

**Exploring Visualisation as a Strategy for Improving Year
4 & 5 Student Achievement on Mathematics Word
Problems**

By

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Abstract

Solving mathematics word problems is more difficult for many students than solving comparable number only problems. Given the wide use of word problems in class teaching and in assessments there is potential for students not to achieve to their full ability. This study aimed to investigate if students' comprehension of mathematics word problems, their accuracy in choosing the correct operation, and the number of word problems solved correctly could be increased through using drawings and mental visualisation. This mixed method intervention study involved 10 Year 4 and 5 students in an inner city New Zealand school. Two separate interventions were used with different groups of five students identified as being at risk of low achievement in mathematics. Each group was involved in three intervention sessions to help with solving mathematics word problems. The first included instruction in creating drawings, and the second using mental visualisation. The study data included pre- and post-tests, verbal student reflections, and student drawings. Results showed that both groups made improvements during their interventions in the number of problems solved, the number of operations chosen correctly, and in their ability to identify and write the equation described in the word problems. Both groups also increased their achievement in number only problems. There were no significant differences between the results gained by students in the different interventions. The findings suggest that visualising word problems is an effective strategy for solving mathematics word problems and is an important step as a part of a mathematics word problem solving process. Implications for teachers include that creating representations is important for students' understanding in mathematics and crucially, that creating both internal representations (visualisations) and external representations (drawings) can and needs to be taught for maximising achievement.

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Chapter One

Mathematical word problems can be difficult for students to solve (Hegarty, Mayer, & Monk, 1995; Verschaffel & De Corte, 1993) with many achieving worse on word problems than comparable number only problems (Carpenter, Kepner, Corbitt, Lindquist, & Reys, 1980; Cummins, Kintsch, Reusser, & Weimer, 1988). In this thesis, the expression ‘word problem’ is used to refer to any mathematics problem that is presented using words rather than numbers only. With the wide use of mathematical word problems in today’s classroom teaching and resources and the use of assessments which often include many word problems, it is crucial that students have strategies to be successful word problem solvers. Literature suggests that it does not appear to be the mathematical knowledge or computation skills that are the issue but other contributing factors including the language used in the problems and the ability to comprehend the problems which may affect achievement (Cummins et al., 1988; Hegarty et al., 1995; Kytala & Bjorn, 2014). If it is student comprehension of the question that is the issue, this raises questions regarding whether reading comprehension strategies can assist students faced with mathematics word problems. These strategies include visualising. When readers visualise they connect the text with their prior knowledge and experiences to create meaningful images in their mind (Ministry of Education, 2006) which can be an effective way to aid comprehension and better understand texts (De Koning & van der Schoot, 2013). Comprehension strategies seem ideal for helping students to solve word problems and it is this broad idea that I wanted to investigate in this study (i.e., can visualising be used to increase achievement on mathematics word problems?).

This chapter begins by explaining the motivation for completing this study before then backgrounding the use of mathematics word problems in New Zealand classrooms and in assessments. Some of the difficulties that students have with word problems are discussed as

well as some of the approaches that have been used to help students improve their ability to solve word problems.

1.1 Motivation for Completing This Study

Mathematics has always been a subject I have enjoyed, from being a student at school to study at University, and into my teaching. I have taught for 17 years in primary schools across Year 0 to Year 8. Helping students to have the same level of enjoyment and success out of mathematics has always been a goal for me, but to my frustration, not always an achievable one. The different successes and difficulties students face with mathematics have always been a challenge for me, in relation to trying to find what will work for each student. One common area I have observed students struggling with has been when solving mathematics questions set in real life problems, especially when in the form of word problems. I often assumed this difficulty was related to reading difficulties and have tried various approaches to assist students including identifying key words and writing questions that include the students in the question. However, students' achievement on word problems has continued to be an area of concern for me. As I moved into management in schools and began looking at school-wide data and a bigger picture, the achievement of students on assessments that were heavily word based became a major concern for me, to the point where the fairness and relevance of the assessments needed to be explored. I felt that year after year, groups of students were getting lower results than their class work suggested they should be getting. What could be done about it? Were there different approaches that could help students to be more successful with word problems and therefore achieve to higher levels? While doing courses as part of post-graduate study, it became clear there were many ways to modify word problems and many approaches that worked for different types of questions or groups of children. I wondered if there was one approach that could be easily utilised by teachers and students, and be applied to the many different types of word

problems used in mathematics teaching and assessment. It was this wondering that led to this study...the hope of producing something that could benefit students (and teachers) and raise their achievement, and enjoyment, in mathematics.

1.2 Why Word Problems are used in School Mathematics

Competence in mathematics is important for students to enable them to be successful participants in society. Being numerate is defined in the Numeracy Project (Ministry of Education, 2007a) as having “the ability and inclination to use mathematics effectively – at home, at work and in the community.” (back cover). It is important for students to relate what they are learning in class to what they may do in their lives in the real world so that the learning is *real* and *has meaning*. This is an important factor for learning to be transferred to long-term memory, a crucial step if knowledge and concepts are to be retained (Sousa, 2008). Students need to be able to approach a range of mathematical problems “in a range of meaningful contexts” (Ministry of Education, 2007b), a factor important for quality teaching and learning in mathematics (Anthony & Walshaw, 2007; Bickmore-Brand, 1990; Mid-Continent Research for Teaching and Learning, 2010; Ministry of Education, 2007a).

While the aim of using context-based work is for students to be solving problems set in authentic contexts, this is often carried out in schools and particularly in assessments through the use of word problems (Jonassen, 2003). Word problems involve mathematics concepts being set in a situation often through the use of a “shallow story context” (Jonassen, 2003, p. 267). Students are exposed to word problems through teacher-made worksheets, as evidenced by the thousands of these available on the Internet, textbooks including the Figure It Out series (Ministry of Education, 2010a), and in assessments. Such assessments, provided for New Zealand teachers and frequently used in New Zealand, include the electronic assessment tool for

teaching and learning (e-asTTle) (Ministry of Education, 2010d), Progressive Achievement Tests (PATs) (New Zealand Council for Educational Research, n.d.), and the diagnostic interview (NumPa assessments) from the Numeracy Project (Ministry of Education, 2010b).

The New Zealand Curriculum (Ministry of Education, 2007b, see, for example, p. 26) and *Numeracy Development Project* (Ministry of Education, 2008, see, for example, p. 4) built on previous documents including *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992, see, for example, p. 11) and *Developing Mathematics Programmes* (Ministry of Education, 1997, see, for example, p. 18) in encouraging the wide use of problem solving and solving mathematics in real contexts in New Zealand classrooms. The use of real life contexts is also promoted as an important part of mathematics teaching in the *Effective Pedagogy in Mathematics: Best Evidence Synthesis* (Anthony & Walshaw, 2007) as is the use of a range of representations including graphs, diagrams, and pictorial imageries (e.g., p. 126). The Numeracy Project (Ministry of Education, 2008) promotes students' mental solving of problems (e.g., p. 2) and for teachers to use a wide range of representations to help students make sense of mathematics concepts (e.g., p. 4).

In their early years of mathematics education, New Zealand students are encouraged to make many external representations to learn and show mathematics concepts. This often involves the use of hands-on equipment which students physically manipulate to show and solve problems. As students move further through their schooling, the use of equipment lessens and more emphasis is put on using number properties only (Education Review Office, 2006). In the middle years of primary schooling, Years 4-5, mathematics begins to involve more use of symbols, rules, and formulae, as students move towards greater understanding and use of number properties. Although less reliance on algorithms and formulas than in traditional teaching of mathematics is a stated goal of the Numeracy Development Project (page 2, Book 3), in these

middle years some students lose touch with mathematics as a meaningful exercise. This can mean they stop creating or linking new knowledge to previously constructed knowledge and representations upon which to build their new understandings (Hiebert & Wearne, 1985). With the use of symbols increasingly replacing external representations, new knowledge and concepts can be introduced with little relevance to the knowledge and representations that students have previously constructed (Hiebert & Wearne, 1985). Less reliance on using representations means students can start using formulae and symbols that they may not understand, or producing answers that are not realistic, as the students may stop linking the work they do to realistic mental models or representations (English & Halford, 1995). It is this ability to generate realistic mental representations which can be the key to solving problems (for an overview on problem solving models, see Section 2.2), so this important step in any problem solving approach needs to be developed. Rather than having a deep understanding of what they are doing and building knowledge that is transferable to different situations, students may be just following a procedure or series of steps to get an answer.

This section has outlined why word problems are used in mathematics and has begun to look at an idea which will be explored further in Chapter Two, that of the important role of creating representations when solving mathematics problems. The next section goes on to discuss the role of language in mathematics, an important factor in mathematics word problems.

1.3 Language and Mathematics

As Durkin explains, language and mathematics cannot be separated: “Mathematics education begins and proceeds in language, it advances and stumbles because of language and its outcomes are often assessed in language” (Durkin, 1991, p. 1). While mathematics may appear to be about a different set of symbols and numbers, the language factor cannot be ignored and

must be consciously factored in to the teaching of the subject. Sousa (2008) explains that human brains process numbers in different parts of the brain when they are presented in symbol form and in word form. When humans encounter numbers in word form they are converted to the number form before being processed, which adds an extra level of processing needed for solving mathematics word problems.

The language factor can become of even greater consequence when it comes to assessments (Section 1.5). Hipwell and Klenowski (2011) argue that it is crucial that teachers understand the literacy demands of assessments as all assessments test knowledge *and* literacies. They state that many assessments have literacy demands which are beyond students' abilities, thus affecting achievements results.

It has been found that certain groups do significantly worse when language is involved in mathematics learning. These groups include English Language Learners (ELL), students with special educational needs, and students with low and average mathematics levels (Abedi & Lord, 2010). All these groups have been found to benefit when language demands in mathematics word problems are modified and simplified (Abedi & Lord, 2010).

Language demands in mathematics can create difficulties for students and one way this can happen is through the use of word problems. Some of these difficulties, which the use of word problems create for students, are discussed in the next section.

1.4 Difficulties That Word Problems Can Cause

Many students find solving word problems difficult, with students performing worse on arithmetic word problems than the on comparable problems in numeric form (Carpenter, Corbitt, Kepner, Linn & Reys, 1980; Cummins et al., 1988). Students often attempt to solve word problems using only the numbers and an operation (Carpenter et al., 1980) with little thought to

the context or meaning of the problems (Kajimes, Vauras, & Kinnunen, 2010), an approach named the “direct translation” strategy (Hegarty et al., 1995, p. 18). This strategy, often used by unsuccessful problem solvers (Hegarty et al., 1995), involves finding the numbers in the problem and doing what appears to be the most obvious thing (adding, subtracting etc.). The direct translation strategy can be successful for simple word problems, so students will often not develop any other strategy, and then when confronted by more complex problems or those with irregular wordings, they tend to make errors.

Word-based problems place greater cognitive demands on students than number only problems (De Corte & Verschaffel, 1991). Word problems that can be solved through the same mathematics operation can be made more or less difficult through the semantic structure used in the design of the word problems (De Corte & Verschaffel, 1991). The semantic structure refers to the way that word problems are organised and phrased to create meaning. The semantic structure was found to have a great impact on students’ ability to solve word problems and on the strategy they chose to attempt solving them (Verschaffel & De Corte, 1993). Supporting this argument, in a 2010 study involving 36 post-graduate students, it was found that it was the structure of word problems which caused difficulties for the solver and not the exact wording (Thevenot, 2010).

An area which has been consistently identified as problematic with regard to solving mathematics word problems is the link between reading comprehension and reading ability (Cummins et al., 1988; Kyttala & Bjorn, 2014; Pape, 2004; Reusser, 1990; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008). The use of word problems introduces extra cognitive demands on students, such as reading comprehension skills, unknown vocabulary, unfamiliar contexts, and common words used in unfamiliar ways, which can also lead to a new set of difficulties for students (Benjamin, 2011; Chinn, 2004; Harvey & Averill, 2012; Whelan Ariza, 2006;

Zevenbergen, 2000). These additional factors can all increase the cognitive load of the problem (Sweller, 2010) (section 2.8.1), therefore increasing the difficulty of the problem and decreasing the likelihood of a successful solution being found. Specific language-based difficulties that students face with mathematics word problems can include:

- linguistic demands (Abedi & Lord, 2010; Jordan, 2007);
- wording, length, and grammatical complexity (Fuchs & Fuchs, 2007);
- the different usage of a word in a mathematics setting as opposed to everyday life (e.g., table, of, difference) (Sousa, 2011);
- having multiple words with the same meaning (add, join, plus) (Sousa, 2011);
- complex problem layout, which could combine words, diagrams, and tables, set in a specific context and written in a mathematical way (described as reading the semiotic domain of mathematics) (Gee, 2008); and
- comprehension of the problem (Fuchs & Fuchs, 2007; Jordan, 2007).

Students can also struggle with the semantic structure of word problems (Verschaffel & De Corte, 1993). This can lead to problems with not knowing which operation is required to solve word problems with subtle wordings and orderings changing the meanings of words (De Corte & Verschaffel, 1991) or difficulty of the problem (Sweller, 2010). Two examples follow of how the semantic structure and the way language is used can alter the difficulty of a word problem. ‘Less than’ is usually associated with subtraction but the problem ‘*Sam has six marbles which is four less than Tom*’ requires addition to find the number of marbles Tom has. This is often where the direct translation strategy leads to errors. Similarly, *Sam has six more marbles than Tom. If Sam has twelve marbles how many does Tom have?* and *Sam has twelve marbles; if Tom has six less*

than Sam how many does Tom have? require the same operation to solve them but the first one is more involved and more difficult.

This section has outlined many difficulties that students have when solving mathematics word problems. When word problems are used in assessments, these difficulties can have greater consequences in relation to student achievement. The next section discusses the use of word problems in assessments and related issues.

1.5 Word Problems and Assessments

A major issue with the use of word problems in mathematics is in assessments (Abedi & Lord, 2010; Hipwell & Klenowski, 2011; McNamara & Ryan, 2011). Whilst word problems have been used for thousands of years, as evidenced by their presence in early Greek, Roman and Egyptian artefacts (De Corte & Verschaffel, 1991), the significance and impact of their usage in New Zealand has become greater due to the increase in standardised assessments and the introduction of Mathematics National Standards (Ministry of Education, 2010c). With a focus on teaching mathematics using real life settings (Ministry of Education, 2007a, 2007b) work and activities are provided so students can relate to, and therefore make connections to the situations and see that mathematics has relevance to everyday life. In classroom teaching situations this can be achieved through using real life situations and practical activities as well as word problems. However, in assessments it is more difficult for this to happen due to practical reasons including time and money factors (Knifong & Burton, 1985), so the reliance tends to be on word problems to provide contexts (Jonassen, 2003). Four commonly used assessments in New Zealand schools - the Numeracy Project GLoSS and JAM assessments (Ministry of Education, 2010b), the PATs (NZCER, n.d.) and e-asTTle (Ministry of Education, version 5, 2010d) - all utilise word problems to present mathematics concepts upon which children are

assessed. With the introduction of National Standards (Ministry of Education, 2010c) in New Zealand the consequences of assessments, including mathematics assessments commonly using word problems, have increased. Schools can have concerns about how results are used and interpreted given that incorrect inferences about the results can be made (Mehrens, 1998) with all schools reporting results and with newspapers publishing these (e.g., Dominion Post, 2013). It is crucial, therefore, that assessments that are used are fair (giving all students a chance) and reliable (including testing what they are testing) (McNamara & Ryan, 2011; Ministry of Education, 2005). Students need the skills and strategies to ensure they are able to show their true mathematical ability, and not be affected through “silent assessors” (Hipwell & Klenowski, 2011, p. 135). Silent assessors are described as other factors that can affect achievement apart from the intended area being assessed such as the language or context of questions.

Given the challenges students face with solving word problems and the prevalence of word problems in many assessments, helping students solve word-based problems is a major challenge for teachers (Verschaffel & De Corte, 1993). To enable students to succeed and show their true ability, they need to be able to understand the context and nature of the problems in the assessments. If they cannot understand these problems, it is unclear whether the assessment result reflects the students’ *mathematics ability* or their *reading ability*. Assessments that rely heavily on word problems could be unfair, particularly for ELL students and students with specific literacy difficulties as ideally a mathematics assessment needs to assess mathematics abilities, not language level or reading ability (Hipwell & Klenowski, 2011; McNamara & Ryan, 2011; Martinello, 2008; Ministry of Education, 2005).

In summary, given the emphasis of word problems in assessments, ensuring students have the strategies to solve mathematics word problems is important so they can achieve to their full

potential. The next section looks at some of the interventions and approaches which have been used to try to help students to improve their ability to solve mathematics word problems.

1.6 What can be done to Help Students Solve Word Problems?

There have been many studies aimed at increasing students' capabilities in solving mathematics word problems. Several studies have identified ways in which word problems can be made more accessible for students. These include:

- personalising the word problems, although findings on this have been mixed (Bates & Wiest, 2004; Ku & Sullivan, 2000);
- simplifying the language demands (Abedi & Lord, 2010);
- having the problems written in the student's first language (Sousa, 2011);
- using specific computer programmes (Glenberg, Willford, Goldberg, & Zhu, 2012; Kajimes et al., 2010); and
- specific interventions designed to improve students' solving of word problems including teaching representation, and key word identification skills (Xin & Jitendra, 1999; Zhang & Xin, 2012).

Studies have also involved specific student groups including English Language Learners (e.g., Orosco, Swanson, O'Connor, & Lussier, 2013) and low achievers (e.g., Kajimes et al., 2010). Schema-based instruction has also been used to teach students to identify patterns in word problems and then apply an appropriate equation to solve the problems (Xin, 2008), although this was found to be problematic as there have been over twenty five different types of word problems identified (Mayer, 1981) which students need to identify and then relate to the appropriate format to solve (Mayer, 1982).

Many of the above studies focused on changing the word problems and providing effective ways for teachers to help students access word problems during class teaching sessions. These modifications are not actions that can happen during formal assessments, especially in standardised tests, so it is important that students have strategies for dealing with word problems that can enable them to be successful in any setting including assessments.

A New Zealand-based study involving Year 5 students (Quirk, 2010) focused on using the reading strategy of reciprocal teaching to improve students' comprehension of mathematics word problems. This study found positive results and promoted the use of a known reading teaching strategy to help with improving mathematics achievement. Quirk found that the lower achieving students were less successful than their higher achieving peers and speculated that the lower achieving students were possibly so focused on decoding the text that they struggled to then identify the mathematics described in the word problem.

Two meta-analyses of word problem interventions for students with learning problems in mathematics (Xin & Jitendra, 1999; Zhang & Xin, 2012) identified representations as an effective tool for helping students with solving word problems, advocating that students need to be able to represent word problems in multiple ways. The first meta-analysis of twenty five studies from 1986-1996 (Xin & Jitendra, 1999) found the use of representations was the second most effective intervention style behind assistive technology interventions. In a follow-up meta-analysis of a further 39 studies published between 1996 and 2012 (Zhang & Xin, 2012), representations were found to be the most effective intervention style to increase problem solving ability.

1.7 Summary

Mathematics word problems are widely used in New Zealand classrooms for teaching and assessment. With evidence highlighting that word problems create difficulties for students, it is crucial that students are taught how to solve word problems and given effective strategies to do so independently. This study aims to investigate if the use of representations, specifically internal representations through visualising and external representations through drawings, can be effective strategies for achieving this.

Chapter Two explores the literature about representations (Section 2.1) and their role within teaching (Section 2.1) and problem solving models (Section 2.2). What is known about the use of learner generated drawings (Section 2.3) and visualisation (Section 2.4) is then outlined. This chapter ends with the research questions being introduced.

Chapter Three describes the methodology and method used and the reasons behind the decisions made. The study design (Section 3.2), data collection tools (Section 3.3), and data analysis (Section 3.4) are outlined. The selection criteria for participants (Section 3.5) and intervention (Section 3.6) are explained before ethical considerations (Section 3.7) and validity concerns (Section 3.8) are discussed.

Chapter Four presents the results of the data collected which included test scores, students' verbal reflections, and drawings. This is followed by Chapter Five which presents a discussion of the findings and the implications of these.

Chapter Two

This chapter provides a definition of representations as used in mathematics education. It then presents a summary of the literature around two areas of external representations, specifically learner generated drawings, and internal representations in the form of visualising.

2.1 Representations

A representation in mathematics is defined as a thing which is produced that symbolises, stands for, is associated with, or otherwise represents something else (Palmer, 1977, as cited in Goldin & Kaput, 1996). Representations can be internal such as a mental model or image, or external including diagrams, equations, graphs, computer software, pictures, and symbols (Goldin & Kaput, 1996). The area of mental representations was widely ignored during the period where the dominant school of thought was linked to behaviourist theories (Gardner, 1985), but with the rise of cognitive science, mental representations are increasingly being recognised and seen as an important part of learning (Gardner, 1985).

Having internal or mental representations of situations or concepts is seen as crucial to understanding. For example, English and Halford (1995) note that “the essence of understanding a concept is to have a mental representation or mental model that faithfully reflects the structure of that concept” (p. 57). Developing students’ internal representations, and therefore understanding, of mathematical concepts is the main goal of teachers (Goldin & Kaput, 1996) but the use of external representations by teachers to model and guide in new learning, and by students to explain and show their understanding, are important aspects of teaching and learning (Goldin & Shteingold, 2001). Noting the importance of creating representations, the New Zealand Ministry of Education (2008) states “A student’s ability to illustrate their mental strategy with materials is evidence of strong understanding of the number properties involved”

(p. 5). This relationship between using representations and number properties is evident in the Strategy Teaching Model within the Numeracy Project (Ministry of Education, 2008, p. 5).

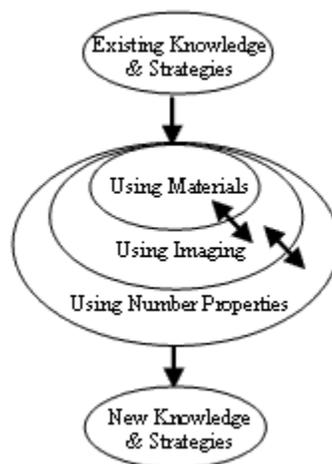


Figure 2.1: Numeracy Development Project Strategy Teaching Model

(Ministry of Education, 2008, p. 5)

The Numeracy Development Project promotes the use of both external (Using Materials) and internal (Using Imaging) representations in the process of achieving new knowledge and understanding (Figure 2.1). Visualising, or using imaging, is an important part of the Strategy Teaching Model which places using imaging as a crucial transitory stage when moving children from using manipulatives to using number properties. This process is advocated for use as each new concept is taught, but as students get older, it appears that teachers often make less use of equipment and greater use of number properties only, which limits students' chances to generate their own representations of each concept (Education Review Office, 2006).

For representation techniques to be effective, explicit instruction and modelling of them are needed (Ministry of Education, 2008; Van Meter & Garner, 2005; Zhang & Xin, 2012). It is not sufficient for learners just to be told to produce a representation, whether it be a picture or a diagram: "Students' progression through the Using Materials to the Using Imaging stages of the

model is unlikely to be successful without targeted input from [teachers].” (Ministry of Education, 2008, p. 5).

The quality of the mental models produced relates to the quality of their understanding of a concept (English & Halford, 1995); therefore, it is important for students to be able to produce quality mental representations. Mulligan (2002, 2011) found that when producing representations it is important to have structure that enables the students to solve the problems. She found that higher achieving students used more symbolic structures with more recognisable mathematical structure than lower achieving students. Successful word problem solvers also tended to produce representations that reflected the mathematical relationship in terms of structure and magnitude expressed in the problem (Orrantia & Munez, 2013). In contrast, students who were low achieving in mathematics tended to produce inconsistent and often crowded representations with repeated or unnecessary features which imposed overload on working memory. These students often produced representations which did not help problems to be solved. It was also found that these same students had poorly developed visual memory and associated visualisation skills. In summary, the studies found that students who struggled to visualise or produce helpful representations tended to be low achievers (Mulligan, 2002, 2011).

The benefits of student generated representations and of teachers developing understanding of these are explained by Anthony and Walshaw (2009): “Effective teachers acknowledge the value of students generating and using their own representations, whether these be invented notations or graphical, pictorial, tabular or geometric representations” (pp. 23-24).

In summary, a crucial element in developing students’ understanding of mathematical concepts is the ability of students to generate representations that show the concept. This process often begins with external representations, using equipment, and moves to an end goal of

students having strong mental models of concepts. For teachers, it is important to teach and model to students how to do this.

2.2 Problem Solving Models

This section explains several problem solving models and highlights the important role that visualising can have when students are involved in the process of solving mathematics problems. Bruner (1965) proposed a developmental model which saw children progress through three levels of representation as they learn: the enactive stage where concrete objects are directly manipulated (e.g., physically joining sets of blocks together); the iconic stage where mental representations and visualisation become key (e.g., generating a picture of the blocks and joining them together); and thirdly, the symbolic stage where children can use symbols rather than images of the objects (e.g., using the symbols $3 + 4$ to join sets together). This theoretical model has been used in the development of the New Zealand Numeracy Project strategy teaching model (Section 2.1) which was adapted from Pirie and Kieren's (1989) Recursive Theory of Mathematical Understanding. Bruner's developmental theory also provided a basis for the development of problem solving models which highlight the importance of creating internal representations. Some of these will be discussed in greater detail in the following section.

Kintsch and Greeno (1985) discussed a two-step model for solving mathematics word problems. This model was based on the written words, the text base, and from this creating an abstract problem model, which could be used to solve the problem. Reusser (1990) added an intermediary step to this model, the situation model. Reusser felt the initial model involved jumping from the text base to having a useful mathematical equation without any thought for the situation of the model. It was also felt that the initial model may be effective for competent problem solvers or students who had a strong understanding of different types of word problems

but was not effective for modelling to younger students or low achieving students. The situation model (Reusser, 1990) involves the problem solver creating a mental model of the situation of the problem which can then be used to create the mathematics knowledge needed to solve the problem, called the mathematical problem model by Reusser. Reusser's model placed greater emphasis on the context of the problem than previous models and highlighted the importance of understanding the problem in its setting.

English and Halford's (1995) views are consistent with Reusser's thoughts that more emphasis needed to be given to the linguistic elements than just the mathematical elements when solving mathematics word problems. They proposed a three-step approach for solving computational problems. These three steps are the problem-text model where the reader constructs a superficial representation of the text as part of the process of comprehension. Next is the problem-situation model where the reader forms a mental representation of the problem. This involves the reader mapping the information in the text onto a familiar situation that they have experienced or can relate to. Lastly, the reader translates the problem-situation model into a mathematical model from which they can solve the problem. They also state that if any one of the three stages is incorrect, then they will have difficulty in solving the problem.

The first two steps identified by Lucangeli, Tressoldi, and Cendron (1998) in their five-step problem solving model are comprehension of the relevant information in the text and the capacity to generate a good visual representation of the situation and the data. With visualising being a key strategy for comprehension, this highlights the importance of students having strong internal representation of both the situation described in the question and of the related mathematics question.

The quality of the problem-situation model is the key to success (English & Halford, 1995) with it being at this step where a mental model is created which brings the information from the text together and creates a relationship between the data in the problem and the missing data (the answer) (Lucangeli et al., 1998). The better the mental model that students can generate, the more accurate they can be in producing the mathematical model and then in solving the problem. This step involving the generation of a mental model is often the step that is left out in the teaching of problem solving (English & Halford, 1995; Gervasoni, 1999) and it is left up to students to do this stage on their own, but many students are not able to do this effectively (Arnoux & Finkel, 2010).

With the use of representations being a large part of problem solving models as shown in the models above, the next sections explore what is known about the areas of learner generated drawings (external) and visualising (internal).

2.3 Learner Generated Drawings

Learner generated drawings as a strategy to help learning have been shown to be effective in a range of studies in different countries and curriculum areas. Van Meter and Garner (2005) reviewed 15 articles which documented drawing as a strategy used in classrooms and 15 studies across all curriculum areas including sciences, reading comprehension, recall, and mathematics. They found inconsistencies between the empirical studies and the applied literature articles, and findings included that there was a lack of studies and information around instruction in the use of drawing as a strategy, calling for further research in this area. In this section, a range of aspects of learner generated drawings are discussed. Some areas show consistent results across studies and other areas show mixed results.

2.3.1 Drawings Increase Comprehension and Learning from Texts

The use of drawings has been found to be effective for increasing comprehension and learning of information in a range of circumstances including content area texts (Van Meter, Aleksic, Schwartz, & Garner, 2006), reading comprehension (Schwamborn, Mayer, Hubertina, Leopold, & Leutner, 2010), science texts (Leopold & Leutner, 2012), comprehending complex animations (Mason, Lowe, & Tornatora, 2013), and with specific groups including dyslexic readers (Wang, Yang, Tasi, & Chan, 2013). When reviewing the literature on using drawings to increase comprehension, De Koning and van der Schoot (2013) summarised that using drawings, especially under supported and instructed conditions, has a positive impact on students' reading comprehension. They suggested that the use of an external representation helps comprehension as readers can lessen the demands on their working memory and divide the demands over verbal and visual working memory, consistent with Dual Coding Theory (Sadoski & Paivio, 2013) (Section 2.6) which enables the reader to focus on important information in the text. They caution that most studies that have looked at using drawings to increase comprehension have compared the drawing strategy against a control situation and not against other comprehension strategies. However, one study that did compare the use of drawings to other reading strategies produced favourable results for using drawings to improve comprehension (Leopold & Leutner, 2012). In two experiments in Germany involving 161 Grade 10 students, Leopold & Leutner (2012) found that drawing was a more effective strategy for increasing students' comprehension of a science text than two other text-focused strategies – main idea selection and summarisation. These experiments directly compared drawing to other strategies as opposed to other drawing or non-drawing conditions and found drawing to be effective.

2.3.2 Age of Students vs. Effectiveness of Drawing as a Strategy

Several studies have found that the age of the students involved can affect the effectiveness of using drawings as a learning strategy; however, there have been conflicting findings in other studies. In their study in America involving 135 4th and 6th grade students, Van Meter et al. (2006) found that creating drawings benefitted the 6th grade students but not the 4th grade students. However, the authors warned against generalising these findings and suggested further research was needed. Similar to Van Meter et al. (2006), van Essen and Hamaker (1990) found that the use of drawings was effective for 5th grade students but was not an effective strategy for younger students with 1st and 2nd grade children showing no improvement. The authors felt that the younger students did not see any benefits from using drawings so they did not use drawings to help solve problems whereas 5th graders could decide that a drawing would help them solve the problem. However, from this study it is hard to generalise on the effectiveness of using drawings to solve word problems for the younger students as they chose not to use drawings when assessed independently.

In contrast, in a study involving eleven classes (106 experimental students and 138 control students) in Hungary, Csikos, Szitanyi, & Kelemen (2012) found that 3rd grade students improved their word problem solving through the use of drawings and that they could be taught how drawings could assist them. This is in contrast to previous studies which found drawings were less effective with younger students.

2.3.3 Quality and Type of Drawings

The quality and type of drawings produced by students has been found to have an impact upon the learning that occurs. In a study involving 196 German 9th graders, Schwamborn et al. (2010) found that the better quality and more accurate the drawings the students produced were,

the better the students' understanding and retention of the content. Similarly, in a study completed in Italy involving 199 7th grade students, Mason et al. (2013) found that the quality of the drawing affected the level of immediate and long-term comprehension with richer and more accurate drawings leading to better comprehension of the texts being read.

With a specific focus on mathematics, several studies have surmised that the style of drawing can affect students' success with solving word problems. Students using schematic representations were more successful than students producing pictorial representations (Edens & Potter, 2010; Hegarty & Kozhevnikov, 1999). The studies conducted with 214 American 4th and 5th graders (Edens & Potter, 2010) and thirty three Irish 6th grade students (Hegarty & Kozhevnikov, 1999) had similar findings which showed that the use of schematic drawings that represented the relationship described in the word problem led to more success when solving word problems and students who produced more pictorial representations were less successful. Edens and Potter (2010) suggested an area for further investigation was to focus on students' mental imagery and the mental models they produce. In contrast, Hegarty and Kozhevnikov (1999) ended with a warning that encouraging students to visualise the problem will probably not help and that students should be instructed in how to represent the relations between objects in the problem and not include irrelevant pictorial details.

In a study which adapted the method and instruments of the Hegarty and Kozhevnikov study, Ahmad, Ahmad Tarmizi, and Nawawi (2010) worked with 381 Form Four (equivalent to New Zealand Year 10) students from eight schools in Malacca, Malaysia. They found that students favoured using schematic representations over pictorial representations. The authors concluded that further investigation into how visualisation can become a part of an effective pedagogy and strategies to include visualisation in mathematics teaching and learning are needed.

2.3.4 Using Drawings is More Effective on Open Ended Activities

The use of drawings has been found to be more effective when working on open activities (e.g., problem solving, free recall questions) than closed activities (e.g., multiple choice). Van Meter (2001) found that students who engaged in drawings scored higher than control participants on free recall questions but not on multiple choice tasks. In the study involving 100 5th and 6th Grade students, the author recorded that students who completed drawings engaged in more self-monitoring behaviours to create accurate drawings. This led to an increase in time spent on tasks which led the author to question whether the extra time spent on task was sufficient to explain the increase in results. Another finding from De Koning and van der Schoot's (2013) literature review was that drawings help students create internal visual representations and that using drawings was most likely to help with deeper understanding (for example, drawings helped with problem solving but not with multiple choice activities).

2.3.5 Generating Illustrations vs. Viewing Illustrations

Many believe that the process of generating an illustration is more effective than simply viewing an illustration for helping students to comprehend texts and solve mathematics word problems (Berends & van Lieshout, 2009; Dewolf, Van Dooren, Cimen, & Verschaffel, 2013; Van Meter et al., 2006). Dewolf et al.'s (2013) and Berends and van Lieshout's (2009) studies both involved providing illustrations with word problems and found that the illustrations to support the students create a mental model of the problems had little impact and in certain circumstances had a negative impact on the solving of word problems. These studies align with Van Meter et al.'s (2006) findings from their American study involving 135 4th and 6th grade students, which found drawing a picture was more effective for helping students learn new information than inspecting an illustration. Theories that can help to explain these findings

include Cognitive Load Theory (Sweller, 1994) (Section 2.8.1) which suggests there is a limited amount of cognitive resources for processing information, so to be effective as much as possible needs to be directed towards the actual solving of a problem rather than towards other factors that require processing (e.g., looking at and processing illustrations).

2.3.6 Summary

There is much evidence that supports the use of learner generated drawings as an effective strategy. These studies and others have all found that by producing an external representation in the form of a drawing, learning can be enhanced for students in a range of subject areas and most importantly in the areas of reading comprehension and mathematics word problems. These results seem to align with Chi's (2009) hypothesis about a hierarchy of effectiveness for activities. Chi suggested that interactive activities (e.g., talking and engaging with others about the topic as you do something) were better than constructive activities (e.g., producing something like a drawing or a concept map) which were better than active (e.g., moving objects or copying what someone else has done) which in turn were more effective than passive activities (e.g., viewing something). Drawing as a constructive activity rates highly on Chi's suggested framework and if combined with a teaching and an interactive element could be even more effective.

By being required to make a visual representation of the content, learners are involved in a process of integration where the written cues need to be closely aligned to the visual production (Van Meter, 2001). To create an image in their mind or on paper, students need to pay close attention to all information in the text/question so through this process engage in more self-monitoring which increases the detection of comprehension errors (Van Meter, 2001). Through creating an image which closely matches the information in the question, students have to

integrate the written representation with their prior knowledge which leads to greater understanding of read texts (Van Meter et al., 2006). The theory that drawing can increase comprehension and learning is explained by the Generative Theory of Drawing Construction (Van Meter & Garner, 2005) (section 2.7). By referring back to the verbal representation and constructing an accurate picture students are forced to pay closer attention to what the text says. This in turn leads to more attention to the specific details and an increase in the comprehension of the text (Van Meter, 2001).

Whilst it has been shown in many different settings that generating a drawing can increase students' accuracy of learning new information or solving problems, there has been little research completed about this area in a New Zealand setting. Also, significantly less is known about the use of internal representations and mental visualisation and this is discussed in the following section.

2.4 Internal Visualisation

This section discusses internal representations and specifically visualisation. A widely used reading comprehension strategy, and a part of many problem solving and teaching models (Sections 2.1 and 2.2), there appears to be less research on visualisation, especially in mathematics, and there are calls for more exploration into how it can be more effectively taught and used in mathematics pedagogy.

Visualising, imaging, or mental imagery is the ability to create, use, interpret, and reflect upon a picture, image, or diagram in one's head (Arcavi, 2003) and is identified as an effective strategy for increasing comprehension (De Koning & van der Schoot, 2013; Ministry of Education, 2006). Sadoski and Paivio (2013) describe its importance as a strategy for developing students' literacy: "The research literature can be summarised conclusively; spontaneously-

occurring mental imagery is a natural and important part of literacy, and educating students in the strategic use of mental imagery is a successful practice in various aspects of literacy learning” (p. 115).

To be able to produce a representation learners need to be able to read the information and access their prior knowledge related to the context, firstly ‘seeing’ what they know in their head, that is to visualise what they understand the content to be. Creating visual images in their mind enables readers to make links to their prior knowledge and experiences, and to have deeper comprehension of what they read including at an inferential level (Davis, 2007). Sadoski and Paivio (2013) add to this when they state “Without the activation of mental representations, no meaning can be present” (p. 50).

When summarising the literature on internal visualisations in reading, De Koning and van der Schoot (2013) surmised that students who are trained in the use of mental imagery improve their reading comprehension when compared to students who do not receive any training in a comprehension strategy, though when compared with other reading comprehension strategies the results are less convincing. De Koning and van der Schoot (2013) surmised that creating a mental representation of what is described in a text helps students to better understand the text, and when instruction in how to create mental representations is included the chances are increased that an accurate mental representation can be constructed. The authors also concluded that even though either an external or an internal representation can improve comprehension, the end goal is the automatic construction of internal mental representations. The authors noted that internal and external representations have usually been used and tested separately and a hypothesis from the authors was that the two forms could have complementary roles and this was an area that required further investigation.

Creating visual images is an important part of literacy and especially reading comprehension; therefore, it seems logical that this step should be useful in the comprehension and solving of mathematics word problems. With comprehending word problems identified as a barrier for many students (Fuchs & Fuchs, 2007; Hegarty et al., 1995; Kajimes et al., 2010; Kenney, Hancewicz, Heuer, Metsisto, & Tuttle, 2005; Sousa, 2011), then the use of a reading strategy to assist students when solving mathematics word problems merits investigation. Visualising, whether it be internal or external, is an effective strategy for improving a reader's comprehension of a text (Davis, 2007; De Koning & van der Schoot, 2013; Frey, Fisher, & Berkin, 2009; Graham, 1990; Ministry of Education, 2006; Park, 2012; Zimmerman & Hutchins, 2003) and therefore it is worth investigating if visualising can help students solve mathematics word problems.

Representations are a way of showing mathematics concepts, be it a graph, table, equation, diagram, or drawing and to be able to produce a representation students need to understand what the problem is. An external representation (such as a drawing) can be produced if first an internal, non-verbal representation, a mental image, can be produced (Van Meter et al., 2006). Creating external representations enables students to share what their understanding is and to discuss what they have produced (Goldin & Kaput, 1996) as well allowing others to 'see' their thinking (Diezmann & McKosker, 2011).

Visualising, or using imaging, is an important part of the Strategy Teaching Model in the Numeracy Project (Ministry of Education, 2008) (section 2.1). This teaching strategy places using imaging as a crucial transitory stage when moving children from using manipulatives to using number properties. Being able to produce representations of their mental images is also identified as a way that students can show a deep understanding of a concept. Sousa (2008) describes a concrete-pictorial-abstract approach as crucial for helping students to progress with

learning concepts. The pictorial stage involves providing or creating visual representations that will help students to visualise mathematical operations during problem solving.

As noted earlier, low achievers in mathematics tend to have poorly developed visual memory and visualisation skills (Mulligan & Mitchelmore, 2009, as cited in Mulligan, 2011). Without the ability to visualise the problems and mathematics content involved, students are going to struggle to solve word problems.

In their summary of the literature around visualisation and mathematics learning, Booth and Thomas (2000) found that results on the effectiveness of visualisation were inconsistent and there were opposing views on the effectiveness of the strategy. They surmised that mental imagery can be valuable as part of a process but if there is too much reliance on visualisation it can affect performance. In a review of papers presented at the Annual Conference of the International Group of the Psychology of Mathematics Education, Presmeg's (2006) final recommendations included the need to find what makes mental imagery effective in mathematics and to investigate effective pedagogy utilising visualisation in mathematics education. Similarly, Arcavi (2003) concluded his report on the role of visual representation in mathematics with comments that visualisation is a central issue in mathematics education, and though warning about thinking visualisation will be a cure all, he stresses the need to better understand visualisation and how it can help in the teaching and learning of mathematics.

While seen as an important, and successful, strategy for helping with reading comprehension, the literature on visualisation is mixed as to the effectiveness of it as part of teaching and learning in mathematics education. However, there are calls for more research into visualisation to help gain a deeper understanding of it and any possible role visualisation can have in mathematics education.

2.5 Reluctance to Use Representations

Whilst the use of representations has been shown to be effective, it has been noted that students appear reluctant to use representations to solve mathematics problems unless specifically directed to, even though teachers often use them in their teaching. Reasons for this include the perception that diagrams are a teacher strategy for teaching (Uesaka, Manalo, & Ichikawa, 2007) and that visual reasoning is considered of low value (Arcavi, 2003; Eisenberg, 1992, as cited in Kaldrimidou & Ikonou, 1998). Even after participating in an intervention on using drawings, van Essen and Hamaker (1990) noted that 1st and 2nd grade students still chose to not produce drawings when working independently (Section 2.3.2). In my personal experience, many students often seem reluctant to produce any representation and appear to think it means they are not good at mathematics if they have to do so.

2.6 Processing Information in Multiple Ways

If a person can decode material in two different ways (for example, understanding the words and creating a picture), then the chance of remembering the material is increased. This idea is called dual coding theory (Sadoski & Paivio, 2013). Dual coding theory states that people can retain ideas in two ways, with verbal associations and visual imagery, and if information is processed in both ways, comprehension or retention can be increased. The idea that mental images can aid learning is supported by this theory through the generation of visual information to support the written word which when read can be processed as verbal data. This theory informs this study with the belief that by reading the question and then creating a mental image of the content, by processing the information in more than one way, the students' comprehension of the question will be greater and their ability to solve it accurately enhanced.

2.7 Using Drawings to Aid Learning

The use of drawings to aid learning has been shown to be effective (section 2.3). Learner generated drawings have been shown to have greater impact on higher order assessments but to have less impact on lower order assessments which involve direct recall or multi-choice options (Van Meter & Garner, 2005). The production of a picture seems to support a deeper connection between content where inferences and reorganisation occurs but not necessarily straight recall. This seems to sit well with the open style of mathematics word problems where deeper processing and reorganising of the text are required.

This theory of how constructing a picture leads to deeper understanding of to-be-learned content is explained by the Generative Theory of Drawing Construction (Van Meter & Garner, 2005) which builds on the Generative Theory of Textbook Design (Mayer 1993, as cited in Van Meter & Garner, 2005). By referring back to the verbal representation and constructing an accurate picture students are forced to pay closer attention to what the text says. The process of creating the illustration to match the text leads to more self-monitoring behaviours and overall deeper comprehension and longer retention of text content (Van Meter, 2001).

Solving mathematical word problems involves greater cognitive demands and processing than solving number only problems. This processing is carried out in the working memory part of the brain. A brief outline is given in the next section of working memory.

2.8 Working Memory

Working memory can be defined as “a temporary storage system under attentional control that underpins our capacity for complex thought,” (Baddeley, 2007, p. 1), “the system or systems that are assumed to be necessary in order to keep things in mind while performing complex tasks such as reasoning, comprehension and learning” (Baddeley, 2010, p. 136) and “the retention of a

small amount of information in a readily accessible form, which facilitates planning, comprehension, reasoning and problem solving (Cowan, 2014, p. 217). There is a potential relationship between working memory and learning (Cowan, 2014) and an understanding of working memory and specifically not overloading the working memory can help for learning to occur (Cowan, 2014; Farrington, 2011). Factors that can affect working memory include trying to hold too much information in mind, distractions, and doing two things at the same time (Alloway & Alloway, 2012) as well as the way in which information is presented (Sweller, 1994, Section 2.8.1).

Raghubar, Barnes, and Hecht (2010) reviewed 29 studies which related working memory and mathematics and while not making causal claims about mathematics ability and working memory they had several relevant conclusions. They found more was known about the phonological loop than the visuospatial element of working memory as well as suggesting that the visuospatial processing may be more important to early mathematics learning. They also found that the way work is formatted and presented affects the way the brain processes mathematics. Working memory is important for performance in mathematical tasks. The holding of information while solving mathematical word problems is likely to occur in working memory, and it is this link which can lead to a possible link between working memory and mathematics achievement. The use of pen and paper more than likely reduces the demands on working memory.

In a study involving 91 3rd grade students in America, Swanson, Moran, Bocian, Lussier, and Zheng (2012) found that some strategies that can be successful in helping students improve mathematics problem solving can increase the load on working memory and so only those students with sufficient working memory capabilities will benefit. The authors also hypothesised

that a low working memory capacity may be an underlying reason for why students struggle with mathematics word problems.

A second study, conducted by Kyttala, Aunio, Lepola, and Hautamaki (2014) found that visuospatial working memory had a direct effect on students' performance in word problems. This study involved 116 Finnish kindergarten and pre-school students. With these younger students the authors found that verbal working memory did not have a major role in word problem solving but visuospatial working memory appeared to play an important role. These findings suggest students, particularly younger ones, create mental models that enable them to solve mathematic word problems.

2.8.1 Working Memory Capacity

The idea that working memory has a capacity is a widely recognised phenomenon although there is disagreement about what the limits are (Farrington, 2011). Working memory capabilities can be affected by factors including age and expertise or knowledge (Cowan, 2014). When completing tasks, the working memory can only hold or process so much information before aspects start to be forgotten. Depending on the style of instruction and how work is delivered, the demands on working memory can be altered. Cognitive Load Theory (Sweller, 1994) is concerned with minimising the cognitive load placed upon students so as close to ideal conditions can be in place to allow learning to occur. Sweller identifies three types of cognitive load – intrinsic, extraneous, and germane. Intrinsic cognitive load relates to the demands of learning the actual content. There is little to be done about lessening the level of difficulty of learning a specific concept. Extraneous cognitive load refers to the way in which new learning is presented; instructional approaches and design can lessen the extrinsic load of a task. Germane cognitive load refers to the way that learners create schema and make new learning automatic

and permanent. Sweller concludes that when the intrinsic and extraneous loads are too much and the working memory is overloaded then true learning cannot happen. The use of word-based mathematics problems significantly increases the extraneous cognitive load demanded of students as the processing of the language, reading comprehension, selecting the correct operation and then processing the numbers is a far more involved process than purely working with numbers. The intrinsic cognitive load of solving $4 + 3$ remains constant but the format it is presented in (i.e., $4 + 3$ as opposed to a word problem) can change the demands on students. Students solving word-based problems may or may not solve the problem but with such a high cognitive load a potential issue is that an effective method for solving future word problems will not be developed (Sweller, 1994). With all working memory being used to remember words and numbers and contexts, little room is left for the development of an effective problem solving strategy which can then be transferred to future examples. This *understanding* is what is key to learning being transferred to long-term memory and being available for future use (Sweller, 2011).

Overloading the working memory can lead to lower achievement and a failure to learn new information. Important consideration needs to be given to how work is presented to students and to the strategies that students have which can lessen cognitive demands placed on students.

2.9 Summary

It has been shown in numerous studies that the generation of pictures helps with learning new information and increasing comprehension (Csikos et al., 2012; Edens & Potter, 2010; Mason et al., 2013; van Essen & Hamaker, 1990; Van Meter et al., 2006). Less, however, is known about visualisation and many authors have called for further research in this area; in particular, the role visualisation has to play in mathematics pedagogy needs to be better

understood. A better understanding of visualisation could assist New Zealand mathematics teachers in particular to maximise the effectiveness of their mathematics and numeracy teaching. Similarly, the use of drawings has not been investigated in a New Zealand setting. This study aimed to investigate if using drawings could help New Zealand Year 4 and 5 students solve word problems as well as investigating if mental imaging or visualising could have the same impact as the use of drawings has been shown to have. Considering the above literature, this study is based on several theories: 1) word problems are more difficult for many students; 2) one of the main reasons students struggle with word problems is text comprehension; and 3) visualising is an effective way of improving text comprehension. This study aimed to investigate if visualisation, students producing an internal representation, was sufficient for improving students' ability to solve word problems as opposed to needing an external representation to be produced. It was an additional goal to investigate if a strategy could be developed that was practical for teachers and students to be able to use in everyday mathematics lessons and which was transferable to any type of word problem or situation.

2.9.1 Research Questions

This study aims to investigate the following research questions:

- Are learner generated drawings effective in helping New Zealand students improve their achievement when solving mathematics word problems?
- Can visualisation increase students' achievement when solving mathematics word problems?
- Is visualising or the use of drawings more effective in helping students to solve mathematics word problems?
- Can one strategy help students solve word problems of any operation and structure?

- Can a strategy be devised which teachers and students can use easily?

This chapter has outlined the use of representations and in particular learner generated drawings and visualisation. Studies have shown that learner generated drawings have helped students in many countries and this led to exploring this strategy in a New Zealand setting. The work on visualisation is mixed as to its effectiveness and this led to questions exploring if mental representations (visualisation) can be as effective as external representations (drawings). In the next chapter the study methodology and method used to explore these research questions are described.

Chapter Three

Methodology and Method

This chapter introduces the research approach chosen and explains why a mixed methods approach was suitable for this study. The study design is then outlined and a timeline of the study provided. This is followed by a section on the data collection tools and how the data were analysed. The selection of the participants and the method used follow. The chapter concludes with a discussion of validity and trustworthiness of the study design and data collected.

3.1 Research Paradigm

The philosophical worldview that guides this study is social constructivism. Building on the work of Vygotsky (1987), social constructivism presents learning as occurring as a result of participating with others. This view portrays learning as occurring through interactions with other people and through sharing ideas. As this study was investigating a teaching and learning strategy, it attempted to replicate conditions often used in many classrooms (i.e., working in small groups in an interactive way). There was a deliberate attempt to use an intervention that would be practical, manageable, and able to be easily adopted for use in classrooms by teachers.

The approach taken for undertaking a research study is chosen in order to best answer the research questions and to ensure the data to be collected will be sufficient, valid, and reliable. To introduce discussion about methodological decisions for this study, three main research paradigms will be explained in this section: quantitative, qualitative and mixed methods research.

Quantitative research is often used to test a hypothesis or theory through the collection and analysis of data. Quantitative research relies on the collection of quantitative data (i.e., numerical

data) and is often concerned with predicting and looking for “probabilistic causes” (Johnson & Christensen, 2008, p. 35) (an event may have caused an outcome). Quantitative research is built on a positivist view of research. Positivism is “the idea that only what we can empirically observe is important and that science is the only true source of knowledge” (Johnson & Christensen, 2008, p. 391). The positivist approach has been identified as having drawbacks when it is applied to the study of human behaviour, particularly “in the contexts of the classroom and school where the problems of teaching, learning and human interaction present the positivistic researcher with a mammoth challenge” (Cohen, Manion, & Morrison, 2011, p. 7). When studying people, a naturalistic approach to research is promoted (Lincoln & Guba, 1985), which includes the qualitative approach to research.

Qualitative researchers believe that the truth is what is experienced by individuals and that these truths are socially constructed and are defined by the environment in which they occur (Cohen et al., 2011). Qualitative research is often set in the subject’s natural setting and enables subjects to express their perspectives (Johnson & Christensen, 2008). These are reasons why supporters of qualitative research believe this approach is the best when studying people and is particularly relevant in school settings (Cohen et al., 2011). Qualitative research provides exploratory approaches to research (Johnson & Christensen, 2008), often looking to generate new hypotheses, rather than starting with a hypothesis to test. Qualitative research is often used when little is known about an area and more is wanted to be known or found out. It can be effective for exploring a smaller number of participants but in greater detail than quantitative research. The data in qualitative research can be from words and images and can be collected in many ways including using interviews, observations, and drawings (Johnson & Christensen, 2008). There can often be more than one interpretation of qualitative data, which can be both a strength and a weakness of this approach (Cohen et al., 2011).

Both qualitative and quantitative research approaches have strengths and weaknesses, which can affect their effectiveness (for a detailed explanation of these see Johnson & Christensen, 2008, pp. 441-442). For example, quantitative research can be effective for establishing cause-and-effect relationships but is less effective for exploring people's perspectives of specific phenomena. Qualitative research can provide detailed information about participants' personal views, but because of the usually small sample used in qualitative research, results can often not be generalised beyond the study participants (Johnson & Christensen, 2008).

Mixed methods research combines the two approaches of quantitative and qualitative research. Mixed methods research attempts to use the strength of each separate approach and eliminate the weaknesses to best answer the research questions. In the past, some researchers have argued that the two approaches should not be mixed due to their differences in philosophy, called the "incompatibility thesis" (Johnson & Christensen, 2008, p. 33). However, during the 1990s researchers began to reject this viewpoint and argue for a more pragmatic approach that stated that both quantitative and qualitative research are important and can be used successfully in the same study. The pragmatic viewpoint is built on the idea that what works is more important than sticking strictly to a philosophy or set of guidelines (Johnson & Christensen, 2008).

Lincoln and Guba (1985) discussed the need for validity in research and the concept of triangulation of data by using different methods and data collection modes. They compared this to a fisherman using nets. The fisherman may have two or more nets with holes in them but if he overlaps the nets he may end up with one good net. Researchers who use mixed methods often adhere to a "pragmatist philosophy" (p. 442) in which they combine research components to come up with the best approach to answer their research question (Cohen et al., 2011; Johnson &

Christensen, 2008). Newby (2010) explains pragmatism as “essentially problem solving” (p. 46) where there is a problem (a research question) to which an answer is needed. Combining methods can help to achieve this. Mixed methods research seeks to address the what (numerical and quantitative) and the how and why (qualitative) types of research questions (Cohen et al., 2011), important when a researcher wants to find out the explanation for the different outcomes and have a deeper understanding of their research findings.

Like qualitative and quantitative research, mixed methods research has recognised strengths and weaknesses. The strengths include the use of words and pictures to add meaning to numerical data, the ability to answer a wider range of research questions, and the ability to combine the best parts of a wide range of research methods and the triangulation of data (Newby, 2010). The weaknesses include the increased requirements for researchers to know about the different methods and approaches used in a mixed methods study, and that mixed methods studies can be more expensive and time consuming (Johnson & Christensen, 2008).

A mixed methods approach was chosen for this study as the research questions involved looking for a link between the interventions and an increase in a score (causation) as well as looking at the students’ feelings and attitudes about mathematics word problems and their thought processes when solving the problems. The collection of numerical data from test scores (quantitative), the pictures drawn by students, and the students’ verbal explanations of how they solve problems (qualitative) were all important to the overall findings of this study.

3.2 Study Design

The mixed methods research approach to this study allowed for the collection of data using a range of approaches (for further detail see Section 3.3 and Table 3.1). An experimental research design was chosen for the study. This involved a pre-test post-test design (Section 3.2.1)

with intervention sessions between these. Experimental research involves the manipulation of one variable to determine a cause-and-effect relationship (Johnson & Christensen, 2008). In this case the variable was the strategy being trialled (i.e., for Group 1 visualisation and drawing, for Group 2, visualisation). Two interventions were planned with the same testing procedures for each intervention. Intervention One involved the students in Group 1 being instructed in the use of visualising and drawing. Intervention Two involved a second group of students being instructed in the use of visualising only. This was to allow comparisons between the two approaches. Each group was to have three intervention sessions. Three sessions were intended as the researcher wanted to see if any changes in behaviour could be achieved in a short period of time, with time restrictions of busy teachers being an important consideration if such interventions are to be easily implemented in school settings. The intervention sessions were designed to replicate a classroom mathematics teaching session with a small group lasting thirty minutes, keeping the situation as natural for the students as possible (Newby, 2010). Even though the students were removed from their classes for the sessions, they were still in their own school, in a classroom, and it is usual for New Zealand students to work outside of their classroom for a range of reasons. In order to keep the students' mathematics programme as usual as possible, the intervention sessions were planned to happen outside of the students' usual mathematics lessons. The intervention sessions involved the researcher modelling the desired strategy and then the students having opportunities to use the strategy. The students were also involved in sharing their solutions and thought processes with the researcher and each other.

3.2.1 Pre-test post-test control group design

The pre-test post-test control-group design is a strong experimental research design (Johnson & Christensen, 2008) as it allows for variables which may influence the results. The pre-test (Test 1) was used before the intervention to collect baseline data of the students' ability

solving parallel number only and word problems. These data were then compared to the post-tests conducted after the intervention for each group (Test 2 and Test 3). The final test (Test 4) was used three weeks after the final intervention session to examine whether any gains made by the participants had been retained.

This experimental design controls situations that may threaten internal validity of the experiment such as testing, history, and maturation (Johnson & Christensen, 2008). Testing refers to any change that might occur in students' scores during a second sitting of a test due to having already sat the test and being more familiar with the style and content of the tests. History refers to any other events that might happen between a pre-test and a post-test which could affect scores such as the classroom teaching or a homework task. Maturation includes any physical or mental change that may occur within individuals over time including learning, aging, and fatigue. Including a control group helps to account for these conditions as both the intervention and control groups will have been affected by the same conditions outside of the intervention sessions (Johnson & Christensen, 2008). To further ensure that conditions for the groups are even students can be randomly assigned to one of the two groups. The groups can also then be checked to ensure they are similar in the number of girls and boys, English language learners, Year 4s and 5s, scores on the pre-test etc. This form of matching is another way of providing control in an experiment (Cresswell, 2012).

Group 2, the visualising only group, was intended to and did act as a control group during Group 1's, the visualising and drawing group, intervention. The control group allowed for a comparison between the intervention conditions and typical conditions (Johnson & Christensen, 2008). Therefore, the control group was exposed to the same classroom conditions as the intervention group which increases the likelihood that any increase in scores for the intervention group was as a result of the intervention. During Group 2's intervention, Group 1, acted as the

control group, receiving no additional support and experiencing the same classroom conditions as Group 2.

3.2.2 Design of Intervention Sessions

Each intervention was designed to have three sessions and for each session to follow the same pattern. There were to be three sessions, preferably on a Monday, Wednesday and Friday of the same week to have the sessions completed in a short time frame. A description of the design of Intervention One will follow.

The sessions were designed to begin with the researcher modelling one mathematics word problem by reading the question, explaining what was being visualised through the use of a think-aloud technique, and then drawing a picture while talking through the thinking behind the drawing (For examples of the researcher modelling, see Appendix 3). The students in the group were to then complete their own examples for three other word problems, one question at a time. Each student would complete their own drawing which they felt would help them solve the problem and then attempt to solve the problem by writing down any working and the answer on the sheets (Appendix 7). Apart from being instructed to create a drawing or representation, the students would not be instructed in how to solve the problems. All students would attempt to solve the same three questions in each session. After each problem, two to three students were to be asked to explain their drawing. This meant every student explaining a drawing in every session at least once. The researcher would ask questions to have students elaborate or to find out why they had made certain decisions.

At the end of each intervention session the students would complete a written reflection (Appendix 4) and two to five students were to be interviewed by the researcher to get their verbal reflection on the session. The verbal reflections were to enable the researcher to question specific

points that had arisen during the session or obtain elaboration regarding the answers they had given.

Intervention Two with Group 2 was designed to follow the same structure as Intervention One with the only difference between the two interventions being the strategy being introduced. The modelling by the researcher and the instructions to the students were to focus on only visualising the context of the mathematics word problem and to not involve drawings. The questions used during the session were to be the same as for Group 1, as would the written reflections and interviews at the end of each session. During Intervention Two, Group 1 was to act as a control group that received no further support.

To gauge the effectiveness of the intervention, data collected from the interventions included test scores, question analysis, think-aloud sessions, the representations the students created, and from written and verbal reflections. These different data collection methods are discussed in the next section.

3.3 Data Collection

Research data were collected in a variety of ways (Table 3.1). There were four tests carried out by all students (Section 3.2.1). From these tests, the questions were also reviewed to analyse students' success with each type of word problem. There were two think-aloud sessions, pre- and post-interventions, where the students were video recorded individually solving word problems and explaining their thinking and any representations they had produced. To collect other supporting data the intervention sessions were video recorded, the students completed written reflections and oral reflections of each session were also video recorded. All work completed by students including drawings and mathematical working out was collected. Pictures completed by students were collected and photographed and oral explanations of their pictures were video

recorded. Each data gathering tool is now discussed in turn. Discussion includes rationale for the choices of method, tool development, and data collection.

3.3.1 Tests

Tests are commonly used as a way to measure the performance, attitudes, and perceptions of research participants (Johnson & Christensen, 2008). The tests used in this study were “experimenter-constructed” tests which were made to ensure they covered the specific areas that were needed (Johnson & Christensen, 2008, p. 202).

An initial test was designed which included 10 number only questions ($12 - 5 = \underline{\quad}$) and 10 word based problems (Beth had fourteen oranges and gave six to her brother and his friends. How many oranges did she have left?). These included three addition, three subtraction, two multiplication, and two division problems. The range of operations (+, -, x, ÷) were used as a focus of the study was to see if students could choose the correct operation needed to solve word problems. The initial draft test was trialled on ten Year 4 students at a school other than the study school with similar location (i.e., inner city), decile rating, and population diversity. After the first trial, the number only questions (e.g., $6 + 7 = \underline{\quad}$) were changed to include two change unknown questions (Carpenter, Fennema, Franke, Levi, & Empson, 1999) (e.g., $7 + \underline{\quad} = 12$) to provide more direct comparison between the word problems. Upon analysis of the trial results it was also discovered the word problem set had more division problems than the number only problems. As the students found the division problems more difficult, this suggested the word problem section would have been more difficult than the number only problems. The word problem test was modified to have a similar balance of operations to the number only questions. The wording was also changed to have the word problems matched in difficulty with the number

only problems (e.g., fifteen divided by three, was changed to fifteen divided by five, to match a divided by five number problem).

The second test draft was then trialled by eight students from the trialling school. Using results from the two testing sessions, a final test (Appendix 1) was created with 10 word problems and 10 number problems with matching mathematical difficulty between the word problems and the number only problems (for example, adding a single digit to a double digit, multiplying by 5, dividing by 10). This structure was intended to allow comparison of results between similar questions in each of the number and word formats. Parallel tests were then created for the pre, post, and repeated post-testing resulting in four tests being made (Appendix 1).

Data collected from the tests included a raw score out of ten for number only problems and for word problems. The times taken to complete the number only problems and the word problems were collected. Also recorded was the number of times that students selected the correct operation for a word problem but made a calculation error. Scores were compared on a group and individual level. Each type of question (addition, subtraction, multiplication, division, change unknown) was analysed in an attempt to see if the intervention strategy was more effective with specific types of questions.

3.3.2 Think-Alouds

As well as being a successful teaching strategy for modelling desired thought processes and behaviours (Davey, 1983; Ministry of Education, 2006; Silbey, 1999), think-alouds, or verbal protocol analysis, are a tool used for collecting data about cognitive processing (Ericsson & Simon, 1980, 1993; McGuinness, & Ross, 2003). Think-alouds involve the participant verbalising everything that they are thinking as they set about solving a problem or completing a

task. While there has been much debate about the validity of verbal information as reliable data, several researchers have shown that verbal data can be as valid as any other data (Charters, 2003; Ericsson & Simon, 1980, 1993; Russo, Johnson & Stephens, 1989). Opposition to the reliability of verbal data includes the claim that merely by talking or being asked to verbalise thinking, the thinking processes change. However, Ericsson and Simon (1980) showed that when participants concurrently verbalise and complete the task this is not an issue.

Even though there are conflicting views on the usefulness and validity of think-alouds, Russo et al. (1989) conclude that “on the basis of our own experience with verbal protocols and other process-tracing data (e.g., eye movements and manual responses), we believe that nothing can match the processing insights provided by a verbal protocol” (p.767). Charters (2003) adds to this by stating that think-aloud research has a “strong theoretical foundation and confirms its value as a way of exploring individuals’ thought processes” (p. 80).

Prompting during think-alouds needs to be kept to a minimum (Charters, 2003; Ericsson & Simon, 1980) to lessen the chance of altering what the participants say. If prompting is necessary then any prompt used needs to be general such as what are you thinking now, or keep talking (Charters, 2003; Draper, n.d.) so as not to bias the participant by adding any external information to their internal processing (Johnstone, Bottsford-Miller, & Thompson, 2006). Russo et al. (1989) also state that minimising prompting during the think-aloud and ensuring effective instructions assists with the validity of data collected.

Think-alouds were used in this study for two purposes: 1) as a teaching strategy for the researcher to show the desired behaviours, and 2) as a data collecting tool for students to explain their thinking and therefore to enable the researcher to analyse the thought processes of the students. The researcher used the think-alouds in each of the intervention sessions with each

group when modelling one question at the beginning of the session. Formal think-alouds were used by the students in a pre-testing situation to collect baseline information on the participants' ability to visualise and explain their thinking, and a second time following the completion of the interventions to allow direct comparison of any change in behaviour or ability. These think-alouds were analysed for the details provided, and the accuracy with which the mathematics concept was identified.

Think-alouds can be concurrent (completed as the task is being completed) or retrospective (completed after the task has been completed) (Ericsson & Simon, 1980). Both concurrent and retrospective think-alouds were used in this study at different times and for different purposes. Concurrent think-alouds were used as the students solved word problems in their heads and explained their thinking as they went through the processing of doing so. All students completed a pre and post think-aloud under these conditions. Concurrent think-alouds were used by the second intervention group to explain what they were visualising during their intervention sessions. Retrospective think-alouds were used by all students when explaining the drawings they had made to help solve a word problem. The retrospective think-alouds were used during the first group's intervention to explain what they had been thinking when they drew their pictures. Two prompts were used if needed to keep the students talking. These were 'keep talking' and 'what are you thinking now?' (Charters, 2003; Draper, n.d.). These were used to keep the students talking without influencing what they might have been thinking or doing.

A verbal protocol was developed based on one used by Johnstone et al. (2006) (Appendix 2) to ensure that all students received the same instructions and conditions regardless of the fact that they were interviewed individually. This protocol was a script which the researcher read to the students to explain what was being asked of them. It had the instructions and word problems on it.

The researcher modelled the expected behaviours/thinking processes using think-alouds in each of the three teaching sessions (Appendix 3). As well, think-alouds were used by the students during the intervention sessions to explain the processes they were going through as they either explained their thinking or explained their drawings.

3.3.3 Reflections

The students' thoughts about word problems and the strategies being used were needed to help answer the research questions specifically if a strategy could be devised which students could use easily so reflection times were built into the sessions. The reflections were used to collect the participants' thinking and to see if there were any changes in students' thoughts, attitudes, or perceptions about the strategy or their own abilities. At the end of each intervention session the participants completed a short reflection on the session. These were either in written form by completing a questionnaire or in a verbal form in an interview which was recorded.

Written Questionnaire

Questionnaires can be used for a vast array of purposes and can be a useful tool for collecting data (Cohen et al., 2011). For this study, the questionnaire was being used to collect data on what the students were doing and how they were feeling about the word problems and the strategies. A short written questionnaire (Appendix 4) was designed and trialled by two students from the trialling school. The trialling involved the students expressing if they understood the wording of the questions but the questionnaires were not filled in following a session. The students responded along a continuum using three faces (sad, ambivalent, happy) as markers (Appendix 4). The questionnaire also had space for comments to explain choices or allow the participants to write any comments they thought relevant. This questionnaire could provide both quantitative data (positioning on continuum) and qualitative data (comments).

Verbal Interview

Verbal interviews are a widely used and flexible tool for collecting data (Cohen et al., 2011). The recorded verbal reflection used in this study followed an interview guide approach (Johnson & Christensen, 2008). This involves having questions written before the interview which explore specific areas (similar to the written reflection) but allows certain areas to be explored deeper or a certain point to be elaborated upon as required. The interviewer is able to redirect/refocus the interview through the set questions if needed. The set questions were the same as the questions used for the written reflection (Appendix 4) which provided a starting point for the interviews.

3.3.4 Representations and Drawings

Collecting external representations in the form of drawings or pictures is a valid data gathering method (Zweifel & Van Wezemael, 2012) and can be especially useful to add to data collected using other tools such as interviews (Kose, 2008). As visual images are constructed in specific settings or in response to specific events, care must be taken when they are analysed not to view them without consideration of those factors. Care must also be taken when they are viewed to remember the viewer will have their own interpretation of them. It can often be helpful to have a written or verbal text to accompany images (Cohen et al., 2011).

Representations were used in both pre-testing and post-testing think-aloud sessions for all students and were supported by oral explanations of what the drawings showed. They were also used in the intervention sessions with Group 1 being instructed in the use of pictures to support them in solving word problems. Students were not told what to draw but to draw whatever they thought could help them solve the problems. As well as being used as a teaching and learning tool, the representations produced were collected to be used as data.

For the purpose of this study, representations were classified as pictorial, schematic, notational, or a combination of these. The definitions used in the study for the different types of representations follow. Pictorial drawings present the problem in a realistic manner and include expressive or extraneous information that reflects the setting of the problem, but do not contain information that will help solve the problem (Edens & Potter, 2010). There is no reference to the numbers. Schematic drawings include key information from the problem which may be represented in a way which shows relationships or proportions between the objects in the problem (Edens & Potter, 2010; Hegarty & Kozhevnikov, 1999) and use abstract or graphic symbols rather than realistic pictures. Again there is no reference to the numbers from the word problem. Notational representations include using the actual numerals and symbols (Mulligan, 2002). Drawings could also be a combination of more than one of these categories.

Examples drawn from the study data showing the five key drawing types follow. Note in the explanations the key differences between the categories:

Figure 3.1 shows a pictorial representation. This drawing realistically depicts the situation described in the word problem but makes no mention of the numbers or relationships described in the word problem.

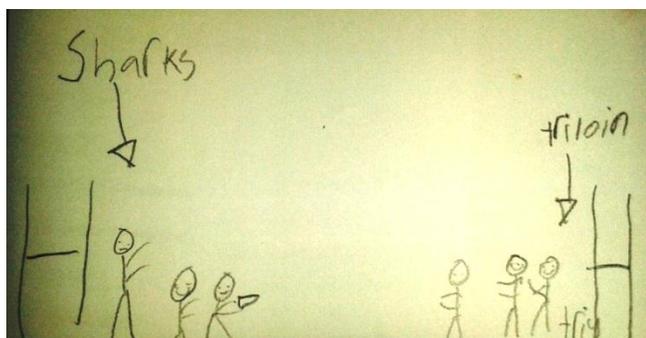


Figure 3.1. Example of Pictorial Representation

In Figure 3.2 a schematic representation is shown. This representation involves the animals in the question being represented using abstract symbols and the relationship described in the question is evident with five in each cage. There is no mention of the numbers.

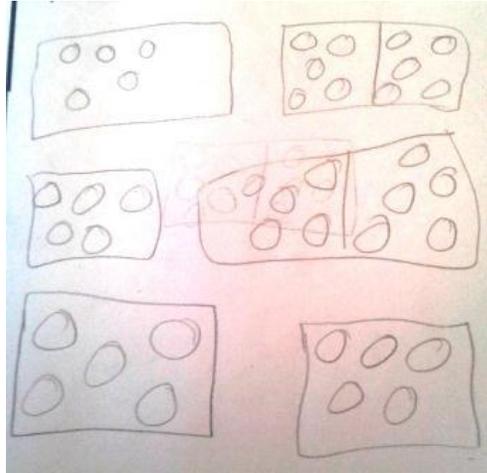


Figure 3.2 Example of Schematic Representation

In Figure 3.3 an example of a notational representation is presented. Only the numbers from the problem have been used to create an equation with no other representation present.

$$32 + 8 = 40$$

Figure 3.3. Example of notational representation

Figure 3.4 is an example of a pictorial-notational representation. It involves a picture which realistically represents the context described in the question. The picture would not appear to help to solve the problem but in this instance an equation involving the numbers from the question is also used.



Figure 3.4. Example of pictorial and notational combination

In Figure 3.5 a schematic-notational representation is shown. The representation depicts the relationship described in the word problem (which pictorial representations do not) leading to an equation. Nine cans have been drawn with the number 4 used to show how many balls are in each can. This has been used to illicit an equation. What differs from the schematic representation is the use of the number 4 in each can rather than drawing four balls.

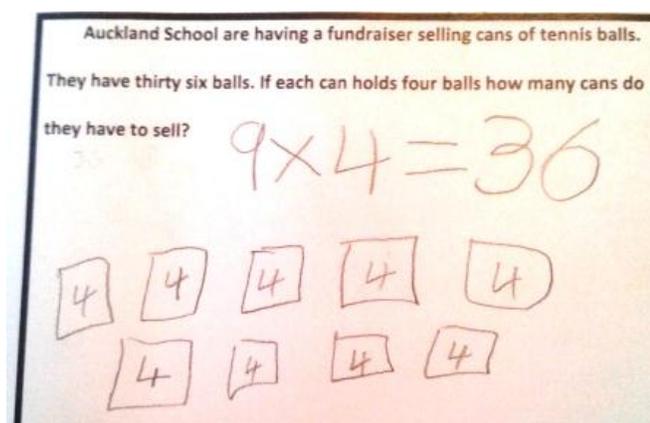


Figure 3.5. Example of Schematic and Notational Combination

This section has outlined the data collecting tools used in this study. The next section goes on to detail how the data collected using these tools were analysed.

3.4 Data Analysis

Tests - The quantitative test score data were analysed in four areas: 1) how many number only problems were solved correctly; 2) how many word problems were solved correctly; 3) how many times the correct operations were used when solving word problems as indicated by a correct equation or representation which matched the correct equation; 4) the time taken to complete both the number only and word problems. These test scores were collated and compared on a group level using the baseline data collected prior to the interventions. The data were analysed using SPSS Version 20 (IBM, 2011).

Types of questions - A second level of analysis occurred on the data collected from the tests. This involved looking at the questions answered and compared separately the successful solving of each type of problem (+, -, \times , \div).

Think-alouds - The first part of the think-aloud, which involved the students describing everything they were thinking when solving the word problem, was analysed for the content of the students' thinking and whether the students described the context of the word problem or went straight to working with the numbers. The second part of the think-aloud, involving the students making a representation that would help them solve the word problem, was also analysed for the content of the representations and whether they were pictorial, schematic or notational (numbers only). The post intervention think-aloud results were compared to the baseline data collected prior to the intervention sessions.

Reflections - The interviews and reflection sessions at the end of each intervention session and after the final testing session were video recorded. Student responses were analysed to identify common themes across the students' responses.

Representations - In the intervention sessions students were recorded explaining what they had produced and why. All representations in the testing session and the think-alouds sessions were analysed by categorising them as pictorial, schematic, notational, or a combination of these (Section 3.3.5) as well as comparing representation type to successful solutions. The representations were analysed in relation to whether they captured the mathematical concept in the problem, or if they were purely a literal representation of the situation, and the extent the picture may have affected the accuracy of the answers. The baseline data were compared to the post-testing data in Test 4 to explore any change in the type of representations the students produced.

3.5 Participants

Convenience and purposeful methods of sampling (Creswell, 2012) were used to identify potential participants of this study. This school was selected for two reasons: 1) a relationship already existed between the researcher and the school, and 2) the sample maximised relevance to the researcher enabling findings to be relevant for future teaching. The participants were from an inner city, decile 6, New Zealand school.

The age group used for this study was Year 4-5. This age group was chosen as it was these levels where the school had identified in previous years that students appeared to struggle with an increased amount of language and word-based problems they encountered in mathematics. It is also at Year 4 that students begin to be assessed with more formal assessments including e-asTTle, which can have many word problems.

There were three teachers who taught Year 4 and 5 students in the school. All three were invited to participate in the study. Two teachers returned permission forms and put forward the

names of students, so these were the two classrooms that students were chosen from for the study.

The two teachers were asked to identify students in Year 4 or 5, who had some basic facts knowledge, but who struggled to solve word problems, and in the teachers' view, possibly achieved lower when word problems were involved than on number only problems (possibly signified by a lower than expected e-asTTle score (Ministry of Education, 2010d)), and who, in the teachers' view, would benefit from access to another strategy for solving word problems.

Initially sixteen students were identified. From these sixteen a group of twelve were selected on the basis they had all completed e-asTTle (Ministry of Education, 2010d) tests earlier in the year. Working with a sample of twelve students would provide two groups of six whilst also allowing for potential withdrawals. The twelve chosen students were approached and the study explained to them. Information and consent forms (Appendix 5) were then given to the students for them and their parents. Ten of the families and students agreed to be part the study. One family opted not to participate and one family did not return the forms so the student was not included. Of the ten students there were two Year 5 and 8 Year 4 students; four were male, and six were female (Table 3.1).

Table 3:1

Description and Characteristics of Students Taking Part in Study

Name*	Group	Gender	Year Level	Class	Ethnicity	Other
Jack	1	Male	5	B	Maori	ADHD, SLD
Molly	1	Female	4	B	Samoan	ELL
Anna	1	Female	5	B	Maori	
Rebecca	1	Female	4	B	Maori	
Max	1	Male	4	A	Korean/NZE	
Frank	2	Male	4	A	NZE	
Matt	2	Male	4	B	Samoan	ELL, SLD
Amber	2	Female	4	B	Chinese	ELL
Amanda	2	Female	4	B	Tanzanian	ELL
Lucia	2	Female	4	B	NZE	Dyslexia

- Pseudonyms used were chosen by participating students

Note: NZE – New Zealand European

ADHD – attention deficit hyperactivity disorder

SLD – specific learning disability

ELL – English Language Learner

3.6 Intervention and Testing

This section outlines the process used for the intervention and testing (Table 3.2). Once selected, all students took part in four testing sessions over the course of the study. There was a pre-test before any interventions took place (Test 1) to collect baseline data. Group 1 received the first intervention as designed and then following this a second test took place (Test 2). Group 2 then received their intervention as designed and at the conclusion of their three sessions another test occurred (Test 3). Three weeks after Group 2's final session a post-test took place (Test 4). There were two pre-testing data collection activities. The first of these was a pre-test (Test 1) of twenty problems with similar mathematical demands, ten number only problems and ten comparable word problems (Appendix 1). The students were each timed separately for completing the number only questions and the word problems. The second activity involved

video-recording students individually solving two word problems using a think-aloud protocol (Appendix 2). For the first word problem the students were asked to verbalise everything they were thinking as they solved the problem. For the second they were asked to draw a picture or representation of whatever would help them to solve the problem (Appendix 6). The researcher asked each student follow-up questions as needed to clarify their responses.

After the pre-testing sessions and baseline data had been collected, students were allocated into two groups using an alphabetically ordered list. After this initial grouping students were reorganised to ensure that the groups were as equal as possible in regards to gender, pre-test scores, which class they were from, and the times taken to complete the word problems.

The two groups were exposed to different versions of the intervention (Section 3.2.2). Group 1, the visualising and drawing group, had their intervention first. The focus for this group was on visualising the word problems and then creating an external representation. After being asked to visualise what they could 'see' in their heads, students in Group 1 were asked to produce an external visualisation of the word problem. The students were not instructed as to what to produce but to produce whatever representation would help them. Group 2, the visualising only group, had their intervention following the completion of Group 1's intervention. Group 2's intervention involved them only being instructed to visualise the word problems and then solve the problems in whatever way they saw fit. Intervention Group 2 was not instructed to produce any representations. This was to allow for some comparison between producing a drawing and not producing a drawing which would show if the act of internal visualisation is helpful when attempting to solve word problems.

Each intervention session followed the same planned pattern (Section 3.2.2) and all were video-recorded. At the end of each intervention session the students completed a written

reflection (Appendix 4) and as many students as possible were interviewed depending on factors which included the time available at the end of the session and whether the students had other events to get to.

The timing for the sessions over the space of one school week did not happen as planned due to having to fit in with the teachers' and school's timetables and events. However, the three sessions were completed over a time frame of six days, so the initial aim of completing the session over a short time frame was still achieved.

After the three intervention sessions the five students from the intervention group individually completed a second think-aloud activity (Appendix 2 and Appendix 6) solving two new problems, which enabled comparison against their initial efforts. All ten students then completed a post-test (Test 2) to enable results to be compared to the scores from the pre-test (Test 1). Group 1 had no further intervention.

There was a gap of two weeks before the second group began their intervention due to the school having an event in this period. The second stage of the study involved Group 2, the visualising only group, completing a new intervention with the researcher. The intervention sessions followed the same pattern as for Group 1 with three sessions over six days. The same sequence was followed and the same nine word problems were used. The difference between the interventions was that Group 2 was not modelled the drawing part of the process (For example of the teacher modelling, see Appendix 3). They were encouraged to visualise the problem and would verbalise this before solving the problem, without being prompted to draw. The researcher started each session by modelling one question. Through use of think-alouds, the researcher modelled what he was thinking, specifically what images were being visualised.

At the completion of their three sessions, Group 2 completed the think-aloud activity for a second time and all ten students again completed a further post-test (Test 3). During the instructions for the third test, Group 1 was instructed to read the question, make a drawing or representation that would help them, and then solve the problems. Group 2 was instructed to read the question, visualise in their head what the problem was about and then solve the problem.

It was noted during Group 1's intervention that the written reflections were very time consuming, the students needed a lot of support to finish them, and the responses given by the students did not add meaningful information. Due to the issues noticed during Group 1's intervention, the written reflections were not used with Group 2. The verbal reflections provided far greater insights than the written ones so only verbal reflections were used with Group 2.

Three weeks after the third testing session a final testing session (Test 4) was held to monitor retention of the strategies taught and to see if the earlier improvements noted were maintained. Following Test 4 there was a reflection session which was recorded. The students had a last opportunity to reflect and offer their thoughts about word problems, using drawings and visualisation. Further think-alouds were not collected because the purpose of the final testing session was to assess how successful the students were in solving the problems rather than how they were solving them.

Table 3:2

Timeline of Intervention and Data Collection

Intervention One		Intervention Two
Group 1-Visualising and Drawing	Day	Group 2- Visualising Only
<p>Baseline data collected (Test 1) Test scores – word problems, number problems time taken (quan), type of question analysis (quan) Representations collected (qual & quan)</p> <p>Think-aloud 1 Verbal explanations collected (qual & quan) representations collected (qual & quan)</p>	1	<p>Baseline data collected (Test 1) Test scores – word problems, number problems time taken (quan), type of question analysis (quan) Representations collected (qual & quan)</p> <p>Think-aloud 1 Verbal explanations collected (qual & quan) representations collected (qual & quan)</p>
<p>Intervention session 1 Written and Verbal reflection (qual) Representations collected (qual)</p>	3	
<p>Intervention session 2 Written and Verbal reflection (qual) Representations collected (qual)</p>	7	
<p>Intervention session 3 Written and Verbal reflection (qual) Representations collected (qual)</p>	8	
<p>Test data collected (Test 2) Test scores – word problems, number problems time taken (quan), type of question analysis (quan) Representations collected (qual & quan)</p> <p>Think-aloud 2 Verbal explanations collected (qual & quan) representations collected (qual & quan)</p>	9	<p>Test data collected (Test 2) Test scores – word problems, number problems time taken (quan), type of question analysis (quan) Representations collected (qual & quan)</p>
<i>School had event on so two week gap between interventions</i>		
	22	<p>Intervention session 1 Written and Verbal reflection (qual) Representations collected (qual)</p>
	23	<p>Intervention session 2 Written and Verbal reflection (qual) Representations collected (qual)</p>
	27	<p>Intervention session 3 Written and Verbal reflection (qual) Representations collected (qual)</p>

<p>Test data collected (Test 3) Test scores – word problems, number problems time taken (quan), type of question analysis (quan) Representations collected (qual & quan)</p>	29	<p>Test data collected (Test 3) Test scores– word problems, number problems time taken (quan), type of question analysis (quan) Representations collected (qual & quan) Think-aloud 2 Verbal explanations collected (qual & quan) representations collected (qual & quan)</p>
<p>Test data collected (Test 4) Test scores – word problems, number problems time taken (quan), type of question analysis (quan) Representations collected (qual & quan) Final reflection session (qual)</p>	50	<p>Test data collected (Test 4) Test scores– word problems, number problems time taken (quan), type of question analysis (quan) Representations collected (qual & quan) Final reflection session (qual)</p>

Note – qual – qualitative data collected

quan – quantitative data collected

3.7 Ethical Considerations

Full ethical approval was granted by the Victoria University of Wellington Ethics Committee (Ethics Approval: 20756, 9 April 2014). This study followed the NZARE ethical guidelines (New Zealand Association for Research in Education, 2010).

Once ethical approval had been granted by the ethics committee, permission was sought from the principal of the school to be allowed to work in the school. Then the classroom teachers were approached and after having the study and what would be expected explained, permission was sought from them.

Active consent was sought from both the students and their parents (Johnson & Christensen, 2008). Active consent means that the parents and students had to sign and return the forms agreeing to be part of the study, as opposed to *passive consent* (Johnson & Christensen, 2008) where participants only return forms if they wish not to be part of the research. The students and their parents were given information packs outlining what the purpose of the study

was, and what the students would be asked to do, and asked for permission (Appendix 5). The students also had the study explained verbally to ensure they understood what was being asked of them. Part of the conditions that all participants, including the principal, teachers, students, and parents agreed to, was the right to withdraw consent at any time up to a given date when all data had been collected. Pseudonyms are used in the report to ensure confidentiality for the participants.

The ethical treatment of research participants and the right to protection from physical and mental harm is the most important issue confronting researchers (Johnson & Christensen, 2008). To have the minimal impact on the participants, the researcher consulted with teachers when working out session times to ensure the least interruption to the students as well as ensuring they were not missing out on what they or the teacher deemed to be important or fun. All sessions were outside normal class mathematics sessions to ensure the participants did not miss out on their normal mathematics instruction which their peers were receiving.

3.8 Validity and Trustworthiness

The use of a pre-test post-test control group design offers a strong research design as it allows for extraneous variables that could impact on the study (Johnson & Christensen, 2008). The parallel tests used in the four testing situations were designed to be equal as they would ensure that the data collected at different stages could be compared.

The triangulation of data using a variety of data collecting tools can help researchers to ensure their data are reliable and trustworthy (Johnson & Christensen, 2008; Newby, 2010). The use of the representations and students' comments in this study helped to corroborate the data gathered from the tests and strengthen the analysis.

A concern raised around the use of think-alouds is that by using the process of verbalising to explain thought processes, the thought processes can be altered (Charters, 2003). Ericsson and Simon (1980) express that the use of concurrent think-alouds limits the impact that the think-alouds have on thought processes and so any data from think-alouds collected with care can be “a valuable and thoroughly reliable source of information” (p. 247).

Another consideration in all types of research is the Hawthorne effect (Newby, 2010). This effect is concerned with the possibility that just by being part of an intervention or knowing they are being observed, participants put in a greater effort which could lead to an improvement. There has been criticism of the original research upon which the Hawthorne effect is based and the widespread acceptance of this phenomenon (Rice, 1982); however, this issue is a consideration relevant to this study.

3.9 Summary

This chapter has outlined and justified the research paradigm and methods used in this study. Also discussed were the participants, the method, and the data collecting tools used as well as how the data were analysed. Ethical considerations as well as validity and trustworthiness of the design and data were also discussed. The next chapter will introduce the study results drawn from analysis of the data collected using these tools.

Chapter Four

Results

This chapter presents the analysis of the qualitative and quantitative data collected during this study. It begins by reporting on the students' thoughts on mathematics word problems before and after the interventions. The chapter then moves on to the quantitative data collected at the different stages of the study to show the extent to which the interventions altered student ability to answer number only and word problems. Following this, further analysis of qualitative data is presented exposing the students' thoughts on using drawings and visualising, and the styles of drawings used by the students.

4.1 Students' Perceptions of Word Problems Before the Interventions

At the start of the first intervention session for each group, the students were shown two problems, one a word problem and the second the matching number only question. Every child identified the word problem as being the more difficult to solve. When asked to explain why they felt this there were four main themes to their responses: knowing which operation to use, challenges finding the numbers, greater cognitive demands, and the structure and semantics of word problems. Examples of each follow.

Students reported difficulty regarding which operation was needed to solve the word problems:

Jack (Group 1): It doesn't really tell you if it's times or what.

Lucia (Group 2): ... it's easier there (pointing to number only question) because you see what it is and just write it down but that (pointing to word problem) you have to read it, see what it's telling you because it could be times, divided by and that... (points to number only question) you just see the plus sign.

A second difficulty students identified in the word problems was the presence of the words, with some students not easily identifying that the word problems contained numbers:

Matt (Group 2): It's different because it has no numbers.

Lucia (Group 2): Um, it's kinda like Tim has seven tennis balls and it's words and you have to kind of... it's easier there (pointing to number only question) because you see what it is and just write it down.

A third area identified by the students was the extra thinking and time that they felt was involved in solving word problems compared to number only problems. The students identified that steps are needed to be taken before being able to solve the problem:

Rebecca (Group 1): You have to find the numbers in it and know which one [operation] it is.

Max (Group 1): It sort of takes a little bit longer cause you don't really know what kind of ...it's easier if you do this one (pointing to number only question) cause then you can just figure it out.

A fourth difficulty which the students discussed during the intervention sessions was the difficulty with the semantics of the problem. Often words could have different meanings to what might be expected due to the way the word problems were structured. This was especially a problem with the words 'less' and 'more' which appeared in addition and subtraction problems:

Max (Group 1): Because I thought it was a takeaway but it was really a pluswell, because he had less, Sau had *less* than his brother so I thought I had to take away.

Matt (Group 2): Well, when it's *less* I don't know if it's a take away or a plus.

Evidence from this study shows that all students felt that word problems were more difficult than number only problems and they realise that extra work is needed to solve word problems than number only problems. The comments also show that students are aware of some of the difficulties that mathematics word problems present (e.g., finding the numbers and understanding the problem). These results are consistent with the extra cognitive work needed by

having mathematics presented in word problems, a format which increases the extraneous cognitive demands and the overall cognitive demands on students (Sweller, 1994). These study results add to what is known about the cognitive demand associated with word problems by showing that students recognise that more work is required in relation to finding the operation and the numbers in the word problems. During the initial pre-test the researcher observed many of the students working out basic facts, whether by counting on in their head, by using their fingers, or using simple drawings. This suggests that even relatively simple equations in number only form such as $7 + 6$ seem quite demanding for the students in this study, so once the mathematics content of basic facts is put into a word problem format the cognitive demand on the students is increased which means accuracy and learning can be difficult (Sweller, 1994).

In summary, before the interventions the students in this study felt word problems were difficult to solve and identified several factors which they felt made word problems more difficult than number only problems. The next section discusses the students' perceptions of word problems after they had taken part in the interventions.

4.2 Students' Perceptions of Word Problems After the Interventions

In the final video-recorded reflection session, eight of the nine students present identified that they felt they were better at solving word problems following the interventions. Comments from the students indicated increased confidence and enjoyment when solving word problems:

Jack (Group 1): You can finally understand them.

Lucia (Group 2): After I did this I found that word problems are easier now.

Frank (Group 2): This whole thing has actually helped me a lot as before I didn't really understand word problems and now I understand them.

Max (Group 1): I find that drawing pictures is fun and it helps you to make it a lot easier to find out the problem.

Amber (Group 2): I found that I kept on working it out in my head. It seems a lot easier now. Even though it was really hard it was fun. I'm starting to like maths now.

However, one student felt he still struggled with the word problems and that understanding the questions was still a problem specifically understanding specific words (e.g., piles).

This largely positive change in attitude by most of the students about their ability to solve word problems is a significant step forward, particularly as, in my experience, confidence and a positive attitude can be important for effective learning. Rather than looking at word problems and seeing what is difficult about them or feeling they may have reasons not to be able to solve them, these results suggest that the students are now more likely to look at word problems with a positive attitude and the belief that they have a strategy they can use to help them be successful.

4.3 Quantitative Results

Quantitative test score data were collected from all ten students at four points during this study (Section 3.3). The full test results (Appendix 8) show raw scores for each individual child for each of the four testing sessions as well as group averages for the two intervention groups.

Tables 4.1 and 4.2 show paired t-test comparisons over time between Test 1 (pre-test) and Test 4 (post-test) for Intervention Group 1 (Table 4.1) and Intervention Group 2 (Table 4.2) for the number of correct number problems, number of correct word problems, and the number of operations chosen correctly. These two tests were chosen as they best show the long-term effects of the intervention with Test 4 taking place three weeks after the final intervention session. The paired sample t-test provides information about the development of students over the entire eight week period. Group 1, the visualising and drawing group, showed significant improvement for

the number of number problems solved correctly ($t(4) = 6.5$, $p=0.003$), and the number of word problems solved correctly ($t(4) = 5.416$, $p= 0.006$), and there was a trend for the number of operations chosen correctly ($t(4) = 2.654$, $p=0.057$).

Table 4.1

Paired t-test Comparison Between Test 4 and Test 1 for Intervention Group 1 (Visualising and Drawing)

	Number	Mean Difference between test 4 and test 1	Std. Dev.	t	P
Number problems	5	2.6	0.894	6.5	0.003
Word problems	5	4.4	0.812	5.416	0.006
Correct operations	5	2.6	0.98	2.654	0.057

Table 4.2 shows similar data for intervention Group 2. Group 2, the visualising only group, showed significant improvement for the number of number problems solved ($t(4)=4.472$, $p=0.11$) and there was a statistical trend for both the number of word problems solved correctly ($t(4) = 2.764$, $p = 0.051$) and for the number of operations chosen correctly ($t(4) = 2.449$, $p = 0.070$).

Table 4.2

Paired t-test Comparison Between Test 4 and Test 1 for Intervention Group 2 (Visualising Only)

	Number	Mean Difference between test 4 and test 1	Std. Dev.	t	P
Number problems	5	2.0	1.0	4.472	0.011
Word problems	5	3.2	2.5	2.764	0.051
Correct operations	5	1.8	1.6	2.449	0.070

An independent t-test was used to compare the progress of the two groups over time (Table 4.3). Results showed that there was no significant difference between the two groups with regard to change over the time period from Test 1 to Test 4. In summary, these results suggest that both approaches that the students were instructed in (for Group 1, visualising and drawing, for Group 2, visualising only) enabled the students to make significant progress over the course of the interventions but there was not a significant difference in progress made between the two approaches.

Table 4.3

Comparison Between the Two Groups from Test 1 to Test 4

Variable	Group	Number	Mean Change	Std. Dev.	t	df	P
Number problems	1	5	2.60	0.89	1.000	8	0.347
	2	5	2.00	1.00			
Word problems	1	5	4.40	1.82	0.849	8	0.421
	2	5	3.20	2.59			
Correct operations	1	5	2.60	2.19	0.653	8	0.532
	2	5	1.80	1.64			

In order to examine the mean change between the two groups over specific time periods independent t-tests were used (Table 4.4). The mean change in the number of correct number problems, word problems, and the number of operations correctly chosen between Test 1 and Test 2 (covering group 1's intervention) was not significantly different between Group 1 (intervention group) and Group 2 (acting as a control group) (number problems, $t(8) = 1.835$, $p = 0.104$; word problems, $t(8) = 1.855$, $p = 0.101$; correct operations, $t(8) = 1.964$, $p = .085$) but there was a trend towards a statistically significant difference in all three areas (i.e., $p \leq 0.10$).

Table 4.4

Comparison Between Groups from Test 1 to Test 2

Variable	Group	Number	Mean Change Test 2 – Test 1	Std. Dev.	T	df	P
Number problems	1	5	1.6	1.14	1.835	8	0.104
	2	5	0.0	1.58			
Word problems	1	5	3.4	2.51	1.855	8	0.101
	2	5	0.0	3.24			
Correct operations	1	5	1.8	1.48	1.964	8	0.085
	2	5	0.0	1.41			

A second independent t-test was used to compare the progress between the groups over the period of time covering Group 2's intervention sessions, Test 2 and Test 3 (Table 4.5). Results show that the mean changes in the number of word problems and the number of operations chosen correctly between Test 2 and Test 3 were statistically significant and there was not a significant difference for the number only problems. Evidence for this is provided by the data which was for word problems, $t(8) = -3.317$, $p = 0.011$, for the correct operations, $t(8) = -2.449$, $p = 0.040$, and for number only problems, $t(8) = -1.725$, $p = 0.123$.

Table 4.5

Comparison Between the Two Groups Between Test 2 and Test 3

Variable	Group	Number	Mean Change Test 3-Test 2	Std. Dev.	t	df	P
Number problems	1	5	-0.6	1.51	-1.725	8	0.123
	2	5	1.00	1.41			
Word problems	1	5	0.80	0.84	-3.317	8	0.011
	2	5	3.00	1.22			
Correct operations	1	5	0.80	0.84	-2.449	8	0.040
	2	5	2.00	0.71			

What Tables 4.4 and 4.5 do show is that both groups made their biggest improvements over the period of their intervention. This is a positive indicator as it suggests that it was the interventions which had an impact.

A paired t-test was used to compare the number of correct number problems with the number of correct word problems (Table 4.6) for each of the groups. As a major concern was the disparity between students' achievement on number problems and word problems this t-test was used to see if the gap between the two lessened between Test 4 and Test 1. For Group 1 there was a statistical trend towards improvement ($t(4) = 2.250$, $p=0.088$). For Group 2 there was no statistical significance ($t(4) = 0.862$, $p = 0.438$).

Table 4.6

Comparison Between the Number of Word Problems and Number Only Problems Solved Correctly from Test 4 to Test 1

	Group	Number	Mean Difference between test 4 and test 1	Std. Dev.	t	p
Difference Between Number Problems and Word Problems	1	5	1.80	1.789	2.250	0.088
	2	5	1.20	3.114	0.862	0.438

An independent t-test was used to compare the difference between the two groups and see if there was any statistical significance between the approaches used for each group (Table 4.7). The gap in improvement between the number of word problems and number problems solved correctly from Test 1 to Test 4 was statistically the same for Group 1 versus Group 2 ($t(4) = 0.60$, $p = 0.718$).

Table 4.7

Comparison for Group 1 and Group 2 for the difference between Number Problems and Word Problems Solved Correctly, Test 4 to Test 1

Variable	Group	Number	Mean Change Test 4 – Test 1	Std. Dev.	t	df	P
Difference Between Number Only and Word Problems Solved Correctly	1	5	1.80	1.789	0.374	8	0.718
	2	5	1.20	3.114			

The groups were timed as they completed the four tests with times being recorded for both the completion of the number only problems and for the word problems (Table 4.8). With regards to the time taken to complete the word problems there was no statistical difference at any stage between the groups or within each group. This is significant with regards to the interventions as it means that the time the students took to complete the tasks did not increase even though they were asked to add an extra step or two to the process. For the time taken to complete the number only problems there was no significant change but there was a trend with both groups completing the number only problems quicker.

Table 4.8

Summary of Times taken to Complete Word Problems

Variable	Group	Number	Mean Diff Test 4- Test 1	Std. Dev.	t	df	P
Time taken to Complete Word Problems	1	5	-1.014	6.274	-0.361	4	0.736
	2	5	-2.448	8.815	-0.621	4	0.536
Time taken to complete Number Only Problems	1	5	-1.456	1.014	-3.212	4	0.033
	2	5	-2.044	1.523	-3.001	4	0.040

Overall, the quantitative results show that both groups made improvements in the areas of the number of number only problems solved, the number of word problems solved, and the number of operations chosen correctly. These results also show that there was no significant change in the time the students took to complete the word problems. Both groups of students made their biggest improvements over the period of their intervention.

4.4 Think-Aloud Comparisons

Results of the students' descriptions of what they visualise and think when solving word problems and their use of drawings to solve word problems will be presented in turn. During the think-aloud activities, students were encouraged to describe everything they were thinking as they solved a word problem. In the testing session, three of the students solved the word problem correctly (two from Group 1 and one from Group 2). A further two chose the correct operation (subtraction) but made a calculation error (both from Group 2). During these sessions, only three of the ten students made any reference to the setting or characters in the word problem when explaining their thinking. The other seven students went straight to the numbers and made equations without any reference to the setting of the problem. This result may have been because the students were not used to doing think-alouds but may also indicate that the students may not see a connection between word problems and real life contexts.

The same think-aloud exercise, with a different question, was repeated for each group after the completion of all of their intervention sessions. On the second assessment, seven (increased from three) of the students solved the problem correctly (three from Group 1 and four from Group 2) with another one student (from Group 1) choosing the correct operation but not correctly solving the word problem. During their second think-alouds nine students (increased

from three) described the setting of the problem making reference to four aspects of the context (i.e., Tammy, netball, goals or scoreboards) (four from Group 1 and five from Group 2).

With an increase in students describing the setting and solving the problem correctly, these results suggest a link between the two areas. Encouraging students to see the word problem as a real event, this may lead to it making more sense and enable the students to solve the mathematics more often.

The second part of each think-aloud involved the students using a drawing to help solve a word problem. The students gave a retrospective think-aloud explaining what their picture showed and how it helped them to solve the problem. In the first think-aloud session three students solved the problem correctly (one from Group 1 and two from Group 2). A further three students used the correct operation but made a calculation error (two from Group 1 and one from Group 2). These were nearly identical results to the first question where the students visualised only. These results suggest that there was no difference in whether students visualised or drew pictures prior to the interventions.

During the second think-aloud session six students solved the problem correctly using a picture to support them (four from Group 1 and two from Group 2). No extra students chose the correct operation. Group 2, being the visualising only group, had received no instruction on how to use drawings and this may explain the difference in the results, suggesting that receiving instruction is an important step to make these strategies successful, consistent with findings of Van Meter and Garner (2005) and Zhang & Xin (2012).

Overall, the think-aloud comparisons from before and after the interventions suggest that asking students to use internal or external representations can help them to solve mathematics

word problems. Another possible finding is the importance of teaching the strategies of internal visualisation or use of external representations.

4.5 Types of Representations Generated by Students

The drawings used by students were categorised into pictorial, schematic, notational or a combination of these (Section 3.4.5). There was a clear shift in the content and style of the drawings produced during the think-alouds before and after the intervention sessions (Table 4.9). Purely pictorial drawings, which were used by three students in pre-testing, were not present at all in the post-testing sessions. In the pre-intervention think-aloud session only five students included any reference to the numbers used in the word problem in their drawings. Of these five, two students solved the problem correctly. In the post-intervention session, seven of the students used the actual numbers described in the problem in their representation and of these, five students successfully solved the problem. In the pre-testing session six students produced a pictorial or schematic presentation with four using the numbers only. In the post-intervention session, every student produced a schematic representation with seven also including the numbers. These results (Table 4.9) show a clear movement in the students being able to see the word problem as a mathematics problem. The students identified the importance of the context of the problems and how the numbers fit into this. Rather than producing a purely literal representation of the setting or just selecting the numbers described in the word problem they were able to integrate the two and there was an associated increase in accuracy in their answers.

Table 4.9

Categorisation of Pictures used in Think-Aloud Sessions

	No picture	Pictorial	Schematic	Notational	Pictorial/ Notational	Schematic/ Notational
All pre-intervention	1 (0)	3 (0)	1 (1)	4 (2)	1 (0)	0
All post-intervention			3 (1)			7(5)
Group 1 Pre-test (Drawing)	1 (0)	1 (0)	1 (1)	1 (0)	1 (0)	0
Group 1 Post-test			1 (1)			4 (3)
Group 2 Pre-test (Visualising)		2 (0)		3 (2)		
Group 2 Post-test			2 (0)			3 (2)

Note: Numbers in brackets indicate successful solutions to word problems.

Empty cells indicate no examples of that category were used.

Test scripts from Test 1 and Test 4 were also analysed to see what the students produced independently (Table 4.10). Answers were categorised as to whether they were notational (answer only or if an equation was used), or whether representations were pictorial, schematic or a combination of these.

During Test 1, 63% of the word problems only had the answer written down with a further 21% having an equation and the answer. There was a considerable change in these areas during the final test with only 19% having an answer only given and 40% of the word problems were solved with an equation and the answer. Group 2, the visualising group, made a large change in this area, more than trebling the number of equations they generated. Overall, there was a change from only 25 problems having an equation or appropriate working in Test 1 to 64 problems being solved using an equation or appropriate working. This important step of being able to identify the mathematical equation described in a word problem is part of the problem solving

models proposed by English and Halford (1995), Lucangeli et al. (1998), and Reusser (1990) (Section 2.2).

There was a similar change with the number of representations generated by the students with only 16 produced in Test 1 and 38 used during Test 4. The style of the representation (pictorial, schematic, notational) did not appear to affect the students' success in solving the problems with similar success rates for each category.

Overall, the results suggest that the ability to identify the equation is an important skill and, from the results in this section, it did not appear to matter what type of representation was produced (pictorial or schematic); the key was the presence of the numbers and the equation, although this point will be discussed in greater detail in a later section (Section 4.10).

Table 4.10

Analysis of Representations Produced by Students Comparing Test 4 to Test 1

	Notational				Problems where an equation or working out was present	Pictorial	Schematic	Number of problems solved correctly
	Answer only	Equation and answer	Pictorial/Notational	Schematic/Notational				
Test 1								
Group 1	32 (12)	13 (3)	0	4 (2)	17 (6)	0	1(1)	18
Group 2	32 (12)	7 (4)	0	3 (1)	8 (5)	0	8 (1)	18
Total	64 (24)	20 (7)	0	7 (3)	25 (11)	0	9 (2)	36
Test 4								
Group 1	3 (2)	13 (10)	14(11)	16 (14)	40 (33)	3 (3)	0	40
Group 2	16 (11)	22 (14)	4 (4)	1 (0)	27 (23)	1 (0)	6 (5)	34
Total	19 (13)	40 (24)	18 (15)	17 (14)	64 (56)	4 (2)	6 (5)	74

4.6 Analysis of Types of Questions

As a goal of the study was to investigate a strategy that students could use in all situations and with a wide range of questions, analysis was carried out by question type across the tests. When being analysed, the questions were split into addition, subtraction, multiplication, and division. They were also split in start unknown, change unknown, or answer unknown (section 3.4.2).

In the first testing situation students had less success with word problems where the start was unknown or the change was unknown (Table 4.11) than end unknown problems. This difficulty was not evident in the number only problems where the students correctly solved change unknown problems at the same rate as end unknown problems. Students also struggled with subtraction and addition problems where they found the wording was unclear regarding what was needed (e.g., Tom has 14 marbles. This is 6 more than Kate. How many does Kate have?).

Table 4.11:

Analysis of Success by Question Type Comparing Test 4 to Test 1

Type of Question	Start unknown subtract or add end unknown	Addition end unknown	Mult, answer unknown	Subtract end unknown or add start unknown	Mult, answer unknown	Div end unknown	Addition, end unknown	Subtract, change unknown	Div, end unknown or mult, change unknown	Sub, end unknown
Test 1										
Group 1	2	1	4	0	1	2	3	3	2	2
Group 2	0	0	3	1	1	4	4	1	1	1
Total	2	1	7	1	2	6	7	4	3	3
Test 4										
Group 1	1	5	4	4	5	4	5	3	4	5
Group 2	3	4	4	2	2	3	5	4	3	5
Total	4	9	8	6	7	7	10	7	7	10

There was a clear improvement for every type of question (Table 4.11). The questions solved least frequently were subtraction problems which used the word more (often associated with addition). It seems that the use of certain words and the way that the problems are structured may still cause difficulties for students. This result is consistent with previous work (i.e., Abedi & Lord, 2010; De Corte & Verschaffel, 1991; Fuchs & Fuchs, 2007; Sousa, 2011) in that structure and language cause difficulties for students when solving mathematics word problems. Overall, however, the results in this section suggest that the use of the strategies in this study can be helpful for students with solving word problems of all operations.

4.7 Students' Thoughts on Using Pictures

At the end of each intervention session during the first intervention, the students were asked to complete a short written reflection (Appendix 4) and 2-3 students were interviewed to obtain replies which had greater depth than the written reflections and to focus on a specific point (e.g., a comment by a student, something observed by the researcher) (Section 3.3.4).

Four main themes were identified when the students' responses were analysed. These themes were that students felt drawings could help them more when they perceived word problems to be of higher difficulty; creating a drawing helped students to understand the word problems better; using drawings lessened the demand on their brain; and drawings helped them to be more accurate with their working when solving the problems.

The first theme from the students was that they found that drawing pictures was useful when they perceived the problem to be difficult:

Max (Group 1): Well if the question is hard, I would like to draw a picture but sometimes when I don't draw a picture, um, it's a little bit easy but usually I like drawing pictures.

Anna (Group 1): I prefer not to [draw a picture] but if it's hard it's sort of easier to draw a picture.

Group 2, the visualising only group, had several students who used representations or pictures to help solve problems even though they were not instructed in this strategy. When asked to explain the times they chose to use pictures, the students had similar responses:

Lucia (Group 2): When they [the questions] are harder and they confuse me.

Amanda (Group 2): Because I actually use them [drawings] when something is hard.

These results suggest the approach the students used was dependent on the students' perception of the difficulty of a word problem. These comments from the students support a researcher observation from the intervention sessions that the students were more likely to use a drawing to help solve multiplication and division problems than when solving addition and subtraction problems. However, during the intervention sessions, addition and subtraction problems were often solved incorrectly. When a problem was addition or subtraction, the students often moved straight to writing down the numbers (presumably perceiving the problem to be easier) and doing what they considered the most obvious operation. The researcher observed several examples where students would write the equation and then add in a more pictorial drawing. The drawing appeared to be added as the students had been told they should do a drawing. This supports findings by van Essen and Hamaker (1990) that students tended not to use drawings if they believed they knew the answer straightaway, even if they were wrong. Some examples of this taken from the intervention sessions follow. These examples (Figures, 4.1, 4.2, 4.3 & 4.4), demonstrating some of the more pictorial drawings produced for addition and subtraction problems as opposed to the more schematic drawings used in multiplication and division problems are included below. For example, in Figure 4.1 and Figure 4.2 the equations were written and then a drawing made to match the equation. The researcher observed that the

drawings were not used to help solve the problem as they were added after the problems were solved. There is little in either of these two drawings that would help to solve the problem. In Figure 4.1 there is no reference to a key part of the problem (i.e., how many pages had been read?) and in Figure 4.2 the given answer has been used in the drawing. Both of the solutions in these examples were wrong.

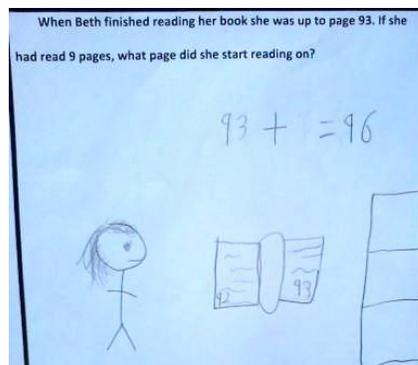


Figure 4.1. Example of drawing from subtraction problem

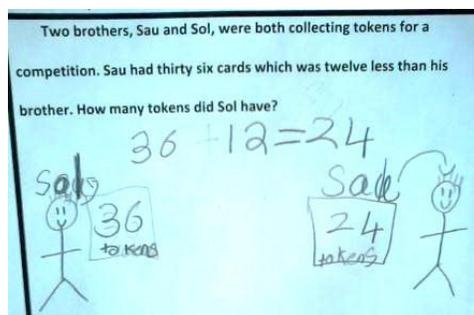


Figure 4.2. Example of drawing from subtraction problem

In contrast, Figures 4.3 and 4.4 are examples of multiplication problems in which the drawings have been used to help solve the problem. For these examples, it was observed by the researcher that the drawings were done before the equations were written. The two examples below produced correct solutions to the word problems. The drawings used in Figures 4.3 and

4.4 are examples of more schematic representations than the more pictorial representations of the situation used in Figures 4.1 and 4.2.

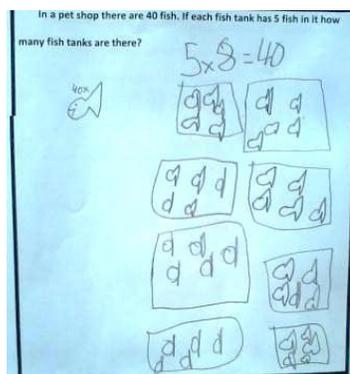


Figure 4.3. Example of drawing from multiplication problem

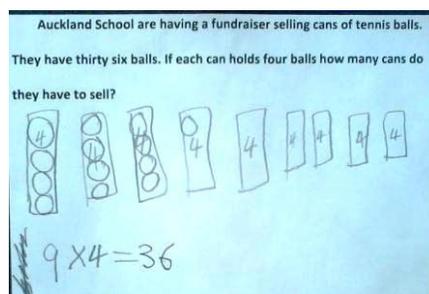


Figure 4.4. Example of drawing from multiplication problem

The approach by the students to only use drawings when they perceived they would be of benefit led to errors during the intervention sessions as the wording and structure of several word problems were not straightforward. This was done to represent the types of word problems students encounter (e.g., ‘less than’ being used in an addition problem). Students using pictures when they perceive a problem to be more difficult is consistent with van Essen and Hamaker’s (1990) findings that students use drawings when they perceive it to be advantageous to them to do so and if they could not see the benefit of using a drawing they would not do so.

A second theme identified through analysis of students' responses was that drawing pictures helped the overall comprehension of the problem (understanding what was being asked) and then figuring out what the mathematics operation and equation involved were. These are two of the steps identified in English and Halford's (1995) approach to problem solving – creating a picture of the situation, then identifying the mathematics involved (Section 2.2). The following quotes highlight the students' feelings about understanding the questions and how this helps them figure out the mathematics in the problem:

Rebecca (Group 1): It helps me to see the numbers and understand the problem better.

Molly (Group 1): It helps me understand it [the question] more.

Max (Group 1): It [drawing] sort of helps me a lot cause then you get a better picture in your head about what to do...because it helps you figure out the maths and solve the problem as well.

Amber (Group 2): It helps me kind of figure out what, um, the thing's actually trying to tell me and it just seems easier.

After intervention session 2, Jack (Group 1) described how drawing pictures helped his thinking:

I think it's kind of good cause it helps you get a picture and it's kind of relaxing instead of getting all stressed out trying to figure it out without pictures...it takes some space out of your head so you get enough time to think.

This final response by Jack (i.e., freeing up space in his brain) was also identified by several other students during the final reflection session following Test 4. Eight of the nine students present on that day identified drawing a picture as a strategy that helped them. When asked to explain, an answer given by three students related to how using drawings lessened the demand on their brain:

Amber (Group 2): You can actually see it there (points to the desk indicating on paper) and you can, you don't have to, um, because you might lose count in your head, or otherwise when it's right in front of you down there, you can actually see what's going on.

Jack (Group 1): It [solving word problems] pushes your brain to the limit if you...because really if there's no picture you kind of have to visualise it really hard, which pushes your brain to the limit, which basically means you won't be able to think straight, so when it's right in the middle you won't have to think as much.

Lucia (Group 2): It's just easier to look down and see you know my way of seeing the question.

These responses support the idea that considerable cognitive demands are placed on students by word problems and having strategies which can lessen what needs to be retained in the working memory can benefit students. As basic facts had not yet been memorised by this group, the demands of solving them places a considerable load on working memory so the extra demands from the format of word problems (i.e., remembering the context, the character, the numbers etc.) can create working memory overload and lead to mistakes, forgetting information and new learning/concepts/strategies not being retained (Sweller, 2011). Such conditions can lead to situations where students might be using their cognitive processing purely to solve problems and not internalising the larger concept of problem solving or a specific strategy (Sweller, 2011) (Section 2.8.1).

Many of the student responses above seem to support Sweller's (2011) idea that by reducing the cognitive load through the use of effective strategies students can be more accurate and achieve more learning. Having and utilising a strategy which allows students to record some information seems to enable students to focus on the solving of the problem rather than focusing on remembering all the information. The comments that using pictures helps the students by their needing to retain less information in their brain is also consistent with Raghubar et al.'s (2010)

findings that using pen and paper for solving mathematics word problems can lessen cognitive load on the working memory.

4.8 Students' Thoughts on Visualising

With their intervention focus being on visualising, during reflections Group 2 were asked questions about if and how they felt visualising helped them with solving mathematics word problems. Frank initially felt that visualising did not help him (Group 2, after session 1):

It kind of messes with my brain a bit... just all the words and trying to figure it out and stuff.

This suggests that the visualising of the word problems is creating too much cognitive load for Frank and he is barely remembering pieces of information as opposed to being able to do any computing to solve the problem.

When asked if he could create a picture in his head, Frank suggested that he struggled to see the problems in his head so could not always solve the problems. He also suggested that drawing a picture helps him. This was supported at the conclusion of the study when he reflected:

Frank (Group 2): I think drawing a picture helps because you don't really have to think that much, because it's right in front of you... you can just see the answer.

This raises the question that this strategy may not work for all students and there could be a link between a style of learning or visualisation abilities/visual reasoning and the success of this strategy. Frank's comments also support the idea that producing an external representation of the problem is an important step towards mathematics competence, particularly before students have a solid grasp of using number properties only.

After the final session, five out of the nine students present felt that visualising helped them to solve word problems. This was fewer than the number who thought drawing pictures helped them (8 out of the 9 students). From the students' responses it seemed combining visualising **and** drawing a picture was the most useful strategy for them and the more difficult a problem was the more that drawing a picture helped. On the whole, more students expressed that drawing pictures helped them:

Max (Group 1): I think that I actually like the drawing pictures idea because at first I didn't actually know that drawing pictures is a better way to solve problems but it's probably best to do when the problems are hard.

Amanda (Group 2): I think using pictures is easier when you have harder problems because when I have harder problems I use pictures to help me.

Molly (Group 1): I think drawing pictures is really good and it helps me.

Only one student expressed that visualising was a better strategy for them than drawing a picture. Lucia (Group 2) responded:

Because I can see what I have to do, when I do a picture it just gets all messed up and I can't count them all, and it just goes all messy, so when I visualise I can see it all in my head.

Interestingly, Lucia is dyslexic, so it may not be surprising that she prefers to work out problems in her head rather than using pen and paper. She was also one student who successfully used drawings and representations as a strategy in the initial testing, mostly to help with computation of the problem as opposed to understanding it. Again this suggests that creating a representation is an important intermediary step to be used when work is difficult or new, or mastery has not yet been achieved.

In summary, considering the two separate interventions, whilst half the students felt that visualising was a strategy that helped to solve word problems, more students felt that drawing

pictures was a better strategy and was more helpful to them when solving mathematics word problems.

4.9 Quality of the drawings

The quality or realism of the picture did not seem to impact on the ability to solve the problem. This result is in contrast to previous studies by Mason et al. (2013) and Schwamborn et al. (2010), which showed that the more accurate and better drawn the information, the more learning occurred. The difference here could be due to the difference in learning goals – those studies being about learning new information whereas this study is interested in solving mathematics problems of a type students had worked on previously. What appeared to be more important in this study was the ability to accurately include or extract the mathematical information and operation that the problems presented, regardless of the neatness, format, or realism of the drawing. Examples of this are found in Figures 4.5, 4.6 and 4.7. In these pictures the students have included the fact it took five minutes to clean each room, which has helped lead to an equation and successful solution.

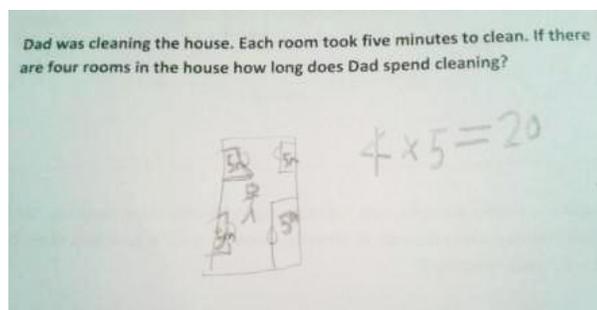


Figure 4.5. An example of a representation which included appropriate information which led to an equation and successful solution

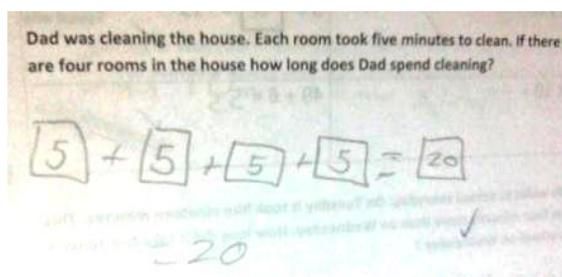


Figure 4.6. An example of a representation which included appropriate information which led to an equation and successful solution

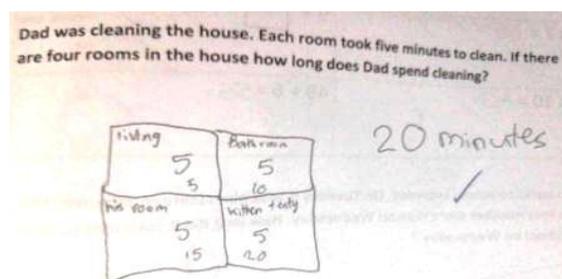


Figure 4.7 An example of a representation which included appropriate information which led to an equation and successful solution

All three examples above have the correct numbers as well as a drawing to represent the situation, with two drawings (Figures 4.5 & 4.7) having four rooms with a 5 in them to show the five minutes cleaning time. The third example (Figure 4.6) has drawn four boxes to represent the rooms and added the 5 minutes to each to enable the problem to be solved. This enabled the students to then solve these problems, two using addition and one using multiplication.

Several other students produced drawings which were accurate to the context of the problem and represented a realistic setting described in the question (Figures 4.8 & 4.9). However, these examples did not include the relevant information described in the problems and were not solved successfully.

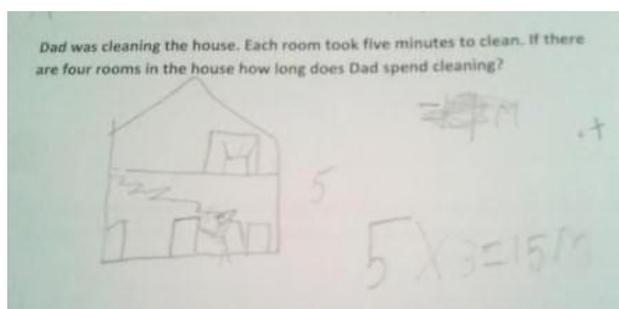


Figure 4.8. An example of a representation which did not include appropriate information and was not solved successfully

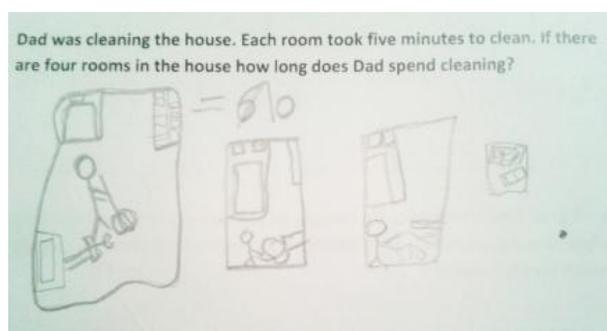


Figure 4.9. An example of a representation which did not include appropriate information and was not solved successfully

The examples in Figures 4.8 and 4.9 also represent the context of the problem accurately but fail to include the numerical data which enable the students to solve the problem. By not including the numerical information the students seem unable to then make the connection to a mathematic equation to solve the problem, the third step in English and Halford's (1995) three-pronged approach to problem solving (Section 2.2). It is this step which the students in Figures 4.5, 4.6 and 4.7 were able to successfully do. This is consistent with Mulligan's (2002) work which found that students who were lower achieving in mathematics often produced more pictorial drawings that did not have the mathematical structure to enable problems to be solved. These findings also align with Edens and Potter's (2010) and Hegarty and Kozevnikov's (1999)

findings that students who produced more schematic drawings were more successful than students who produced pictorial representations. This idea will be explored more in the next section.

4.10 Change in Style of Drawing Towards Schematic and Notational Representations

As described in the previous section, including the relevant numerical information and creating an appropriate equation are important steps in solving mathematics word problems. A noticeable change in the drawings produced by several of the students occurred over the course of their intervention. Initially, several students completed drawings which were pictorial, purely represented the situation and did not include the numerical information described in the word problem. The drawings did little to help the solving of the problems. These students seemed unsure what they were meant to focus on. It was noticeable, but not surprising, that these were some of the lower achieving students, consistent with Mulligan's (2002) findings. Examples from four students follow that illustrate the change from pictorial representations to more schematic representations and the inclusion of the relevant quantities described in the problems.

Figure 4.10 shows Matt's initial effort during the first think-aloud at producing a drawing to help solve the problem. He has simply drawn the eight cards mentioned in the problem, a representation which did little to help solve the problem.

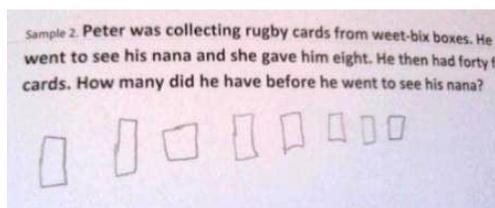


Figure 4.10 Matt's initial drawing during pre-testing

Figure 4.11 shows Matt's effort during his post-intervention think-aloud. He has represented the three friends, with two of the three friends and the number of cards they had accurately portrayed. Although he still failed to solve the problem accurately, the improvement after just three sessions suggests that with further instruction even greater accuracy could be achieved. Matt achieved this improvement in drawing even though he was in the visualising only group and received no instruction in drawing the situations portrayed in the question.

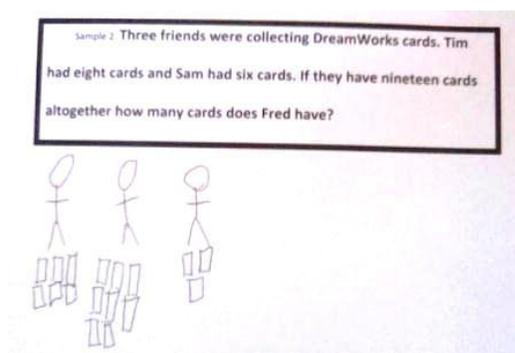


Figure 4.11 Matt's drawing during post-intervention think-aloud.

Figure 4.12 shows Jack's first attempt at producing a drawing to help him solve the word problem. The drawing accurately portrays the situation described in the question on a context level but has no numerical information which could help with solving the problem. When explaining his drawing he also failed to include any relevant numerical information.

Jack: He just got out rugby cards and he showed his nana and she gave him some more. Is it 9? because I don't know how many he had before.

This suggested that Jack did not make a connection between the setting of the problem and the solving of a mathematics problem.

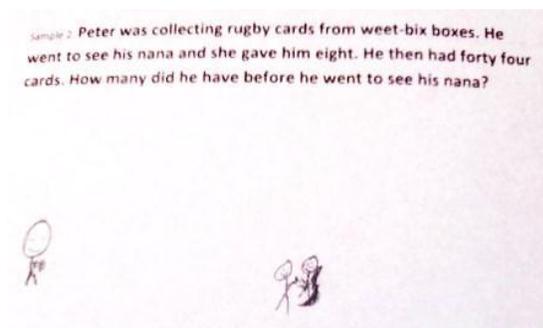


Figure 4.12. Jack's initial drawing during pre-testing think-aloud session

Jack's effort during the second think-aloud session (Figure 4.13) shows a clear improvement in the content of the picture. He has two friends with their cards numerically represented in their hands. He has also identified the correct operation for the first part of solving the problem. While Jack is also unsuccessful in solving the problem, his drawings demonstrated a greater understanding of the problem and positive signs about his ability to identify the numerical information presented.

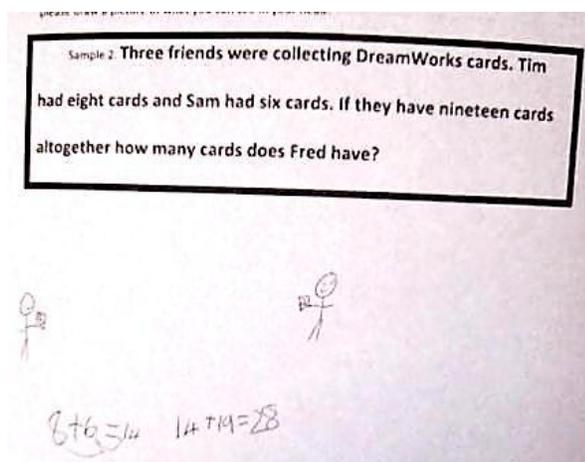


Figure 4.13. Jack's final drawing during post-intervention think-aloud session

Similarly, Jack's oral explanation was far more detailed and whilst the last step is wrong he showed greater awareness of the information in the question:

Jack: Tim had 8 and Sam had 6 so 8 plus 6 are 14 then I added on 9 (sic) to get 28 so I added on 19 to 14.

Molly was unable to produce any representation at all during the first think-aloud session. After two minutes of silence and no drawing Molly was prompted with questions from the researcher which were “What’s going on in your head, what can you see?” and “Can you see part of the problem like how many people there are, what they are doing?” She replied “I don’t know” and that ended the session.

In the second think-aloud session, Molly produced the representation in Figure 4.14. This was a detailed representation which included pictorial elements and schematic elements.

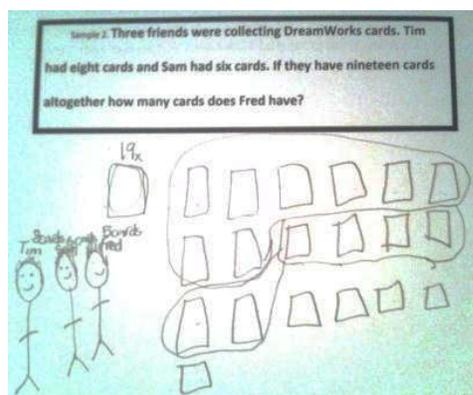


Figure 4.14. Molly’s final drawing during post-intervention think-aloud session

Her verbal description was also presented confidently with the appropriate details and she successfully solved the problem:

Molly: I drew Fred, Tim and Sam. I drew 19 cards. I grouped them all up – Tim had 8 cards then Fred had 5.

Max was a student who solved both problems correctly but he still changed the style of his drawings from a more pictorial style to a schematic-notational style. In Figure 4.15, his first

effort, he draws the number of cards described in the problem (44 and another 8). In his second effort, Figure 4.16, Max has moved onto drawing one card and putting the actual number in it.

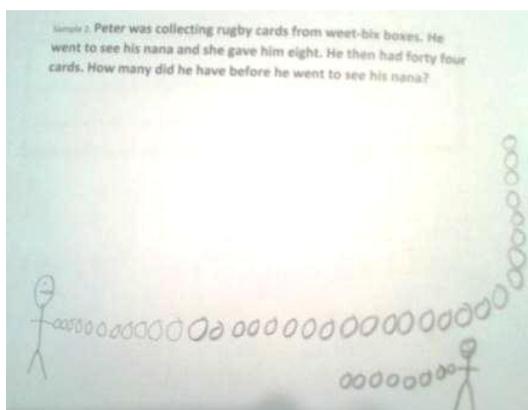


Figure 4.15. Max's initial drawing during pre-intervention think-aloud session

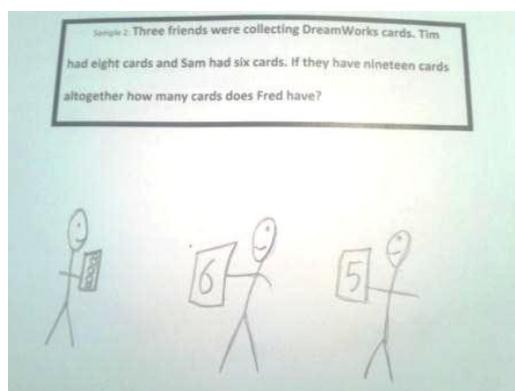


Figure 4.16. Max's drawing during post-intervention think-aloud session

Whilst both approaches were successful, the second approach seems to be less time consuming and easier to work with. It seems to be also more transferrable to word problems with larger numbers, i.e., drawing 3567 cards would not be an effective strategy!

In summary, the results in this section seem to contradict findings described earlier (Section 4.5) based on the overall results, which suggested it did not matter whether students produced a pictorial or a schematic representation. Regardless of the inconclusive findings in

relation to the types of representations produced, what does appear crucial to the ability to solve a word problem is the ability to generate an equation from the word problem.

4.11 Summary

This section has presented the qualitative and quantitative data collected during this study and made links to previous studies. These results include improvements for both groups in the number of word problems and number only problems solved correctly. Whilst there was no statistical difference between the two groups' results, the students' responses indicate that they felt the use of pictures was a more effective strategy than visualising alone for solving word problems. There were mixed results regarding whether the style of drawing had an impact on success, but it appeared that the inclusion of the numbers and the ability to generate the equation described in the word problem were important. In the next chapter, these results will be discussed in greater detail and some of the implications of this study will be outlined.

Chapter Five

Discussion

This study set out to explore if teaching students to use drawings and to visualise when solving word problems could have a positive impact on achievement. The study was a mixed methods research design which collected data from tests, interviews, representations, and reflections. The study involved two different interventions and aimed to compare the two approaches. With student production of external representations having well-published success related to increasing educational outcomes, this study aimed to investigate if these results would be replicated in a New Zealand setting, and to see if visualising could have the same impact as using drawings when solving mathematics word problems.

5.1 Limitations

Due to the small size of the sample, caution should be used when generalising the results beyond this study. Also, as the groups were removed from their classrooms for the interventions, it is unclear whether the approaches are workable and effective as a classroom teaching strategy. As the interventions were completed in a group rather than classroom setting, and the students were involved in discussions, students' learning may have been affected by seeing what other students said or did. In addition, think-alouds may have been a new approach for the students, so some results may be affected by the students learning how to do think-alouds during the sessions, or feeling more confident with what was expected of them by the end of the interventions. These potential factors raise areas worthy of further investigation (Section 5.3.4).

5.2 Discussion of Findings

This study set out to investigate five research questions - Are learner generated drawings effective in helping New Zealand students improve their achievement when solving mathematics word problems? Can visualisation increase students' achievement when solving mathematics word problems? Is visualising or the use of drawings more effective in helping students to solve mathematics word problems? Can one strategy help students solve word problems of any operation and structure? Can a strategy be devised which teachers and students can use easily?

The evidence from this study suggests that both using drawings and visualising are effective strategies for improving students' ability to solve mathematics word problems. Both Group 1, the visualising and drawing group, and Group 2, the visualising only group, made progress in the areas of solving number only problems, word problems, and choosing the correct operation needed to solve the problem (Section 4.3, Tables 4.1 and 4.2). Students also maintained their improvement once their interventions had concluded. A telling factor for the success of the strategies was that both groups made their improvements over the course of their intervention (for Group 1, from Test 1 to Test 2, and for Group 2, from Test 2 to Test 3) (Section 4.3, Tables 4.4 and 4.5). With each group experiencing only three intervention sessions in total spread over one week, the level of progress was considerable and suggests that the intervention is likely to be able to be used by teachers without undue negative consequences for their programme delivery. In addition, with further sessions even greater progress could potentially be made by students.

Of particular interest was whether the disparity between the success with number only problems and word problems could be lessened. At the first testing time before the interventions there was an average difference of 2.8 for Group 1 and 2.4 for Group 2 between the number of

number only problems and number of word problems solved correctly. After the interventions the difference in achievement had lessened to 1.0 for Group 1 and 1.2 for Group 2 respectively. While this change was not statistically significant (there was a trend for Group 1), this was a positive movement and may indicate that the strategies used in the interventions had a positive impact on lessening the gap in students' achievement between word problems and number problems.

A major shift made by the students was in their ability to identify the relevant equation in the mathematics word problems (Section 4.10). Students of Group 1, the visualising and drawing group (more than doubled from 17 to 40 out of 50) and Group 2, the visualising only group (more than tripled from 8 to 27 out of 50) either wrote equations or had appropriate numerical working related to the problems during the final testing session, Test 4. The step of eliciting the mathematics equations from a mathematics word problem is part of problem solving models proposed by many (i.e. English & Halford, 1995; Lucangeli et al., 1998; Reusser, 1990) (Section 2.2). Therefore, this study is important in showing that instructing students in identifying the equation in word problems is an important consideration.

As both groups made improvements over their interventions, it is possible that visualising the context of the problem could be a contributing factor, whether or not an external representation is produced, as shown by Group 2's success, even though they produced fewer representations than Group 1 (Section 4.5, Table 4.10). However, even the most detailed verbal description or accurately drawn picture of the setting in the problem did not help with solutions if the equation described in the problem could not be found. The inclusion of the numbers in any representation produced was found to be important in helping students to identify suitable equations and solve the problems.

In this study, results were inconsistent as to whether it mattered what representation students produced (pictorial or schematic). It appeared that several students made a move from using pictorial representations to more schematic and notational representations which seemed to help them to be more successful (Section 4.10). However, in the test results there was no difference in success rates whichever form of representation the students used (Section 4.5). The key factor in solving the word problems seemed to be whether students could include the numbers from the word problem. The inconsistency of these results makes it difficult to compare this work to previous work done by Edens and Potter (2010), Hegarty and Kozhevnikov (1999), and Mulligan (2002) who had found that students who produced schematic representations were more successful than students who produced pictorial representations. In this study it may be that the focus on visualising the context of the problems (whether mentally or on paper) was sufficient to enable students to be more successful with identifying the equation, providing further evidence that visualising word problems can be an effective strategy.

The results of this study add to previous studies that found students do worse on word problems than comparable number only problems (Carpenter et al., 1980; Cummins et al., 1988) by finding that students identify word problems as more difficult than number only problems and can identify reasons for this (Section 4.1). As shown by the students' responses, the strategies of visualising and drawing, and visualising only were successful in helping the students gain a more positive attitude towards mathematics word problems and their ability to solve them (Section 4.2).

A consideration in relation to the success of the interventions in this study is that the time the students took to solve the problems was very similar before and after the interventions (Section 4.2, Table 4.8). This result suggests that while the students used extra steps in their problem solving process, they spent no longer solving the problem overall while increasing their

accuracy. This adds to the work completed by Van Meter (2001) by suggesting that it is not just extra time spent on the task that led to the improvements but the process of visualising which is of benefit. This is an important result given the crowded curriculum and time issues teachers face and also a major factor for students when completing timed tests such as e-asTTle (Ministry of Education, 2010) and PATs (NZCER, n.d.).

The improvement in solving word problems and choosing the correct operation was anticipated but the improvement in number only problems was not expected. As both groups made the improvement in this area over the course of the intervention sessions and the improvement was seen over such a short time period (less than two weeks for each group), it is unlikely that outside factors contributed to this. One explanation for the result may be that the students transferred the strategy of visualising from the word problems to the number only problem which helped to increase their accuracy. Further investigation would be needed to see if there is a link between visualising and the solving of number only problems.

One limitation of the study findings in relation to the use of visualising only is that it is difficult to compare the two strategies used (purely visualising, and visualising and drawing) as some members of Group 2 (the visualising only group) chose to use drawings to help with their solutions. However, in the think-aloud situations where students were asked not to use drawings there was an increase in the number of students referring to the setting of the problem and in the resultant answers (Section 4.4). This result suggests that visualising alone benefits students when solving mathematics word problems. Further evidence that internal visualising only can help students was found in the analysis of the representations used by the students during Test 1 and Test 4 (Table 4.10). In Test 1, students from Group 2 produced more representations than their peers in Group 1 (11 vs 5). In Test 4, Group 2 students only produced eight representations versus 30 produced by Group 1 members. This result suggests that the production of an internal

representation can be sufficient for students to solve word problems. It also suggests that the use of internal and external representations is part of a process of solving mathematics word problems and may best work when they are used together. It seems that having knowledge of both of these strategies and being able to use them when appropriate could benefit students the most.

One of the issues identified by several students at different stages was that only visualising created too much ‘clutter’ in their brain and that drawing helped to ‘free up space’. This ties in with theories that working memory has a capacity and can only hold so much information (Cowan, 2014). Trying to create a mental picture, work out the equation and then calculate the equation appeared too much to store in the working memory for some students. Students’ comments suggested that the use of drawings to get some of the information down or to help with the calculation of the problem enables the students to be less stressed and be more accurate in their working (Section 4.7). This result is consistent with Raghubar et al.’s (2010) review that found the use of pen and paper likely lessens the load on working memory. It may be that drawings help more with lessening the load on the working memory and with calculation accuracy than in helping with the comprehension of the question. Further investigation could enable exploration of this area.

A strength of the intervention strategies is that they seemed to work across all types of word problems. Students’ accuracy increased for all types of wordings and operations. This is a powerful result in relation to other strategies, including schema-based strategies, that involve knowing the appropriate format to use for the different types of question, as it involves students knowing only one strategy and being able to apply it to all types of word problems. Having just one strategy lessens the demands on students’ thinking as they know they have a strategy which

can help with word problems without spending any thinking or time on deciding which strategy or format is needed for a specific problem.

The area that caused most difficulties for students was the wording and semantics of some problems. Addition problems with ‘less than’, subtraction problems with ‘more than’, and problems where the start value was unknown were more difficult for students to solve than other question types. This is an area that teachers need to be aware of and to spend time helping students.

Combined with the work of Quirk (2010) (Section 1.6), the results from this study provide compelling evidence of the power of common reading strategies used in many New Zealand schools to assist students with solving mathematics word problems. With the large amount of reading involved in mathematics, consideration needs to be given to pedagogy which caters for this, including using reading comprehension strategies. Generally, effective teaching strategies used in one curriculum area need to be explored in other curriculum areas, especially where there is a large crossover in skills such as the reading comprehension skills needed in mathematics word problems.

In summary, there are several key results found in this study:

- Both visualising and drawing, and visualising only are effective strategies for helping students to solve mathematics word problems;
- The time taken to solve word problems did not significantly change post-intervention, yet accuracy improved;
- Students can be taught how to use visualisation in a short period of time with positive results;

- A key to successful solving of word problems is identifying the mathematical equation described, and visualising can help achieve this; and
- Effective reading comprehension strategies can help students be more successful when solving mathematics word problems.

5.3 Implications of the Study Findings

From this study there are implications which can be applied at different levels.

Implications for policy and document writers (Section 5.3.1) are discussed first followed by considerations for initial teacher education (Section 5.3.2). As the focus of the study was finding an effective strategy for use in classrooms by teachers and students, a longer section on implications for classroom teachers follows (Section 5.3.3). This section concludes with a discussion of implications for future research (Section 5.3.4).

5.3.1 Implications for Policy Makers and Document Writers

A focus in policy and resource documents, such as the Numeracy Development Project books, needs to be on the importance of the step of mental imagery, of which visualisation is a crucial aspect. The results in this study show that visualising can help students increase their achievement in a short time and the importance of this needs to be emphasised in official documents. This could be achieved with a greater focus on the Strategy Teaching Model (Section 2.1) and an increased emphasis on the importance of the Using Imagery step.

5.3.2 Implications for Initial and Inservice Teacher Education

During initial and inservice teacher education, emphasis needs to be given to the role that visualisation can play in mathematics education. The Strategy Teaching Model (Section 2.1) has a section on Using Imaging and the role and importance of visualisation needs to be focused. A

second focus during teacher education can be on the transfer of effective teaching strategies between curriculum areas. The use of reading strategies, including visualisation, can be used in mathematics to increase student achievement. This approach needs to be made explicit within initial and inservice teacher training.

5.3.3 Implications for Classroom Teachers

The study has highlighted effective strategies for students to use when solving mathematics word problems, so this leads to many implications and considerations for classroom teachers. A major implication for teachers is that the use of representations in the form of drawings and visualisation helps students solve word problems accurately and that students can be taught how to use these. This means teachers need to plan sessions that teach how to visualise word problems and to draw pictures that can help with word problems. The results, specifically comments from the students, suggest that having both strategies is the most effective approach so students can use what is needed to solve word problems.

An important factor is that students use representations, including visualising and drawings, when they perceive they will be helpful. It is important that teachers show students the benefits of visualising and using drawings so students will value the strategies and use them independently.

It is also important for teachers to show students there is a process for solving word problems, which includes comprehending the question, seeing the question in its context and identifying the mathematics described in the form of writing an equation. Again this process needs to explicitly be taught to students.

A final implication for teachers is recognising effective teaching strategies and looking at how they can be used across all curriculum areas. Specifically, this study has shown how using

an effective reading comprehension strategy can help students to solve mathematics word problems.

5.3.4 Implications for Future Research

From this study, there appear to be several areas worth investigating further. In order to see if the results can be applied to the wider population it would be worth trialling these strategies with a larger sample and over a longer period of time. Completing the study in a range of schools with many more participants would enable the results to be more reliable as an indicator of the general population. Carrying out the study over more sessions would show whether greater progress could be made, and especially on the type of questions where the students in this study still struggled (e.g., those questions with semantic or structural challenges).

With the specific nature of the sample in this study, it would be worth completing the study with older students and with higher achieving students. This could help to see if visualising is a skill which these other groups use more independently or with greater competence.

It would also be worth trialling the approach in classroom settings with the classroom teachers taking the sessions. This would help to ascertain if the approach is manageable and realistic for teachers and students to use as part of everyday mathematics teaching and learning.

This study suggests the use of drawings and visualising have a positive impact when working on word problems involving the number strand. It would be worth exploring other areas of mathematics to see if the approach works across all the strands taught in mathematics. It may also be worth exploring the use of drawings and visualisation in other curriculum subjects to see if there are positive results.

The students in this study increased their number only scores but as this was not a focus the reason behind this is unclear. Exploring whether visualising can be used to increase students' achievement when working on number only problems would be an area worth investigating. Comments from students suggested drawings helped with accuracy and working out, but again this was not explored as a focus of this study. Hence, another area worth investigating is if producing drawings helps with accuracy of working as opposed to the comprehension of the word problems.

5.4 Conclusion

The success for the students of this study with such a short intervention is a positive indicator that the use of internal visualisation is a successful strategy for increasing students' ability to solve mathematical word problems. The strategy of visualising is easy for students to access as it is often something they are encouraged to do when reading. This study shows that a large difference can be made with a simple strategy without negative consequences for teachers or students. Visualising is a powerful strategy for increasing reading comprehension and has the potential to be just as powerful for solving mathematics word problems.

This study has shown that representations, both internal and external, have the potential to help students achieve to their full potential in mathematics. With the high stakes of assessments and the prevalence of word problems in mathematics, the use of drawings and visualising could be an important tool in helping all students to be successful in mathematics.

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Appendices**Appendix 1: Tests**Name _____ Ruma _____ Year 4 5

Solve these problems any way that you can. Please show your working

$15 - 8 =$	$42 - 8 =$
$13 - \square = 6$	$20 \div 2 =$
$15 + 8 =$	$45 \div 5 =$
$5 \times 7 =$	$9 + \square = 13$
$4 \times 10 =$	$49 + 6 =$

Bob walks to school everyday. On Tuesday it took him nineteen minutes. This was four minutes more than on Wednesday. How long did it take him to walk to school on Wednesday ?

Mary had some time working in her garden. She spent eighteen minutes cutting down a tree and she had seven minutes to weed the vegetable garden. How long did she work in the garden in total?

Dad was cleaning the house. Each room took five minutes to clean. If there are four rooms in the house how long does Dad spend cleaning?

Ken and Beth had a running race. Beth won the race by 7 seconds. If Ken took thirty nine seconds how long did Beth take?

Sarah had a money jar full of one dollar coins. She put the coins into piles of ten. If she had eight piles of coins how much money does she have?

There are twelve players in Noah's rugby team. If they have to get into groups of two to do a passing drill, how many groups will there be?

At Lily's gym first competition she got a score of 46 points. The next week she got a score that was eight points more. How many points did Lily get in the second competition?

Eva and Lola were both practising their shooting for netball. Eva got 27 goals which was 7 less than Lola. How many goals did Lola get?

Mum spent five minutes talking to each of her brothers and sisters on the phone. If Mum talked for thirty minutes, how many brothers and sisters does she have?

Katie was taking twenty three students to a soccer tournament. If eight of them are boys, how many girls are going?

Name _____ Ruma _____ Year 4 5

Solve these problems any way that you can. Please show your working

$15 - 6 =$	$63 - 7 =$
$16 - \square = 9$	$18 \div 2 =$
$19 + 8 =$	$25 \div 5 =$
$5 \times 8 =$	$5 + \square = 14$
$7 \times 10 =$	$27 + 6 =$

Sam and Kate are collecting marbles. Kate has twenty five which is seven more than Sam. How many marbles does Sam have?

Brad spent \$8 on his lunch. If he had \$15 left in his wallet how much did he have before he bought his lunch?

In a pet shop there are five rabbit cages. If there are eight rabbits in each cage how many rabbits are there?

Room 7 beat Room 5 by eight runs in a game of softball. If Room 5 got fifty three runs how many runs did Room 7 get?

Billy goes to tennis lesson. Each lesson costs \$10. If he goes to seven lessons how much will that cost in total?

At Sally's basketball game, her team got 14 points. If each goal is worth two points how many goals did the team get?

Lily was trying to get the highest score in her maths tests. One week she got 45 marks and the next week she got 7 marks higher. What was her score in the second week?

Brad and Ted always try to beat each other in everything they do. In a spelling test Brad got 19 which was 6 less than Ted. How many did Ted get?

Kat and her dad were selling thirty biscuits to make some money. If they put five biscuits in each bag how many bags will they have to sell?

Ruma Koromiko and Ruma Nikau are going on a school trip. If there were twenty one students altogether and 8 of them were boys, how many girls were there?

Name _____ Ruma _____ Year 4 5

Solve these problems any way that you can. Please show your working

$12 - 9 =$	$72 - 6 =$
$14 - \square = 9$	$14 \div 2 =$
$17 + 8 =$	$35 \div 5 =$
$5 \times 6 =$	$7 + \square = 15$
$9 \times 10 =$	$38 + 6 =$

Tim and Kate both play soccer. Kate has fourteen goals which is six more than Tim. How many goals has Tim scored?

Bob went to the movies and his dad gave him some money. He spent \$9 on a ticket and had \$16 left to buy some food and drink. How much money had his dad given him?

Sally and Ben bought some flowers for their friends. They bought eight bunches and each bunch cost \$5. How much did they spend on the flowers?

Koromiko and Nikau are going on a trip. Twenty three students from Nikau have brought back the forms which is five more than Koromiko. How many students from Koromiko have brought their forms back?

Steve works for seven hours on Saturday at his job at Countdown. If he gets \$10 an hour how much will he get paid?

In Vanessa's class there are sixteen students. If they have to get into groups of two how many groups will there be?

In the first week at basketball the Te Aro team scored thirty five points. If in the next game they got eight more points, how many points did they score in the second game?

Eva and Beth both collect comic books. Eva has got 21 which is 5 less than Beth. How many comic books does Beth have?

The SPCA had all the puppies in cages. If there are twenty five puppies in total and five puppies go in each cage, how many cages do they need?

Ruma Koromiko and Ruma Nikau are going swimming. There are forty two students altogether but five cannot swim because they have colds. How many students will go swimming?

Name _____ Ruma _____ Year 4 5

Solve these problems any way that you can. Please show your working

$15 - 8 =$	$42 - 8 =$
$13 - \square = 6$	$20 \div 2 =$
$15 + 8 =$	$45 \div 5 =$
$5 \times 7 =$	$9 + \square = 13$
$4 \times 10 =$	$49 + 6 =$

Bob walks to school everyday. On Tuesday it took him nineteen minutes. This was four minutes more than on Wednesday. How long did it take him to walk to school on Wednesday ?

Mary had some time working in her garden. She spent eighteen minutes cutting down a tree and she had seven minutes to weed the vegetable garden. How long did she work in the garden in total?

Dad was cleaning the house. Each room took five minutes to clean. If there are four rooms in the house how long does Dad spend cleaning?

Ken and Beth had a running race. Beth won the race by 7 seconds. If Ken took thirty nine seconds how long did Beth take?

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Mum spent five minutes talking to each of her brothers and sisters on the phone. If Mum talked for thirty minutes, how many brothers and sisters does she have?

Katie was taking twenty three students to a soccer tournament. If eight of them are boys, how many girls are going?

Appendix 2: Sample Think Aloud Protocol

Think aloud protocol for pre and post comparison

I am interested in what happens in our brains when we solve word problems. I am going to ask you to solve some problem and let me listen to how you solve it. You need to tell me everything that goes through your mind.

This is an example of a think aloud. (Researcher demonstrate)

What you say is really important so I am going to record the session as well so I do not forget anything.

Do you have any questions about what we are doing today?

Now I'm going to ask you to read and solve a problem. Just say everything that goes through your mind while you solve the problem.

Please read the problem out loud and then begin

Tammy's netball team won their game on Saturday. They got 23 goals and they won by 8 goals. How many goals did the other team get?

(Ask any appropriate follow up questions to clarify once totally finished)

This time I am interested in what pictures or images you create in your head. What I would like you to do is to read the problem and visualise in your head what the problem is about. Then can you please draw a picture of what you can see in your head.

Sample 2. **Three friends were collecting DreamWorks cards. Tim had eight cards and Sam had six cards. If they have nineteen cards altogether how many cards does Fred have?**

Can you please explain to me what your drawing is about

Appendix 3:

Example of teacher modelling a question at beginning of session.

Group 1 - visualising and drawing group

(Reading the question) Teddy has \$27 to spend on books. If each book costs three dollars how many books can he buy? I'm going to model this one so I've got teddy here (drawing a teddy) ... there's teddy there... so I can see Ted and I can see how much money has Ted got? I can see he's got 27 dollars (adding money amount to picture) so I know he's got 27 dollars and I know that he wants to buy some books and each book is three dollars (drawing a book with \$3 on it) I have to figure out how many of these (points to book) he can get for that (points to \$27) so if he buys one book that's three dollars and I've got another book (draws a book), that would be three... another book (draws a book),that would be three....another book (draws a book),that would be three... another book (draws a book),that would be three...3, 6, 9, 12, 15, I'm not there yet so I need another book (draws a book), another book for \$3, that's 18, another book, 21, another book and another book and that's 27. so how many books, 9. The maths for this one, who knows what the equation is? Nine books is the answer so it is something $\times 3$ equals 27, the other way is 27 divided by 9. You can see how I drew teddy, I drew 27 dollars and I drew a book with 3 on it.

Group 2 visualising only group

I will just model this one first. (Reading question) At a school there are five groups of children going on a trip. If there are seven children in each group how many children are there

altogether? SO it is something I know about. In my head I can picture, I can see a school. There are five groups of children on a trip. We've all be on school trips ,aye, so I've got that picture in my head. I can actually see Te Papa but that doesn't matter but I can see groups going and there is an adult with each group. There's a group of kids but I don't know how many kids there are yet but I know there are five groups so that is one of my numbers (writes down 5) and there's seven kids in each group. I need to figure out if it's a plus or takeaways or times or divided but there is five groups and there is seven in every group so that is going to become a timestable so five groups so seven there, seven there, seven there, seven there, seven there (hand pointing indicating each group) so it's going to be five times seven (writes equation) so I can see that and I can count in fives or if I know my five times table, 5, 10, 15, 20, 25, 30, 35 so there is 35 students.

Appendix 4: Sample Session Reflection Sheet

I enjoyed today's session	   
Comment	
The work we did today is helping me to solve word problems	   
Comment	
I think drawing pictures is a good way to solve word problems	   
Comment	
I can see a picture of the word problems in my head	   
Comment	
Drawing pictures is useful because	
Any other comment :	

Appendix 5: Sample Information Sheet and Permission Letter



April 2014

Exploring Visualisation as a Strategy for Improving Students Comprehension of Mathematics Word Problems

Dear Parent or Guardian:

I am Ray Teahen, a Masters of Education student of Dr. Robin Averill from the School of Educational Policy and Implementation (SEPI) at Victoria University of Wellington. I request permission for your child to participate in a research study to be used for my Master's thesis. I am conducting a research project on whether visualizing skills can help students to solve mathematics word problems.

The study aims to investigate an area that children can struggle with in mathematics – understanding what a mathematical word problem is asking of them. The results will be used to inform our teaching at Te Aro School and through this thesis will inform other teachers. Any findings might be published in an academic journal and shared with other researchers or teachers through a conference or professional development sessions.

The study consists of the following activities:

1. We will ask your permission for your child, as part of a small group, to take part in 5 sessions of 30 minutes over a two week period. This will involve coming out of their class to work in the small group. The most suitable time will be negotiated with the teachers.
2. These sessions will include: (1) the researcher and the small group working together, (2) the researcher modeling a strategy for solving word problems focusing on creating a picture in their mind and then making a drawing of it, (3) the students attempting this strategy themselves
3. The sessions will be video recorded. The video data is for analysis purposes only and will not be shown publicly at any stage.
4. At the end of each session your child will be asked to complete a short written or verbal reflection which will be recorded.

5. Completing three tests: a pre-test before the sessions begin, a post-test after the sessions, and another test four weeks after the sessions end to see if the ideas have been retained. These will take approximately one hour and be done as part of classwork.

The project will be explained to your child in terms that your child can understand, and your child will participate only if he or she is willing to do so. Participation in this study is voluntary. Even if you give your permission for your child to participate, your child is free to refuse to participate. If you consent and your child agrees to participate, he or she is free to end participation at any time before the 1 June 2014, when it is anticipated that all data will have been gathered. If you decide to withdraw from the study please email me, see me or let your child's classroom teacher know.

Only Dr. Averill and I, and possibly a transcriber, will have access to any information from your child. No one else will see the videos or other data. Pseudonyms will be used for each child in the written report so that any responses will not identify individuals.

Should you have any questions or desire further information, please feel free to contact

Ray Teahen

Dr Robin Averill

teahenraym@myvuw.ac.nz

robin.averill@vuw.ac.nz

0210797751

463 9714

This research requires written consent from student participants and their parents/guardians.

This study has been approved by the Human Ethics committee at Victoria University of Wellington (Ethics Approval: 20756). If there are any ethical concerns about the research you should contact Dr Allison Kirkman (Allison.Kirkman@vuw.ac.nz), ph: 04 463 5676, Convener, Human Ethics Committee, Victoria University of Wellington.

Please keep this letter for your information after completing and returning the consent page to me.

Sincerely,

Ray Teahen



Exploring Visualisation as a Strategy for Improving Students Comprehension of Mathematics Word Problems

I realize that by giving my consent for my child to be part of this study

- He/she will complete pre- and post-tests
- He/she will take part in five small group lessons on strategies for visualizing word problems
- Be asked to complete a short written reflection for each lesson or be videoed talking about the session
- His/her work during these sessions will be collected
- He/she is likely to be videoed during the group lessons. The video data is for analysis purposes only and will not be shown publicly at any stage
- I understand that only the research team will have access to the recordings
- I understand the reports will not identify my child
- I understand I can withdraw my child before 1 June 2014

Please indicate whether or not you wish to allow your child to participate in this project by checking one of the statements below, signing your name and returning it to school. Sign both copies and keep one for your records. **Can this form please be returned to your child's teacher by Thursday 8 May?**

_____ I do grant permission for my child to participate in this study

_____ I do not grant permission for my child to participate in this study

I would like a summary of this research once it is completed. Email _____

Signature of Parent/Guardian

Printed Parent/Guardian Name

Printed Name of Child

Date

Appendix 6: Student Sheet for Think Aloud Activities

Please read the problem out loud and then begin talking about what you are thinking

Tammy's netball team won their game on Saturday. They got 23 goals and they won by 8 goals. How many goals did the other team get?

This time I am interested in what pictures or images you create in your head. What I would like you to do is to read the problem and visualise in your head what the problem is about. Then can you please draw a picture of what you can see in your head.

Sample 2. **Three friends were collecting DreamWorks cards. Tim had eight cards and Sam had six cards. If they have nineteen cards altogether how many cards does Fred have?**

Can you please explain to me what your drawing is about?

Appendix 7: Sample Teaching Session Cards

There were forty three animals in one paddock and there were 12 less in the paddock next door. How many animals were in the second paddock?

A dairy was selling bananas in bunches of three. If they had twelve bunches how many bananas are they selling?

Sarah had twenty apples to share with her three friends. How many apples is each of the four friends going to get?

Appendix 8: Summary of Test Results

		Pre intervention						After intervention with Group 1						After intervention with group 2						Post all interventions- three weeks after final session with Group 2					
Name	Group	Numbers 1	Time 1	Word problems 1				Numbers 2		Word problems 2				Numbers 3		Word problems 3				Numbers 4		Word problems 4			
		Correct 1	Time 1	Correct 1	Time 1	Correct operation 1	Calculation error 1	Correct 2	Time 2	Correct 2	Time 2	Correct operation 2	Calculation error 2	Correct 3	Time 3	Correct 3	Time 3	Correct operation 3	Calculation error 3	Correct 4	Time 4	Correct 4	Time 4	Correct operation 4	Calculation error 4
Anna	1	8	2.42	7	15.17	7	0	9	2.34	8	14.13	8	0	7	1.75	8	11.58	10	2	10	1.55	9	9.12	9	0
Molly	1	8	4.33	4	15.5	8	4	10	1.58	7	23.02	8	1	9	1.15	8	23.53	9	1	10	1.42	8	13.33	8	0
Jack	1	3	2.42	2	12.67	2	0	6	3.08	3	29.89	6	3	5	5.5	5	23.33	6	1	7	2.16	6	21.43	8	2
Max	1	7	4	3	13.84	7	4	7	3.4	9	17.07	9	0	9	2.68	10	14.77	10	0	9	2.67	8	14.83	9	1
Rebecca	1	6	3.33	2	18.23	7	5	8	1.47	8	10.33	9	1	7	2.65	8	7.1	9	1	9	1.42	9	11.63	10	1
Group 1	Ave	6.4	3.3	3.6	15.082	6.2	2.6	8.2	2.374	7	18.888	8	1	7.4	2.746	7.8	16.062	8.8	1	9	1.844	8	14.068	8.8	0.8
Amber	2	8	5	3	10.17	7	4	9	3.82	8	8.21	9	1	10	3.08	10	13.83	10	0	10	2.83	9	8.72	10	1
Amanda	2	5	2.93	1	12.18	4	3	3	2.38	1	8.02	4	3	4	2.83	4	10.73	7	3	6	3.02	7	11.73	7	0
Matt	2	4	4.33	4	11.18	5	1	6	2.93	0	7.13	3	3	5	1.83	3	8.08	5	2	7	3.08	5	21.93	8	3
Lucia	2	7	6	6	17.38	8	2	7	4.23	5	11.56	8	3	8	5.5	10	8.02	10	0	8	2.83	7	6.33	8	1
Frank	2	6	6.87	4	24.17	6	2	5	5.4	4	21.07	6	2	8	3.12	6	14.22	8	2	9	3.15	6	14.13	6	0
Group 2	Ave	6	5.026	3.6	15.016	6	2.4	6	3.752	3.6	11.198	6	2.4	7	3.272	6.6	10.976	8	1.6	8	2.982	6.8	12.568	7.8	1