonwords was not further enlarged by the presence of a concurrent multiplication task [F (2, 60) = 8.472, MS_{error} = 191,856, p = 0.02, η^2 = 0.22]: Change detection performance decrement *decreased* with increasing number of nonwords involved, with a greater performance difference between dual- and single-task conditions found for correct change detection with four-nonwords (157ms) than with five-nonwords (-29ms) or six-nonwords (-194ms). These results were inconsistent with the prediction.

Contrary to the accuracy analysis, participants did not perform change detection less rapidly with increasing multiplication demands [F <1]: Mean response times did not differ between correct change detection performed with no-carry problems (1,855ms), single-carry problems (1,869ms) and double-carry problems (1,863ms). Moreover, there was no predicted three-way interaction [F <1], which was inconsistent with the prediction that more change-detection performance decrement will be associated with increasing multiplication complexity.

There were no other significant two-way interactions, all Fs <1.

Figure 17 shows the mean accuracy rates and response times for the phonological change-detection task as functions of number of nonwords and numbers of digital carriers involved in the concurrent multiplication task, in each of the dual-task and single-task conditions.

Figure 17. Mean accuracy rates (left column) and response times (right column) of the phonological change-detection task as functions of numbers of digital carriers and number of nonwords in each of the dual-task and single-task conditions. Error bars represent standard errors.



Discussion

The results of this experiment provide support for a role of phonological representations in carry operations in mental multiplication. Although only marginally significant, the predicted interaction between number of carriers and task conditions was observed in the accuracy data of the multiplication task: A greater performance decrement in the dual-task condition was observed for double-carry problems than for no- or single-carry problems. In line with previous research (e.g., Seitz & Schumann-Hengsteler, 2000, 2002, Trbovich & LeFerve, 2003), these results suggest that phonological representations are required for carry operations in mental multiplication. This is also consistent with the prediction of the Triple Code Model that multiplication relies heavily on verbal processing.

The present results also suggest that the mechanism of the relative involvement of phonological resources in carry operations in complex mental multiplication is similar to that in mental addition: When using exactly the same experimental paradigm and phonological change-detection task, similar patterns of results were observed in the multiplication task in present experiment and in Experiment 3, in which addition problems were used.

The present results were inconsistent with those of Experiment 2, in which the predicted task-by-carry interaction was only observed in the response time data when the same multiplication problems were solved under visual interference. It could be argued that these results might be attributable to different mechanisms being associated with processing and maintaining visual and phonological representations in working memory: Maintaining Gabor arrays might be less resource-demanding than maintaining non-words. Unlike the Gabor arrays, the non-words were sequentially presented, which might take more effort to maintain due to the longer total presentation time and the interference between non-word representations. However, if this were the case, then more predicted task-by-carry interactions should have been observed for addition problems solved under phonological interference (Experiment 3) than for those solved under visual interference (Experiment 1).

One possible explanation for the observation of the predicted task-by-carry interaction in the accuracy analysis of the multiplication task in the present experiment might be that phonological representations are more involved in carry operations in mental multiplication than visual representations. This is consistent with the notion that multiplication is associated with phonological representations (Lee & Kang, 2002). The difference between the current findings and those of Experiment 2 provides support to this notion: In Experiment 2, in which multiplication problems were solved with a concurrent visual change-detection task, the presence or absence of the visual load only affected response times rather than accuracy. However, the predicted task-by-carry interaction was observed in both the accuracy and the latency analysis of the multiplication task in the present experiment, suggesting that, compared to visual resources, phonological resources are much more extensively required in carry operations in complex mental multiplication.

Inconsistent with the prediction, the present results did not provide stronger evidence for a role of phonological representations in mental multiplication than in addition (Experiment 3, in which addition problems solved under phonological load). The only predicted load-by-carry interaction observed in the accuracy data of the multiplication task was only marginally significant. Moreover, the predicted three-way interaction was not observed in either task, which is the same in Experiment 3. Although a significant three-way interaction was observed in the accuracy analysis of the phonological change-detection task, the dual-task performance decrement did not increase with increasing number of nonwords as predicted.

One possible explanation could be that more switching occurred in the present experiment than in Experiment 3: When less phonological resources are available for maintaining carrier information due to increasing phonological interference, more participants might have switched to visually-representing and maintaining carrier information when solving multiplication problems than when solving addition problems. Thus, it is likely that any potential for switching might be enhanced by the heavier demands on working memory are imposed when carrier information have greater values, as is typically the case for multiplication.

Chapter 7

General Discussion

Solving mental arithmetic problems has been theorised to rely heavily on working memory resources (Daneman & Carpenter, 1980; Ericsson & Kintsch, 1995), especially for problems involving carry operations (Hitch, 1978; Logie et al., 1994; Seitz & Schumann-Hengsteler, 2000, 2002). Although the cognitive mechanisms underpinning mental arithmetic performance have been investigated extensively in the past decade, discrepancies in the literature regarding the relative involvement of visual and phonological working memory resources in carry operations are evident. Moreover, the role of working memory resources in carry operations in mental arithmetic in terms of the interaction between working memory load and question complexity (defined by number of carriers in the present study) remains unclear due to the absence of the manipulation of working memory load levels in previous literature. The present research aimed to achieve better understanding of the role of working memory in carry operations in complex mental addition and multiplication in the context of the therotical framework of the Triple Code Model (Dehaene, 1992; Dehaene & Cohen, 1997).

Two research questions guided the present research. The first pertains to the relative involvement of visual and phonological representations in carry operations and/or the storage of intermediate solutions in complex mental addition and multiplication. Some previous studies have suggested that visual and phonological working memory resources are involved in the calculation process of mental addition (Hayes, 1973; Heathcote, 1994; Hitch, 1978; Hubber et al., 2014; Trbovich & LeFevre, 2003) and in mental multiplication (Imbo & LeFevre, 2010; Imbo et al., 2007), while others have not (Logie et al., 1994; Noël et al., 2001; Otsuka & Osaka, 2014; Seitz & Schumann-Hengsteler, 2000, 2002). As discussed in General Introduction, this inconsistency between studies might be caused by any or all of variations in experimental paradigms, working memory stimuli and approaches to data analysis. Moreover, no previous study has manipulated the level of working memory load simultaneously with the number of carry operations, so it is not clear that how working memory load can affect the involvement of phonological or visual resources in carry operations. The present study addressed this lack of clarity in the literature by implementing this manipulation in a dual-task interference paradigm comprising change-detection tasks and mental arithemic tasks.

The second question concerned whether visual and phonological working memory resources are differentially involved in complex mental addition and multiplication. According to the Triple Code Model (Dehaene, 1992; Dehaene & Cohen, 1997), addition is more reliant on visual processing, while multiplication is more reliant on verbal processing. Moreover, compared to simple arithmetic, additional activation of a theoretical magnitude number line is postulated to be involved in solving complex arithmetic problems regardless of operation. Few neuro-imaging studies have included complex addition or multiplication problems and although some previous behavioural studies have compared the involvement of visual and phonological representations in complex mental addition (e.g., Seitz & Schuman-Hengsteler, 2000; Trbovich & LeFevre, 2003) and in complex mental multiplication (e.g., Imbo & LeFevre, 2010; Seitz & Schuman-Hengsteler, 2002), interpretation of their findings is limited by the incomparability of visual and phonological interference tasks employed in these studies. The present study addressed this issue by using a change-detection paradigm for all experiments, which can be effectively used to implement both visual and phonological interference. It was hypothesied that visual processing would be observed for complex addition, while both visual and verbal processing would be observed for complex multiplication.

Working Memory and Complex Mental Arithmetic

Roles of both phonological and visual representations for carry operations in multi-digit mental addition were demonstrated in the present research. Significant task-by-carry interactions showing greater dual-task costs for carry-problems than for no-carry problems were observed in the accuracy analysis of addition tasks in Experiments 1 and 3, in which addition problems were solved under visual and phonological interference. A role for phonological representations in carry operations in mental multiplication was also demonstrated in the present research, evidenced by marginally significant load-by-carry interactions in both the accuracy and latency analysis of the multiplication task of Experiment 4. Only a limited role for visual representations in carry operations in mental multiplication was found; a task-by-carry interaction was observed only in the latency analysis, but not in the accuracy analysis, in the multiplication task of Experiment 2.

Evidence supporting a role of visual representations in carry operations for mental addition was observed in Experiment 1. Specifically, the effect of the number of carry operations involved on the accuracy of addition performance was increased by a concurrent visual load, suggesting that visual representations are involved in the temporary storage of intermediate solutions or carry information in mental addition. These results are consistent with some previous research (e.g., Hayes, 1973; Heathcote, 1994; Hitch, 1978; Trbovich & LeFevre, 2003), but contradicted other studies (e.g., Hubber et al., 2014; Otsuka & Osaka, 2014). As reviewed in General Introduction, the apparent discrepancy between previous studies and the present study might be reconciled by consideration of the selection of visual interference tasks. The use of a visual change-detection task in the present study might have revealed the importance of visual processing in carry operations in mental addition. Unlike the visuo-spatial tapping task employed by Otsuka and Osaka (2014), the concurrent visual change-detection task used in the present study required active storage of visual representations in visual working memory while addition problems were solved, which is likely to have put more load on visual working memory.

It could also be argued that the continuous presentation of entire equations in the present study might have further encouraged reliance on visual working memory. However, it should be noted that the focus of the present research is the role of visual representations in *carry operations and/or intermediate solutions* of mental arithmetic rather than the role of visual representations in mental arithmetic more generally. Thus, what was compared in the present study was the *performance difference* between dual- and single-task conditions. Although the encoding of operands might also require working memory resources, with exactly the same presentation format and duration used in both dual- and single-task conditions, any impact of the continuous visual presentation of addends on the dual-task performance costs should have been eliminated.

The results of the present study support an important role for phonological representations in carry operations or intermediate solutions in mental addition and multiplication. Significant load-by-carry interactions were observed in the accuracy analyses of the addition task of Experiment 3 and in the multiplication task of Experiment 4, with the effect of the number of carry operations on arithmetic performance increased by a concurrent phonological load. This importance of phonological processing in complex mental addition and multiplication is consistent with the findings of Imbo and LeFevre (2010) and Seitz and Schumann-Hengsteler (2000; 2002), but contrast with the conclusions of Imbo and Vandierendonck (2007) and Imbo et al., (2007). Again, the discrepancy may be resolved by a comparison of methodologies used. For example, in the present study, participants were instructed to report the final results orally rather than to type solutions of each column of the addition or multiplication problem as in the study of Imbo and Vandierendonck (2007). The former could have increased the cognitive demand associated with the storage of intermediate solutions in working memory. Conversely, it could be argued that the oral method of responding might have forced participants to use a phonological form of representation, and could have biased them away from using visual representations. However, as argued above, the impact of verbal reporting on the relative involvement of visual or phonological working memory resources in the problem-solving process was eliminated by analysing *differences* between the dual-task and control conditions, in each of which exactly the same presentation modality and report format were used.

Only limited evidence for a role of visual representations in mediating carry operations or intermediate solutions was found for mental multiplication; a significant load-by-carry interaction was observed only in the latency analysis of the multiplication task in Experiment 2. It could be argued that the absence of such interaction in the accuracy analysis could be explained in terms of a speed-accuracy trade-off: As multiplication is theorised to be more difficult than addition (Ashcraft, 1992; Ashcraft & Battaglia, 1978; Campbell, 1995; Geary, 1996; Groen & Parkman, 1972). Participants might have focused on achieving correct products at the cost of speed. However, similar performance was not observed when they solved exactly the same problems under phonological load (Experiment 4), with a significant load-by-carry interaction observed in the accuracy analysis. Thus, unlike phonological load, visual load only has a small impact on complex mental multiplication.

The absence of greater evidence for visual interference effects on multiplication contrasts with the findings of Imbo and LeFevre (2010), in which both phonological and visual resources were found to be required to solve complex mental multiplication problems. It should be noted that however, their conclusions were based on an interaction between presentation format (vertical vs. horizontal) and working memory load rather than on an interaction between problem complexity (defined by the number of digital carriers involved) and working memory load. Thus, it is unclear whether the relative involvement of visual resources in mental multiplication found in their study apparently facilitated by vertical presentation is associated with carry operations or with some other aspect of the process, such as encoding.

It could be argued that horizontal presentation of the arithmetic problems might have encouraged activation of phonological processing, as addition performance was more impaired by phonological load than by visual load when problems were horizontally presented (Trbovich & LeFevre, 2003). However, if this was the case, this argument cannot easily account for the results of Experiment 1, which showed evidence for the involvement of visual working memory in solving horizontally-presented addition problems. Another explanation of the absence of obvious involvement of visual resources in carry operations in mental multiplication could be that verbal report of solutions biased participants away from using visual resources. As discussed above, this possibility can largely be discounted on the basis that verbal report was used in both dual-task and control conditions and only *differences* between the two conditions were taken as evidence for phonological or visual involvement. Therefore, it might be concluded that, compared to phonological resources, visual resources only play a small role in carry operations in mental multiplication.

Another possible explanation for the apparently limited involvement of visual resources in complex multiplication could be that adult participants use alternative working memory resources available for temporary storage of carry information when visual resources were heavily loaded (Hubber et al., 2014). That is to say, although participants may sometime use visual resources in maintaining numerical information, verbal storage might still available as an alternative. So when visual resources are not available (e.g., because they are heavily loaded by another task), adult participants might have chosen to use phonological resources instead. This raises the possibility of switching for the temporary maintenance of carry information or intermediate solutions between visual and phonological working memory. Evidence supporting this hypothesis would be a *reduction* in the performance decrement associated with increasing numbers of Gabor patches in the dual-task condition: Additional loading on visual working memory should not interfere with representations of digital carriers or intermediate solutions that have been phonologically recoded, and should, in fact, release visual working memory resources occupied at lower levels of load (for detailed performance pattern supporting the switching hypothesis, see Appendix C).

In conclusion, through manipulating both numbers of carriers and levels of working memory load, the present study produced findings consistent with what was found in previous studies: A working memory load adversely affected arithmetic performance, especially in accuracy, and this effect was augmented in multi-carry problems (Fürst & Hitch, 1994; Imbo et al., 2007). This interaction between working memory load and number-of-carriers can be explained in terms of competition for limited working memory resources: When there is a carry operation to be performed, its value needs to be briefly maintained in working memory. Values of the carry information or intermediate solutions might not be maintained correctly due to insufficient working memory resources available, caused by a working memory load associated with the simultaneous change-detection tasks. This interference in dual-task conditions will affect accuracy more than speed of arithmetic performance, probably because incorrectly maintained carry information leads to wrong solutions but not necessarily to longer response time, so that the load-by-carry interaction observed in accuracy analysis was more reliable. Nonetheless, it is likely that strong involvement of working memory resources will be reflected in both the accuracy and response time data.

It could be argued that the arithmetic performance decrements observed in the present study might be associated with the numerical magnitudes of solutions rather than maintaining digital carriers or intermediate solutions in working memory: The solutions of arithmetic problems with carry operations are greater in magnitude than those of no-carry problems. Greater magnitude solutions have been shown to result in longer latencies and higher error rates, which is commonly referred as the problem-size effect (see Ashcraft, 1995; Campbell & Xue, 2001 for a review). However, it should be noted that, while the problem-size effect is robust and amongst the most common findings in simple mental arithmetic research, it is not found in complex mental arithmetic involving double-digit operands (e.g., Deschuyteneer, De rammaelaere & Fias, 2005). In the study of Deschuyteneer et al., (2005), they found that double-digit addition problems with correct sums comprising a larger tens and a smaller unit were solved equally fast as problems with correct sums comprising a smaller tens and a larger unit. For example, 23 + 61 = 84 and 32 + 16 = 48 were solved equally fast because they both involved the same retrieval of arithmetic facts 2 + 6 = 8 and 3 + 1 = 4. Thus, the possibility that the problem-size effect would be a confounder in the present study, in which only double-digit arithmetic problems were used can be ruled out. Deschuyteneer et al., (2005) also found that carry problems (e.g., 39 + 48 =) were solved slower and less accurately than no-carry problems (e.g., 34 + 53 =) even when they had the same numerical size of sums (87). These results are consistent with the notion that errors rise more frequently in mental arithmetic when a carry operation is involved (e.g., Fürst & Hitch, 2000), suggesting that for complex mental arithmetic, the greater performance decrement for carry problems than for no-carry problems observed in the present study is probably attributable to the extra step in the problem-solving process (manipulating the carry operation or intermediate solutions in working memory), rather than to the larger numerical sizes of the solutions.

Although the level of working memory load was manipulated, task-by-carry interactions were not further modulated by the level of working memory load, which contradicts the prediction that the effect of number of carry operations on arithmetic performance would be further increased with working memory load. Specifically, no significant three-way interactions between numbers of carriers, levels of load and task conditions were observed in either the accuracy or latency analysis of the arithmetic tasks in Experiment 1, 3, or 4. Although there was a significant three-way interaction in the latency analysis of the multiplication task in Experiment 2, performance of double-carry problems was more impaired under *lower* levels of visual load. Regardless of whether arithmetic problems were solved under visual or phonological load, the results described above showed a similar pattern: The arithmetic performance impairment caused by the increasing number of carriers was not further increased by increasing working memory load. This is difficult to address in terms of a speed-accuracy trade-off or a trade-off between tasks. However, switching between visual and phonological working memory components could potentially explain this phenomenon. For example, it is possible that participants switched to phonological working memory to maintain the representations of the carriers or intermediate solutions, rather than continuing to use visual working memory when it was heavily loaded by the visual change-detection task. Thus, additional loading on visual working memory should not interfere with representations of digital carriers or intermediate solutions that have been phonologically recoded, and should, in fact, release visual working memory resources occupied at lower levels of load. Future study should investigate the possibility of this hypothesis in the domain of mental arithmetic.

Operation-specific Involvement of Working Memory Resources

Taking the findings from Experiments 1 and 3 together, there is evidence supporting a role for both phonological and visual working memory in carry operations in mental addition. These findings did not support the notion that the storage of carriers or intermediate solution information for complex addition is mainly associated with visual representations (Dehaene, 1992; Dehaene & Cohen, 1997; Imbo & LeFevre, 2010), which was postulated under the Triple Code Model proposed by Dehaene (1992). Under this model, three codes are theorised to be differentially involved in numerical representation and processing: visual Arabic codes, auditory-verbal codes and analogue magnitude codes (Dehaene, 1996, 1997). Simple addition is theorised to rely on fact-retrieval that activates verbal code only, while complex addition and subtraction are theorised to rely on the activation of an internal magnitude line, which consequently engages visuo-spatial working memory resources (Dehaene, 1992, 1997; Dehaene & Cohen, 1995, 1997; Imbo & LeFevre, 2010). Thus this model predicts that complex addition performance should only suffer under a concurrent visual load, and not under a phonological load.

Nonetheless, evidence supporting the involvement of both visual and phonological resources in carry operations in mental addition has been found in the present research. Moreover, these findings suggest a similar mechanism for the involvement of both visual and phonological resources in carry operations in mental addition. Similar patterns of evidence regarding the relative involvement of visual and phonological representations in carry operations in mental addition was observed in the calculation tasks of Experiments 1 and 3, in which the same dual-task interference paradigm was used. Specifically, a significant load-by-carry interaction was observed in the accuracy but not the latency analysis of the addition task in both experiments. As discussed in relation to Experiment 2, the successful storage of intermediate solutions or digital carriers is expected to be primarily reflected in calculation accuracy rather than response time. These results therefore suggest that both visual and phonological working memory resources mediate the temporary maintenance of intermediate solutions and carry information in complex mental addition, contradicting the idea that complex addition is solved with the aid of magnitude number line mainly utilising visual processing (Dehaene, 1992; Dehaene & Cohen, 1997). Thus, present findings are inconsistent with operation-specific involvement of cognitive pathways in mental arithmetic.

Furthermore, these findings challenge the strict distinction between fact-retrieval and mental magnitude line proposed by the Triple Code Model, but rather provide support for recent models of arithmetic fact retrieval, which assume a link between visually-mediated mental magnitude representations and verbally-mediated arithmetic facts (e.g., The semantic/symbolic model of Stonianov, Zorzi & Umilta, 2004; Zorzi Stoianova & Umilta, 2005). Similarly, an interaction between fact retrieval and numerical magnitude processing has been proposed by Klein et al., (2014). Specifically, Klein et al., (2014) re-analysed data sets from several functional neuro-imaging studies, and proposed that visually-mediated magnitude processing and verbally-related fact retrieval operate as a functionally integrated circuit in numerical cognition.

Another challenge to the present work might be argument that the observation of phonological working memory resources in complex addition could be due to the practice effect. As introduced in Experiment 1, the same addition problems were solved three times in a fixed order of experimental conditions: alone, under visual load, and under phonological load. Thus, participants might have become familiar with some of the questions, even without realising it consciously. This familiarity might have encouraged more retrieval of answers memorised from the previous session rather than calculation, which has been theorised to activate phonological processing (Delazer et al., 2003).

However, if this has occurred, then the number of participants and percentage of trials included in change detection tasks should increase with the progress of experiment sessions, with more participants and trials should be selected in the phonological change-detection task than in the visual change-detection task performed with addition. This is because change detection trials were selected based on whether or not the concurrent addition problems were solved correctly, thus, any increased accuracy of addition performance due to practice effect should have been reflected in greater numbers of participants and trials being selected for change detection tasks in later sessions. However, this was not the case: Both the number of participants and percentage of trials included in the change detection accuracy analyses *decreased* slightly with the progress of the experiment (61.7% at the second presentation compared with 58.4% at the third presentation). Thus, the possibility that the observed role of phonological resources in carry operations or intermediate solutions in mental addition in the present study was due to practice effect can be ruled out. Rather, it is because carry operations or intermediate solutions were phonologically represented during the problem-solving process, which is inconsistent with the operation-specific involvement notion proposed by the Triple Code Model.

There was evidence for a greater involvement of phonological representations (Experiment 4) than visual representations in carry operations in mental multiplication (Experiment 2). In both of these experiments, the same multiplication problems were solved under concurrent phonological and visual loads respectively. Marginally significant task-by-carry interactions were observed in the accuracy analysis under phonological load, while such an interaction was only found in the latency analysis under visual interference. These results indicate that, unlike phonological resources, visual resources are not strongly associated with the temporary storage of intermediate solutions or carry information in complex mental multiplication.

The present findings on the relative involvement of working memory resources in complex multiplication are partially aligned with the predictions of the Triple Code Model: Although multiplication is assumed to mainly rely on fact-retrieval (activates verbal processing), additional employment of magnitude number line (activates visual processing) might also be required due to increased problem difficulty (Dehaene, 1992; Dehaene & Cohen, 1997). This is because multiplication is often theorised to be a rote verbal-memory task with facts stored in an internal phonological code, which utilizes phonological working memory resources (Cohen & Dehaene, 1996; 2000; Dehaene, 1992; Dehaene & Cohen, 1995; 1997; Lee & Kang, 2002). As discussed in Experiment 3, although solving complex mental multiplication problems has been theorised to require the interaction of fact-retrieval and procedural processing (e.g., guides the sequential execution of calculation algorithms or computational strategies, Delazer, 2003), it should be noted that the primary strategies for complex multiplication might be very different from those for complex addition (Rosenberg-Lee et al., 2011). For example, complex multiplication (e.g., 28 × 4) might be solved through decomposing the problem into simpler problems (e.g., 20 × 4 + 8 × 4) rather than through other procedural strategies normally used for complex addition (e.g., counting, 28 + 28 + 28 + 28). Therefore, retrieval might be used more consistently for complex multiplication than for complex addition, resulting in an absence of visual processing in complex multiplication.

However, practice effects might also have contributed to the apparent absence of visual resources in complex multiplication. In the present study, same multiplication problems were solved three times, alone, under visual load and under phonological load. Although tasks involving multiplication problems were never repeated in the same session, it is likely that the performance of participants (at least some) might have been affected by the repeated presentation of the same problems across sessions, resulting in shorter response latencies due to more automatised rather than quantity-based calculation compared to the first time. Under the Triple Code Model, this would result in a greater phonological processing than visual processing (Delazer et al., 2003), and potentially an absence of involvement of visual resources in the multiplication task. In one neuro-imaging study, Delazer et al., (2003) compared brain activation patterns between familiar and novel complex multiplication problems (two digit multiplies by one digit). Compared with novel problems, solving familiar complex multiplication problems resulted in more activation in the left intraparietal sulcus and the left sylvian fissure, which have been associated with verbal processing. Thus, they concluded that these findings were reflections of the behavioural changes from step-by-step procedures to faster and more automatized processing that are supported by language-relevant areas. Thus, the repetition of multiplication problems in the present study might have affected the observed involvement of phonological and visual resources. This possibility is further discussed in the Limitations section.

In conclusion, the present findings show that, besides fact-retrieval activating verbal processing, solving complex mental multiplication did not additionally require procedural number manipulation strategies or numerical quantity processing involving visual processing, and complex addition are not exclusively performed via the magnitude-related visual route. Although these results contradict the assumptions proposed by the Triple Code Model about complex arithmetic problems, differential involvement of visual and phonological working memory resources between addition and multiplication was observed.

The present findings could possibly be addressed in terms of differences in strategy selection: Both addition and multiplication have been theorised to rely most heavily on the retrieval of well-learned arithmetic facts in adults (Campbell, 2008; Campbell & Xue, 2001). However, complex arithmetic problems not only involve fact-retrieval, but also procedural processes. Thus, it is likely that the extent to which each of fact retrieval and procedural strategies are involved in arithmetic might vary across arithmetic operations (Dehaene & Cohen, 1997). Retrieval might still be used consistently for complex multiplication because alternative computational strategies are less efficient than those for complex addition (Rosenberg-Lee et al., 2011).

There were many strengths of the design of the present study. First, I used within-subject design of experiments that requires all participants to complete all tasks, so that the effect of working memory load on arithmetic performance can be compared between addition and multiplication. Moreover, the possibility of results being affected by variances associated with individual differences (e.g., arithmetic proficiency, working memory capacity) can be ruled out. Furthermore, the within-subject design increased the statistical power compared with a between-group design due to increased sample size (e.g., 24 participants completed addition tasks and the other 24 participants completed multiplication tasks), as all 48 participants completed all of the tasks. However, such a design also suffers carry-over effects, such as the practice effect and fatigue. Although some precautions have been taken (e.g., tasks involving the same arithmetic problems or visual/phonological stimuli were never performed in one session), the practice effect might still have affected the present results. This is further discussed in the Limitations section.

Second, I used comparable visual and phonological change detection tasks as secondary interference tasks, so that whether visual and phonological resources are involved in carry operations or the storage of intermediate solutions in mental arithmetic in the same way or not can be tested. Moreover, change detection tasks are very likely to interfere with representations of digital carriers or intermediate solutions stored in working memory (either visually or phonologically), because the change-detection paradigm requires participants to maintain visual or phonological stimuli throughout the whole calculation process.

Third, the combination of mental arithmetic problems and visual or phonological stimuli in dual-task trials was counterbalanced between participants as different versions, so the findings of the present study are not affected by variance in the complexity of arithmetic problems or visual/phonological loads (e.g., relatively difficult problems always presented under higher levels of load). In particular, 36 randomly selected no-carry, single-carry and double-carry problems were allocated to each level of load (low, medium and high). These varied across different versions of experiments. In total, there were six versions of experiments for each of addition and multiplication solved under phonological and visual load (see Appendix D as an example). Participants were randomly allocated to different versions of experiments, with six participants in each version. Although used as a between subject variable in the analyses, results related to versions were not reported as it was not the focus of the present study.

Fourth, all participants were required to complete a short survey about mathematics anxiety prior to the experiment. Participants showing a high level of mathematics anxiety did not proceed to the experiment. Thus, it is unlikely that mathematic anxiety might have acted as confounder in the present results, as participants' (48 included in the final analysis) responses to the sMARS (short Mathematics Anxiety Rating Scale; Alexander & Martray, 1989) showed that they were only slightly anxious about settings or experience related to mathematics (e.g., a math test).

Limitations

Despite the strengths, there are several practical limitations and design choices that have affected the present study, one of them being substantial variance in the number of no-, single- and double-carry problems. I used only 14 double-carry addition problems and 18 double-carry multiplication problems in the present research, which is only a small fraction of the total number of arithmetic problems used. This limited the statistical power for detecting performance decrement differences between different types of problems. Moreover, the small number of double-carry problems also resulted in fewer trials being involved in the analysis than trials comprising no-carry and single-carry problems, due to the trial selection rules used in the present study. Since participants have shown higher error rates related to double-carry problems, and only trials with correct responded arithmetic problems were selected for analysis, the statistical power was further weakened in the analyses, especially in latency analysis. Equal numbers of no-carry, single-carry and double-carry problems would be used if the same study was conducted again.

A second limitation was that the same arithmetic problems and secondary tasks were repeatedly used due to limited available resources (I had no access to an established laboratory or a participant pool as my campus was not based on the main campus and participants had to be paid and driven to my campus to attend my experiment). Practice effect might therefore have affected arithmetic performance; the same 108 addition problems and multiplication problems were solved by the same participants alone as well as under both phonological and visual load due to restricted available resources. Although the same questions were never presented in the same session, the interval between sessions involving the same problems was not the same for all participants. Thus, it is likely that some participants might have become more familiar with questions as the experiment progress. However, this is only true for multiplication problems, reflected by the increased number of participants and trials included in the change detection accuracy analysis with the progress of experiments, as change detection trials were selected when concurrent multiplication problems were correctly solved (see Table 25).

Table 25. Percentage of trials and number of participants included in change detection accuracy analyses

Task order	Task Name	Percentage of trials included	Number of participants included
1	Visual change detection with addition	61.7%	37
2	Phonological change detection with multiplication	71.1%	44
3	Phonological change detection with addition	58.4%	30
4	Visual change detection with multiplication	78.4%	47

With respect to the interpretation of results of the present study, this practice effect might have encouraged the involvement of phonological resources in complex mental multiplication, as participants became more automatised (Delazer et al., 2003). Thus, although multiplication performance was only slightly improved, I cannot completely rule out the possibility that the absence of a role of visual resources in multiplication accuracy analysis (Experiment 2) is due to the practice effect. If the same experiment was conducted again, with sufficient resources, either different participants or different problems would be used in different experiments to avoid any practice effects.

Third, the substantial data loss caused by the strict trial selection criteria might also have affected the statistical power of the latency analyses of the present results. The strict trial selection rules used in the present study resulted in the elimination almost half of the trials in latency analyses. Excluding this data may have limited the power of statistical analysis of the response time data, resulting in the absence of significant load-by-carry interactions to further support a role of working memory resources in mental arithmetic in Experiments 1, 3 and 4. More participants should be recruited to ensure sufficient trials required to produce stable and reliable latency analyses if the same trial selection criteria were used again.

Fourth, arithmetic operations were not compared directly in the present study. This was because of the strict trial selection criteria mentioned above. For example, if two operations were compared in the same latency analysis, then only trials with both addition and multiplication problems as well as visual and phonological change-detection correctly performed would be selected. This would cause even more loss of statistical power and affect the reliability, which is why such analysis was not conducted in the present research. Although the current results of different experiments can be compared due to the within-subject design used (the same participants completed all tasks), future studies could directly compare addition and multiplication in the same analysis as a within-subject factor (e.g., 2 (addition, multiplication) × 2 (single-, dual-task condition) × 3 (low, medium, high load) × 3 (no, single, double carriers)).

Finally, arithmetic strategy selection was not controlled in the present study. Different strategies (e.g., counting, decomposition of original question into easier sub-questions) has been theorised to be associated with differential involvement of working memory resources. For example, it has been theorised that phonological resources are needed in counting processes (Ashcraft, 1995) and a disruptive effect was observed for counting processes under phonological loads (e.g. Camos & Barrouillet, 2004; Imbo & Vanderiendonck, 2007). Also, strategy selection might vary across participants (e.g., Hecht, 1999; LeFevre et al., 1996a, 1996b) and arithmetic operations (Dehaene & Cohen, 1997). However, the control of strategy selection reported in previous literature is not convincing, as there is no way to verify whether or not participants actually used the strategies as they reported using. For example, some studies used a self-report method, which requires participants to report what strategy they had used after each trial (e.g., Hetch, 2002). Others used choice/no choice method (e.g., Imbo & Vanderiendonck, 2007; Siegler & Lemaire, 1997), with participants free to choose strategies in the choice condition, and forced to use a given strategy in the no-choice condition. These methods are problematic as participants might solve the arithmetic problems in their own way rather than using the allocated strategies, even when trained to do so, there is no guarantee that a given strategy has been used as instructed. Therefore, I did not manipulate arithmetic strategies in the present study. Future studies should investigate the effect of strategy selection on the relative involvement of working memory resources using more reliable methods than self-report or forced to use a given strategy.

Implications for Future Studies

Future research focusing on the role of working memory in the carry operations in mental arithmetic might benefit if the values of digital carriers are taken into account. Values of digital carriers have been shown to have an effect on performance of complex mental arithmetic (e.g., Fürst & Hitch, 2000; Imbo et al., 2007). Problem difficulty has been found to increase with the values of carriers involved; problems involving greater values of digital carriers are solved less accurately and slower than problems involving smaller values. More importantly, this effect is augmented by the presence of a working memory load (Imbo et al., 2007). Specifically, Imbo and Vandierendonck (2007) found interactions of working memory load and values of digital carriers in accuracy analyses of mental addition tasks. Although further analysis showed that such an interaction was only present under loads presumed to affect central executive functioning rather than the functioning of phonological working memory, it is likely that this absence of interaction between phonological load and the value of carriers was due to reduced phonological storage demand: rather than report the final solutions, participants were instructed to type down sums of each column, which decomposed the problem into sequential sub-problems. Therefore, it is likely that similar interactions between visual and phonological loads and values of carriers can also be found in the present study, which might be helpful in having a better understanding of the role of working memory resources in carry operations in mental arithmetic in terms of the value of carriers.

Individual differences in arithmetic proficiency should also be taken into account in future research. The relative involvement of working memory resources in solving arithmetic problems is affected by individual proficiency in mental arithmetic, in terms of the strategies used: the activation of an arithmetic fact retrieval process has been found to be more frequent for highly-proficient participants than for their less proficient counterparts (see LeFevre, Sadesky & Bisanz, 1996; Imbo & Vanderendonck, 2007). Non-retrieval strategies have been theorised to be more working-memory-resource demanding than retrieval, which might consequently lead to a strategy-specific effect on the relative involvement of working memory resources between more- and less-proficient participants. Although a few studies have investigated the relative involvement of phonological and visual working memory, in terms of individual proficiency in mental arithmetic, nearly all such studies defined proficiency levels dichotomously, using subsets of the French Kit, a paper-and-pencil test of arithmetic skill that includes complex addition, subtraction and multiplication problems (French, Ekstrom, & Price, 1963). In these studies, participants were instructed to solve the problems as fast and accurately as possible at the speed of around two minutes per page (e.g., Imbo & Vandierendonck, 2007, 2010; Thevenot et al., 2007, 2010). Participants were defined as ‘proficient’ in arithmetic if their scores for problems correctly solved on an arithmetic fluency test were above the median of all participants. Such operational definitions of proficiency might lead to misinterpretation of results, because the French Kit is a speeded performance test and results might be affected by a speed-accuracy trade-off. For a full understanding of the relative involvement of visual and phonological resources in solving complex arithmetic problems, future research might involve testing individual proficiency in mental arithmetic, defined, for example, by an Item Response model (e.g., Bock, Zimowski, Linden & Hambleton, 1996), in which both problems attributes and individual arithmetician proficiency are taken into consideration, in order to calibrate both problem difficulty and participants’ proficiency.

Future research is also required to investigate the switching hypothesis, arising for present findings, which might be crucial to exploring the theoretical structure of working memory. Using a large number of double-carry problems with both the number and value of digital carriers manipulated in dual-task experiments might be helpful in revealing the possibility of switching storage of carry information between phonological and visual working memory for adult participants. Any switching process might be affected by other factors, such as the relative capacity of phonological and working memory: For example, under visual interference, individuals with larger visual working memory capacities might be less likely to switch to phonological working memory due to having more available visual resources than individuals with smaller visual working memory capacity. Thus, individual differences in working memory capacity should also be taken into consideration when investigating the possibility of switching between different working memory components.

Future research might also benefit from an investigation of the cognitive styles of participants. The term *cognitive style* refers to a theoretical set of psychological dimensions representing consistencies in the cognitive functioning of individuals with respect to processing and representing information (Ausburn & Ausburn, 1978; Richardson, 1977; Riding & Rayner, 1997). A wide variety of dimensions of cognitive style have been proposed by different theorists (see Riding & Cheema, 1991). One of them, which has received the most attention in the field of mathematics education, is the visualiser-verbaliser dimension proposed by Richardson (1977), cited in Pitta and

Christou (2009). This dimension distinguishes individuals who represent encoded information in either mental pictures (imagery) or verbally. Nonetheless, it should be noted that good visualisers should not be assumed to possess poor verbal working memory. In fact, a particular individual might well be visualising as well as verbalising when performing cognitive tasks. The one-dimensional model of cognitive style under which visual and verbal ways of processing information are theorised to be two contrasting poles has been criticised for its bipolar construction (Blazhenkova & Kozhenikov, 2009; Kozhenikov, Hegarty & Mayer, 2002; Kozhenikov, Kosslyn & Shepard, 2005). The research of Kozhenikov and Blazhenkova (2009) demonstrated that there are two types of visualisers: objective visualisers and spatial visualisers. The former are theorised to construct vivid, concrete and pictorial images of objects using imagery, whereas the latter are theorised to represent and transform spatial relations as well as complex spatial transformations among objects (Blanzhenkova & Kozhenikov, 2009).

Just as individuals differ in terms of their potentials, habits and preferences, there is evidence that they also differ in the degree to which they recruit different cognitive resources in performing tasks such as numerical cognition (Zarnhofer, Braunstein, Ebner, Koschutnig, Neuper, Reishofer, & Ischebeck, 2012). It has been suggested that individuals may use different approaches when performing mathematics tasks, depending on their cognitive styles (Gray, Pitta, & Tall, 1997). Thus, cognitive style might have an effect on the relative involvement of working memory resources in solving mental arithmetic problems with carry operations. Specifically, during arithmetic calculation, individuals with differing dominant working memory resources might show different processing preferences. For example, verbalisers might tend to repeat information such as interim operands and solutions by sub-vocal rehearsal during the calculation process, whereas visualisers might prefer to visualise numbers or partial results. Therefore, a preference for selective employment of either visual or verbal working memory might depend on whether an individual is a verbaliser or a visualiser.

Future research could also investigate the generalization of the current findings to other operations: subtraction and division. Both visual and phonological resources were found to be involved in solving mental subtraction problems in the study of Imbo and LeFevre (Exp. 1, 2010). In their study, participants were required to report the remainders of subtraction problems comprising two double-digit operands, with half of them involving a borrow operation, and the other half not. The subtraction task was performed in three conditions: a single-task condition in which subtraction was performed alone; a dual-task condition in which a phonological task was performed simultaneously, and a dual-task condition in which a visual task was performed simultaneously. In the phonological task, participants were required to memorise letter strings of four consonants and repeat them aloud throughout the calculation of three subtraction problems. In the visual task, patterns of four asterisks were used as visual stimuli, which were to be reproduced after the calculation. Compared to the single-task condition, more impaired performance was observed in both dual-task conditions for the subtraction problems involving a borrow operation than for no-borrow problems, so the authors concluded that both visual and phonological resources were required during solving complex subtraction problems. The findings of Imbo and LeFevre (2010) have implications for future research, in terms of how working memory is involved in the calculation process for mental subtraction: If both visual and phonological resources are involved in this process, there might be a possibility that switching of temporarily-stored representations of intermediate solutions of borrowers might occur between phonological working memory and visual working memory when one of them is overloaded, if switching does occur.

Few studies have investigated the role of working memory in division. An exception is the study of Imbo and Vandierendonck, (2007, Exp. 2). In this experiment, only the potential involvement of the central executive and PWM resources were explored in respect of mental division. Therefore, a productive avenue for future research would be to replicate the experiments described in the present research using subtraction and division problems.

Aside from the mixed evidence concerning working memory resources involved in mental arithmetic, the results of the present experiments have shown that, for New Zealand (western-educated) participants, visual processes play an important role in complex mental multiplication involving a carry operation. However, it is likely that cross-cultural differences might also be observed in the role of visual processes in mental calculation. In the study of Imbo and LeFevre (2010), Chinese participants were more affected by phonological interference than by visual interference compared to

Canadian participants. Therefore, they proposed that cultural, and especially, educational differences between Asian and North American countries might also have an effect on the relative involvement of modality-specific working memory resources in mental calculation. For example, Chinese students engage in more rote learning of multiplication facts than is typical in western countries. All participants in the present study were native English speakers from New Zealand, so the present results may reflect a variety of strategies, rather than a reliance on rote verbal memory, which might, in turn, increase the involvement of visual processes (Imbo & LeFevre, 2010).

General Conclusions

Through manipulating both numbers of carriers and level of working memory load, the results of the present series of experiments showed that both phonological and visual working memory resources are required by carry operations in mental addition. Phonological resources were found to play a role in carry operations in mental multiplication, while only a limited role was found for visual resources. These conclusions are evidenced by a significant task-by-carry interaction observed in the accuracy analyses of arithmetic tasks in all experiments, except in Experiment 2, in which multiplication problems were solved under visual interference. The manipulation of numbers of digital carriers in the present research provides further support to the notion that more working resources are required when solving carry problems than when solving no-carry problems.

The manipulation of levels of working memory load did not further increase the effect of number of carriers on arithmetic performance. On the contrary, a reduction of the performance decrement with increasing level of load was observed in working memory load tasks. One possible explanation could be that participants might switch to the other working memory component for the temporary storage of carry information when one of them is heavily loaded.

The present findings only partially support the notion that complex mental arithmetic is solved with the aid of additional semantic elaboration on the mental number line, which is proposed to be reflected by the involvement of visual processing for all arithmetic operations. The present findings did not support the operation-specific involvement of visual and verbal processing, as both visual and phonological processing were observed in mental addition. This could be attributable to several different reasons: First, different strategies could have been used by participants in the problem-solving process, which might be mediated by both arithmetic operation types and individual differences in arithmetic proficiency. Second, the practice effect caused by the same arithmetic problems solved repeatedly by the same participants might have encouraged more activation in phonological than visual processing.

The present findings provide a different view of the theoretical framework of the Triple Code Model in terms of the cognitive pathways underlying complex mental arithmetic involving carry operations. Moreover, findings of the present research have implications for future research in terms of the role of working memory resources in complex arithmetic, with a special consideration of the effect of strategy selection as a function of individual differences in arithmetic skills and arithmetic operations on cognitive processing for complex arithmetic.

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APPENDICES

APPENDIX A

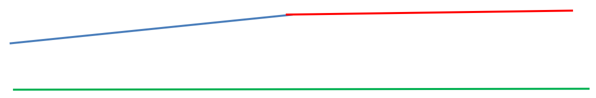
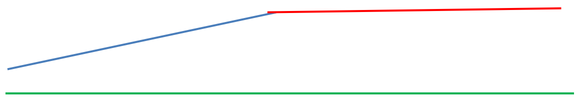
12+53	13+26	13+68
14+48	14+69	13+89
14+97	16+19	14+58
15+32	18+36	16+23
16+67	24+92	17+26
16+75	24+97	17+75
23+19	25+89	23+43
23+35	26+21	26+35
25+32	27+39	27+21
27+34	28+11	28+46
27+91	35+18	29+57
28+53	35+22	32+28
32+58	35+41	32+43
35+21	36+27	34+17
35+56	36+76	35+26
35+56	43+15	36+87
36+41	43+18	38+55
37+21	44+68	43+31
37+46	45+28	45+18
47+12	45+71	46+61
47+61	45+92	48+75
48+56	48+19	52+24
48+71	52+69	53+83
53+18	54+31	54+21
55+37	54+73	54+81
59+86	55+41	65+51
63+13	55+63	65+58
65+34	58+34	67+16
66+26	62+53	67+52
67+25	67+51	68+86
71+85	71+65	73+21
72+15	74+17	73+51
75+38	76+17	82+31
76+42	76+47	82+63
78+31	87+12	82+74
81+16	95+43	85+42

APPENDIX B

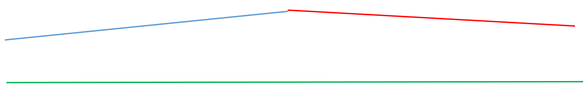
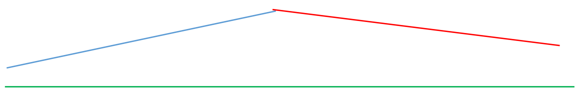
59 × 2	79 × 2	73 × 2
74 × 2	56 × 3	93 × 2
62 × 4	21 × 3	87 × 2
56 × 2	29 × 2	82 × 3
58 × 2	34 × 3	81 × 2
13 × 3	51 × 2	78 × 2
42 × 3	84 × 2	33 × 2
27 × 2	92 × 3	96 × 2
55 × 3	42 × 4	67 × 2
82 × 2	48 × 2	63 × 5
16 × 4	38 × 2	63 × 2
21 × 4	92 × 2	62 × 3
22 × 4	12 × 4	54 × 2
53 × 2	52 × 2	53 × 3
32 × 3	62 × 2	52 × 3
41 × 2	98 × 2	51 × 3
46 × 2	49 × 2	47 × 2
12 × 7	23 × 3	43 × 3
72 × 2	91 × 2	42 × 2
76 × 2	73 × 3	39 × 2
61 × 2	14 × 3	37 × 2
43 × 2	15 × 3	36 × 2
19 × 4	69 × 2	33 × 3
26 × 2	24 × 4	72 × 3
13 × 2	71 × 2	31 × 3
64 × 2	97 × 2	31 × 2
18 × 2	18 × 4	28 × 2
83 × 3	18 × 3	25 × 3
44 × 3	41 × 3	24 × 3
17 × 3	14 × 2	24 × 2
57 × 4	44 × 2	19 × 5
61 × 3	21 × 2	19 × 3
94 × 2	57 × 2	19 × 2
86 × 2	77 × 2	17 × 5
68 × 2	91 × 3	16 × 4
13 × 4	54 × 3	13 × 7

APPENDIX C

Performance Patterns
Switching occurs: (performance decrement stop increasing or decreasing with increasing load)

<table>
<tr><td>Problem/load level</td><td>Low</td><td>Medium</td><td>High</td></tr>
<tr><td>Single-carry problems</td><td colspan="3"></td></tr>
<tr><td>Double-carry problems</td><td colspan="3"></td></tr>
</table>

Or

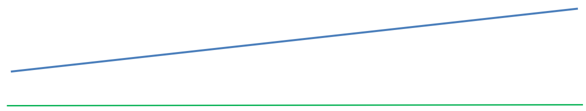
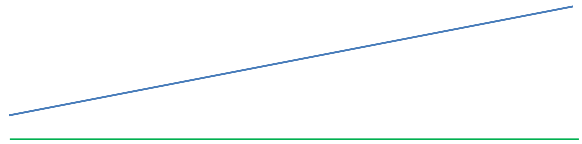
<table>
<tr><td>Problem/load level</td><td>Low</td><td>Medium</td><td>High</td></tr>
<tr><td>Single-carry problems</td><td colspan="3"></td></tr>
<tr><td>Double-carry problems</td><td colspan="3"></td></tr>
</table>

Red: Switching occurs

Green: Single-task condition

Blue: Dual-task condition

Switching not occurs

Problem/load level	Low	Medium	High
Single-carry problems			
Double-carry problems			

Red: Switching occurs

Green: Single-task condition

Blue: Dual-task condition

APPENDIX D

Addition under Phonological load (4, 5 or 6 nonwords)

Version1		Version2		Version3	
12+53	4\s\pattern6	12+53	5\s\pattern6	12+53	6\s\pattern6
14+48	4\d\pattern2	14+48	5\d\pattern2	14+48	6\d\pattern2
14+97	4\d\pattern8	14+97	5\d\pattern8	14+97	6\d\pattern8
15+32	4\d\pattern9	15+32	5\d\pattern9	15+32	6\d\pattern9
16+67	4\s\pattern9	16+67	5\s\pattern9	16+67	6\s\pattern9
16+75	4\d\pattern7	16+75	5\d\pattern7	16+75	6\d\pattern7
23+19	4\d\pattern5	23+19	5\d\pattern5	23+19	6\d\pattern6
23+35	4\d\pattern7	23+35	5\d\pattern7	23+35	6\d\pattern7
25+32	4\d\pattern6	25+32	5\d\pattern6	25+32	6\d\pattern6
27+34	4\s\pattern8	27+34	5\s\pattern8	27+34	6\s\pattern8
27+91	4\d\pattern4	27+91	5\d\pattern5	27+91	6\d\pattern6
28+53	4\s\pattern7	28+53	5\s\pattern7	28+53	6\s\pattern7
32+58	4\s\pattern4	32+58	5\s\pattern5	32+58	6\s\pattern6
35+21	4\s\pattern7	35+21	5\s\pattern7	35+21	6\s\pattern7
35+56	4\d\pattern4	35+56	5\d\pattern5	35+56	6\d\pattern6
35+56	4\s\pattern5	35+56	5\s\pattern5	35+56	6\s\pattern6
36+41	4\d\pattern5	36+41	5\d\pattern5	36+41	6\d\pattern6
37+21	4\s\pattern2	37+21	5\s\pattern2	37+21	6\s\pattern2
37+46	4\d\pattern1	37+46	5\d\pattern1	37+46	6\d\pattern1
47+12	4\s\pattern1	47+12	5\s\pattern1	47+12	6\s\pattern1
47+61	4\s\pattern5	47+61	5\s\pattern5	47+61	6\s\pattern6
48+56	4\s\pattern1	48+56	5\s\pattern1	48+56	6\s\pattern1
48+71	4\d\pattern3	48+71	5\d\pattern3	48+71	6\d\pattern3
53+18	4\s\pattern3	53+18	5\s\pattern3	53+18	6\s\pattern3
55+37	4\s\pattern6	55+37	5\s\pattern6	55+37	6\s\pattern6
59+86	4\d\pattern6	59+86	5\d\pattern6	59+86	6\d\pattern6
63+13	4\s\pattern3	63+13	5\s\pattern3	63+13	6\s\pattern3
65+34	4\d\pattern2	65+34	5\d\pattern2	65+34	6\d\pattern2
66+26	4\d\pattern3	66+26	5\d\pattern3	66+26	6\d\pattern3
67+25	4\d\pattern9	67+25	5\d\pattern9	67+25	6\d\pattern9
71+85	4\s\pattern9	71+85	5\s\pattern9	71+85	6\s\pattern9
72+15	4\s\pattern8	72+15	5\s\pattern8	72+15	6\s\pattern8
75+38	4\s\pattern2	75+38	5\s\pattern2	75+38	6\s\pattern2
76+42	4\s\pattern4	76+42	5\s\pattern5	76+42	6\s\pattern6
78+39	4\d\pattern8	78+39	5\d\pattern8	78+39	6\d\pattern8
81+16	4\d\pattern1	81+16	5\d\pattern1	81+16	6\d\pattern1

Note: s refers to the same condition in change detection, d refers to the different condition in change detection.