Teacher practice in primary mathematics classrooms:
A story of positioning

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Abstract

The past twenty-five years have seen a dramatic increase in the interest given to dialogue between teachers and students, and students and students during mathematics teaching and learning. This interest is evident within the growing body of research and the call for the increased quality and quantity of student discourse in curriculum and policy documents. Recent research in mathematics education is underpinned by the belief that students learn best when they have the opportunity to participate in their own and others' mathematical talk, text, and actions in purposeful and meaningful ways.

This study explores how teachers position themselves and students in their lowest and highest mathematics strategy groups and how that positioning influences the sharing of mathematical know-how. Mathematical know-how within this study comprises teacher and student independence, judgement, and creativity.

Social-constructivist theories of teaching and learning underpin the focus of this study. The importance of teachers and students constructing and co-constructing individual and shared mathematical understandings through dialogically rich interactions with each other and the environment are considered. Positioning theory provides the theoretical lens through which mathematical know-how will be analysed and understood. The constructs of positioning theory important to this research were the teachers' and students' positions, enacted as their rights and duties, the storylines that develop through the positions, rights, and duties and the teachers' and students' social acts which come to have significance and be a social force within the teaching and learning.

The decision to employ qualitative case study methodology arose naturally from the subjective social phenomenon of teaching and learning. The analysis of data generated through video and audio recordings, transcriptions, participant observations, and documents and archival records supported the development of the two cases: teacher affording positioning, and teacher constraining positioning.

The particularised and investigative design of qualitative case study supported the development of an emerging taxonomy of teacher affording and constraining positioning. The taxonomy contributed to the growing body of knowledge regarding student participation by categorising new thinking in regards to the phenomenon of teachers and positioning in mathematics. Teachers in this study afforded the sharing of mathematical know-how from the position of appropriator, procurer, and provoker. The positions of controller, proprietor, and protector were found to constrain the sharing of mathematical know-how.

Significant differences were revealed in how teachers positioned themselves and how their positioning influenced opportunities for student engagement. Higher levels of student talk, text, and actions were evident when teachers positioned themselves to ensure the mathematics was visible, fluid, and contestable. Collaboration between teachers and students, and students and students, was a strong feature of the emerging taxonomy.

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This thesis is dedicated to all the learners and teachers out there whose tummies start to go a wee bit queasy at the thought of mathematics. There is hope for us yet!

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Chapter One: Introduction to the Study

> Our strongest evidence of the potential for higher achievement for diverse students arises out of a range of classroom research programmes that make student learning processes and understandings transparent, and make explicit the kinds of teaching practices and approaches that support student learning processes. (Alton-Lee, 2003, p. 90)

1.1 Introduction

Mathematics teaching and learning in 21st century New Zealand looks and sounds different to earlier and more traditional approaches and experiences. Significant changes have occurred in regard to who gets to talk, who listens, and what gets discussed. Prior to the first efforts to reform mathematics in the 1990s, teachers tended to be positioned as the unchallenged authority and as such had greater opportunity for determining what got shared and by whom (Anthony & Hunter, 2005; Wood, 2002; Young-Loveridge, 2005). Students were usually positioned as passive receivers of instruction and knowledge and were expected to listen to and watch the teacher demonstration, reproduce, and practise (Attard, 2011; Even & Tirosh, 2008; Goos, 2004; Hunter, 2009). Access to mathematical knowledge was often limited to students who were perceived by the teacher to be better prepared to learn mathematics proficiently. Students not perceived to be mathematical tended to experience teaching that was procedural and simplified (Anthony & Walshaw, 2007).

Traditional mathematics classrooms have been superseded by reform-oriented inquiry-based pedagogical approaches and learning experiences. Inquiry-based teaching and learning is socially and culturally framed and should be thought provoking and talk provoking for teachers and students (Boaler, 2009; Walshaw & Anthony, 2008). Teachers and students should be interacting, communicating, reasoning, challenging, and defending (Cobb, 2012; Goos, 2004; J. Hunter, 2009; R. Hunter, 2005; Walshaw & Anthony, 2008). The New Zealand Ministry of Education (MoE, 2007a) now requires teachers to develop classrooms as "learning environments that foster learning conversations and learning partnerships and where challenges, feedback and support are readily available"

(p. 24). The development of such classrooms requires teachers to step back and create the space for students to explore mathematical ideas but not so far back that some students might be “left to drift in the space” created by the teacher (Murphy, 2013, p. 109). Inquiry-based teaching and learning requires a subtle balance between teaching and facilitating.

The move to inquiry-based teaching and learning of mathematics in New Zealand was occurring alongside international trends. For example, Australian teachers were asked to provide opportunities for students to talk about their mathematical thinking within the context of their backgrounds and interests (Australian Association of Mathematics Teachers, 1998). In America, the National Council of Teachers of Mathematics (2000) called for pedagogies that stimulated student fluency, creativity, resourcefulness, insightfulness and understanding and classrooms that endorsed genuine understanding of mathematics. OFSTED (2003) (the Office for Standards in Education in Britain) highlighted the need for teachers to position students to talk about and collaborate on differences, difficulties, and successes in their mathematics learning.

Central to the national and international recommendations discussed above is a vision of teachers and students sharing their mathematical knowledge. Walshaw and Anthony (2008) contend that this vision is dependent on “a shared understanding of the importance of dialogue and the sharing of mathematical ideas" (p. 525). Implicit within this vision is an understanding that the mathematical knowledge shared is relevant, rigorous, challengeable, defendable, and progresses learning. What is required are targeted pedagogical approaches that position all students to share their mathematical know-how in ways that elicit and honour students’ contributions, support them to explain and justify, and advance mathematical thinking (Anthony & Walshaw, 2007; Lampert, Boerst, & Graziani, 2011; Yackel & Cobb, 1996). Students who engage with their mathematics can be thought of as “insiders” (Attard, 2011, p. 69). According to Attard (2011, 2013), insiders feel involved with and included in their learning because they have a place and a voice within their classrooms and the mathematics. Therefore, the ways teachers position students can have powerful and pervasive effects on their learning, learning behaviours, and academic engagement (Davies & Hunt, 1994; Herbel-Eisenmann & Wagner, 2010; Wagner & Herbel-Eisenmann, 2009, 2013; Yamakawa, Forman, & Ansell, 2005).

1.2 Research Aim

This study, *Teacher practice in primary mathematics classrooms: A story of positioning* was conducted within the context of teacher positioning for the sharing of teacher and student mathematical know-how. Its main purpose was to understand the effects and influences of 12 teachers affording and constraining positionings of themselves and the students in their lowest and highest groups.

The research question the study addressed was:
How do teachers in New Zealand primary schools position themselves and students in their lowest and highest mathematics strategy groups so that mathematical know-how can be shared?

Positioning in my study refers to the talk, text, and actions that occurred between the 12 teachers and their lowest and highest mathematics groups. The lowest and highest mathematics groups were selected for two reasons. First, because of my personal experiences as a learner in the lowest group. Secondly because of the continued international interest in ability grouping in mathematics education and the advantages and disadvantages of such grouping.

How the teachers and students act and interact with each other describes and serves to explain their positioning (Davies & Harré, 1990). Important within, and influencing the actions and interactions are the teachers and students rights and duties, the storylines created, and the social acts that come to have significance within the group. Rights and duties, storylines, and social acts, as constructs of positioning are described in detail in Chapter Four.

Pedagogical positioning can be reflexive or interactive (Davies & Harré, 1990; Harré & van Langenhove, 1991). Reflexive positioning occurs when teachers position themselves, and interactive positioning occurs when they position a student or students. Pedagogical acts of positioning could include asking or answering questions, accepting answers or probing for explanations, rejecting or accepting answers or explanations, and allowing cognitive conflict or ensuring politeness norms. This study also determined if teachers position students in their highest and lowest groups differently and the positive or negative effect on learning and achievement any differences might have.

1.3 Background Context of the Study

Twenty-first century students are expected to participate more in their own and their peers' mathematics learning. They are expected to contribute, listen, explain, represent, and challenge what they and others know (Anthony & Walshaw, 2006; Yackel & Cobb, 1996). Students who are able to participate in their own and their peers' mathematics are seen to be developing their mathematical proficiency, and their mathematical identity, efficacy, and generosity (National Research Council, 2001).

Teachers in the 21st century are expected to notice, question, and reflect on students' ideas and explanations, and orchestrate and operationalise opportunities for students to talk (Choppin, 2011; Hunter, 2008; Kazemi, Franke, & Lampert, 2009; Nathan & Knuth, 2003; Smith & Stein, 2011). However, talking merely to sustain a conversation or as a means to achieve student co-operation is not enough (Smith & Stein, 2011). Talking about and listening to mathematics must be centred on powerful ideas (Brophy, 2002, 2006; Cirillo, 2013a; Walshaw & Anthony, 2008). Any decisions made or actions taken by teachers should be "nudging the conversation in mathematically enriching ways" (Walshaw & Anthony, 2008, p. 536).

All mathematical talk should uphold the integrity of students' ideas, test the reliability of the ideas, and synthesise them (Fraivillig, Murphy, & Fuson, 1999). Teachers should plan, monitor, reflect upon, and make changes that "demand students' mathematical talk" (Walshaw & Anthony, 2008, p. 523). They should validate contributions and ask "authentic questions" (Cirillo, 2013a, p. 3) that seek information more than answers with the intent of increasing access to the mathematical knowledge. Students' ideas should be used to shape instruction and to occasion particular mathematical understandings in the classroom. O'Connor and Michaels (1996) contended that responsive teachers "tie together the different approaches to a solution" (p. 65) by providing opportunities for students to share their thinking, listen and attend to the thinking of others, and be listened to by others, the ultimate aim being all students seeing "themselves and each other as legitimate contributors to the problem at hand" (p. 65).

Research regarding student talk has included relationships with teacher beliefs (Askew, 2002; Askew, Brown, Rhodes, Wiliam, & Johnson, 1997; Millett, Brown, & Askew, 2004), teacher pedagogical expectations (Askew, 1999, Nathan & Knuth, 2003; Sfard & Kieran, 2001; Thomas & Ward, 2002; Wood, 2002; Woodward & Irwin, 2005; Yackel & Cobb, 1996), and social justice (Ball, 1993; Boaler, William, & Brown, 2000; Cobb & Hodge, 2002; Zevenbergen & Ortiz–Franco, 2002). Less researched is the relationship between teacher positioning and student talk and the outcomes of that relationship. How teachers position themselves and their students influences the opportunities students have to participate. Such positioning could be referred to as the teacher's pedagogical style (Fried & Amit, 2004) and include a teacher deciding to personally validate a student's answer, ask the individual to validate the answer, or ask other students to validate the answer for the individual. This study sought to understand the effects and influences of teachers affording and constraining positionings of themselves and the students in their lowest and highest groups.

1.4 Rationale for the Study

There were three motivations underpinning the rationale for this research. The first motivation was my experiences as a mathematics learner, the second as a mathematics teacher, and the third as a facilitator of the Numeracy Development Project (NDP, MoE, 2007b) professional development. Each motivation is discussed next.

I did not know I was not good at mathematics until I started school. As a five-year-old I soon realised that my mathematical thinking was incorrect, that is, it did not match my teacher's way of thinking mathematically. I was placed in the bottom group and as Wink (2000) suggested, even as a five-year-old “I knew the difference between the buzzards and the blessed” (p. 89). I remained in the bottom group for the rest of my mathematics career until giving up (officially) halfway through Year 12, my second to last year at secondary school. As a long-standing member of the bottom maths group I believe I was given fewer and less varied instructional opportunities to learn, and these opportunities were more procedurally based and simplified (Anthony & Walshaw, 2007; Boaler et al., 2000) and teachers may have expected less of me (Stein, Smith, Henningsen, & Silver, 2000). I do not recall my know-how being asked for, let alone appropriated, nor was I privy to the know-how of others — other than of course the teachers. When I did seek mathematical know-how from my peers I was accused of cheating and ordered (once again) to the corridor. I believe that I thought differently to the teacher, but as the teacher’s thinking was the only model I had, I soon came to realise that different meant wrong and that I could not do maths. Not surprisingly, I had negative feelings towards mathematics as a subject, myself as a learner of mathematics, and in some ways towards my mathematics teachers.

My experiences as a learner in the bottom group influenced my pedagogical beliefs as a teacher of mathematics. Whenever possible, students worked in heterogeneous co-operative groups to share their different ways of thinking and determine the efficiencies of their strategies. I was careful not to impose my way of thinking for two reasons. First, I wanted students to feel that their ideas were important and relevant to our discussions, and secondly, because I still felt that my way of thinking was wrong.

It was not until I participated in the NDP professional development in 2002 that I realised there was more than one acceptable way of thinking mathematically. We were asked to solve a problem; the different strategies were recorded on the board, and for the first time the strategy I used was accepted. I had not been confident enough to share my strategy but my sense of relief that I had a kindred ‘thinking’ spirit was huge. This experience reinforced the decisions I had made about how I would teach mathematics and had a huge influence on my confidence as a mathematician, so much so that I became an NDP advisor in 2004. My role as an NDP advisor included me supporting schools and their communities to develop their mathematical knowledge and professional practice in the context of their own school setting (Higgins, Sherley, & Tait-McCutcheon, 2007).

This research is driven by my experiences as an unsuccessful learner of mathematics. Through further encounters as a teacher, advisor, lecturer, and researcher, I have become more aware that my lack of success need not have been an on-going cycle. I am sure I could have been more successful had I experienced more effective pedagogies and teachers who positioned me as a contributor of mathematics. This research uncovers those effective pedagogies and builds on existing knowledge of how to ensure success for all students.

1.5 Overview of the Thesis

This thesis is presented in nine chapters. Chapter Two provides the background to this study by introducing a social constructivist model of teaching and learning mathematics and reviewing the curriculum and programme documents and pedagogical tools that guide mathematics in New Zealand. To provide insight into the success of these documents and tools, recent trends in student achievement are explored. Chapter Three examines three constructs of student mathematical know-how: independence, judgement, and creativity. Positioning theory is introduced in Chapter Four alongside the constructs of positioning theory used to analyse this research: positions and positioning, storylines, and the action-social act sequence. The epistemological, ontological, and methodological approaches underpinning this study are described in Chapter Five. Chapter Six focusses on the seven teachers in my study who had a consistent approach (affording or constraining) to positioning students in their lowest and highest mathematics groups. The teachers who had an inconsistent approach to positioning individuals or groups are presented in Chapter Seven.

Chapter Eight presents a taxonomy of teacher affording and constraining positioning derived from the research data. Included in this final chapter are the implications of the findings, recommendations for future research and policy initiatives, and the limitations of this study. This thesis is concluded with a personal reflective statement.

Chapter Two: Background to the Study

> Strategy sharing also has more impact when the teacher's specialised mathematical knowledge and pedagogical content knowledge allows the communication to be nudged in mathematically enriching ways. (Anthony & Walshaw, 2006, p. 30)

2.1 Introduction

In my study, the disciplinary nature of school mathematics is interpreted as the co-construction of mathematical claims, reasoning, and understanding. That is, teachers and students participating collaboratively in discussions in mathematical ways and arriving at agreed understandings (Conner, Singletary, Smith, Wagner, & Francisco, 2014). Teachers and students should have opportunities to develop the identities and practices of mathematicians by considering, shaping, sharing, trialling, reviewing, and reshaping their own and others' mathematical know-how (Anthony & Walshaw, 2006; Boaler, 2002). Both should have opportunities to deliberate over the problems, to model and experiment with alternative ways of thinking, to struggle, persevere, and to enjoy the challenges and successes. The teacher as mathematician has a specific responsibility to embody, experience, and exhibit mathematical enquiry to create an environment in which students can also experience mathematics as mathematicians (Barton, 2009; Watson, 2008).

Students sharing their mathematical know-how is an important issue in mathematics education. There are two aspects to consider when examining what teachers do to facilitate such sharing. The first aspect is the contexts in which the teacher's decisions occur and the mathematical thinking is shared. The second is the decisions teachers make within those contexts in regard to who, the teachers or students, gets to share their mathematical thinking, when, and how.

This chapter explores contexts of teaching and learning mathematics in New Zealand primary schools. The curriculum and programme documents that guide the teaching and learning of mathematics in New Zealand are examined. Also considered is the mathematics and numeracy achievement data of New Zealand students.

Section 2.2 provides an overview of social constructivism and foreshadows the kinds of decisions teachers need to make about how the teaching and learning of mathematics should proceed. A social constructivist framework underpins the NDP (MoE, 2007b), and the New Zealand Curriculum (NZC, 2007a). The development and iterative nature of the NDP (MoE, 2007b) is explored in Section 2.3. Section 2.4 examines the three pedagogical tools that underpin the teaching and learning of numeracy in the NDP: the Number Framework (MoE, 2007b), the Diagnostic Interview (MoE, 2007c), and the Strategy Teaching Model (MoE, 2007d). The rationale behind each tool and their interconnectedness is discussed. Included in Section 2.4 is a review of the structure of the model strategy lessons and the use of routine problems in the NDP teacher resource books. The NZC (2007a) is explored in Section 2.5. Features of the curriculum such as its visions, principles, and values, and the key competencies for learning are described within a mathematical context. The importance of students communicating their mathematical thinking is emphasised throughout Sections 2.3, 2.4, and 2.5. Trends in New Zealand students' mathematical and numeracy achievement are explored in Section 2.6.

2.2 Teaching and Learning Mathematics in New Zealand

Teaching and learning mathematics in New Zealand has been framed by a social constructivist model since the 1990's (Cobb, 2007; J. Hunter, 2006). The model infers that people act and interact, socially and culturally, together and with their environment, to construct individual and shared knowledge (Ernest, 1994, 1996). The focus of social constructivism within mathematics teaching and learning is on shaping ideas and meanings rather than behaviours and procedures (Higgins, Irwin, Thomas, Trinick, & Young-Loveridge, 2005; Thomas & Tagg, 2004). This focus contrasts with earlier traditional approaches where correct answers and rote learning were emphasised (Anthony & Hunter, 2005).

Knowledge can be considered from the personal view of an individual or the collective view of a group (Bobis, Mulligan, & Lowrie, 2004; Brophy, 2006). Personal knowledge is shaped, reorganised, and strengthened by the relationships of the group in which it is created and "individual students contribute to the evolution of classroom mathematical practice as they reorganise their mathematical understanding" (Cobb, 2000, p. 173).

Context and environment are considered important within mathematics teaching because a change in either could lead to a change in what is, or can be, socially constructed. The construction of mathematical understandings occurs because of the dialogically rich social interactions in which students participate (Bobis et al., 2004; Cobb, 2000; R. Hunter, 2006; Wood, 2002). Therefore, teachers need to position themselves and their students to share, discuss, argue, and defend. The definition of social constructivism specific to teaching and learning mathematics, and applied in my study, comes from Ernest (1998): "Social constructivism has adopted conversation as an underlying metaphor for epistemological reasons, to enable the social aspects of mathematical knowledge to be adequately treated within the philosophy of mathematics" (p. 274).

Social constructivism is a complement of the constructivist work of Piaget (1964, 1970) and the sociocultural work of Vygotsky (1962, 1978). Piaget focussed on how individual students construct and reconstruct meaning and understanding for themselves through the interrelated processes of adaptation and mental organisation. Students interact with their environment in ways that are consistent with their existing schema (assimilation) and in ways that require them to modify an existing schema or create a new one (accommodation). New knowledge is assimilated with, or accommodated onto, existing knowledge (von Glasersfeld, 1990, 1993). Working from previous learning and engaging with existing knowledge are important principles advocated by Piaget (Bobis et al., 2004; von Glasersfeld, 2000).

Piaget (1964, 1970) claimed that direct instruction could inhibit a student's development and recommended teachers observe students' interests and provide appropriate materials for them to construct knowledge with, and through, their own actions. Where Piaget took an individualised approach to knowledge construction, Vygotsky posited that students needed to be actively and interactively involved in their own and others' learning. Students construct, co-construct, and reconstruct knowledge through their interactions with, and the influences from, their environment and others (Cobb, 2007). Vygotsky (1962) believed that “the only good kind of instruction is that which marches ahead of development and leads it, it must be aimed not so much at the ripe, as at the riping” (p. 104). Cobb (1994) points out that social constructivism is underpinned by both individual cognitive and interactive social perspectives and as such each perspective tells “half of a good story” (p. 17).

Within his sociocultural frame, Vygotsky (1978, 1987) also emphasised the importance of cultural tools or artefacts in learning. Examples of cultural tools include dialogue, written words and symbols, equipment, and norms. The cultural tools that students appropriate are not inherited; instead, they are embedded and embodied in the existing and current social practices of their group or class (Cazden, 2001). Used individually or collectively, cultural tools mediate progress and enhance understanding (Cobb, 1995). Therefore, cultural tools and artefacts are social as is the process of appropriating them.

Constructivist and sociocultural theories highlight the conditions required for learning to occur, what could be learned, and the processes through which learning could occur (Cobb, 1994). Four important aspects of mathematics learning arise from a sociocultural and constructivist theoretical combination. The first is the crucial role that activity plays in mathematics learning and development (Cobb, 1994). The second is the shared understanding of an individual functioning within a social activity (Rogoff, 1990). The third is the culturally and socially situated nature of individual and shared learning (von Glasersfeld, 1992), and the fourth important aspect is that communication is a key component of learning (Sfard, 2000).

Learning, within a social constructivist framework, is a social process, negotiated in collaboration with other students and the teachers who are part of classroom learning communities (Bobis et al., 2005; Brophy, 2002, 2006). Mathematical learning is “both a process of active individual construction and a process of enculturation into the mathematical practices of wider society” (Cobb, 1994, p.13). For interactions, collaborations, and negotiations to occur between students and teachers, an environment rich in communication and the sharing of mathematical know-how are essential (Bobis et al., 2005; R. Hunter, 2006; Wood, 2002). Students must be positioned at the centre of an active, interactive, and constructive process of learning (Cobb, 2007; von Glasersfeld, 1992).

Limitations of social constructivism include the problematic nature of student talk whereby not all students or groups of students have the same opportunities to talk and the dilemma that not all student talk progresses learning, with some being off topic or mathematically incorrect (Brophy, 2006; Nuthall, 1999, 2004). Nuthall (2004) suggested that teachers have a responsibility to ensure all students have opportunities to participate in mathematical talk focussed on their shared experiences. Student talk needs to be based around a shared task rather than a textbook reading to ensure the mutuality of the experience. All students need to participate to ensure the same understandings are shared. Learning based on shared experiences has been found to be mutually supportive and students have been better positioned for understandings to become taken-as-shared. Secondly, teachers need to ensure the mathematics shared by students and appropriated by others is clear, distinct, and correct.

The dichotomous positioning of social constructivist and transmission teaching styles is not helpful because teachers rarely teach through one approach (Anthony & Hunter, 2005; Brophy, 2006). Both social constructivist and transmission styles of teaching could be required within the same lesson. A social constructivist approach does not mean that teaching with telling is unacceptable or that students would work independently of teacher structuring or scaffolding (Brophy, 2006; Cobb, 1994). A transmission approach does not mean that students cannot co-construct knowledge independently of the teacher or pursue questions of interest different to those of the teacher. More important than teachers attempting to teach in particular ways is their conception of when one approach might be more beneficial to students than another. Brophy (2006) observed that social constructivism should be about teaching and learning and social constructivists should be asking “what approaches to teaching will optimise the students’ construction of knowledge?” and “what is the nature of knowledge and how is it constructed and validated?” (p. 530).

Social constructivist perspectives of teaching and learning were applied in this study to investigate how teachers’ positioning of themselves and their students influenced opportunities for students' mathematical strategies to be shared in the highest and lowest strategy groups in New Zealand primary classrooms. The focus given to students in the lowest and highest groups in this study foreshadows a need to also consider the structure of ability grouping students for instruction and its possible impact on teacher positioning and students achievement. Grouping for instruction is explored in the following section.

2.2.1 Grouping for instruction

The dilemma of grouping students by achievement or mixed-ability "is one of the most contentious issues in education" (Boaler, 2014, p. 1). Ability grouping, according to the NDP (2007d) "allows students to work on problems that tightly match the next progression in their learning trajectory" and " provide intense situations for dialogue and new learning, increasing students' potential for success" (MoE, 2007d, p. 12).

Research has identified both advantages and disadvantages of this ability grouping. The advantages include practicality, targeted teaching, and student opportunities. Grouping by ability was found to be a more practical group arrangement, as teachers' believed it allowed them to pitch work at an appropriate level for students (Bartholomew, 2003; Blatchford, Hallam, Kutnick, & Creech, 2008). Boaler, Wiliam, and Brown (2000), agreed that in ability groups, students could be more readily given appropriate work within their zone of proximal development and the level of work could be altered to meet shifting zones. According to Higgins (2002), teachers believed they taught ability groups in more focussed ways that provided greater opportunities for students to explain their problem solving strategies.

The disadvantages of ability grouping in mathematics, particularly for those in the lowest group include disparate teacher instruction, interactions, and expectations, inequality, and impeded student self-belief. Research has shown that placing students into ability groups can create a set of expectations for teachers that overrides their awareness of individual capabilities (Bartholomew, 2003; Boaler, et al., 2000; Murphy, 1988; Oakes, 1985; Zevenbergen, 2003).

Students in lower ability groups tended to follow narrower protracted curriculums that were procedural and simplified, less interactive, with fewer instructional opportunities to learn (Boaler., et al 2000). They were more likely to be allocated the least effective teachers and there appeared to be an underlying belief that anyone (such as teacher aides or parent helpers) could teach the lowest group (Oakes, 1985). Instructional time was lost getting lower ability groups started, their instructional time was more likely to end earlier, they lost more time during transitions and interruptions and were more likely to have periods of time when they had no tasks to complete (Murphy, 1988). Teachers spent more time managing behaviour in lower groups and students exhibited higher rates of off-task behaviour. Bartholomew (2003) identified that some mathematics teachers valued the experience and contributions of the lowest group less than the highest group. With the lowest group the teacher was also very authoritarian in manner and with the top group more friendly and chatty.

Not surprisingly, students in the lowest group tended to have lower self-belief. They saw themselves as not being able to succeed to the same level as their peers and constructed themselves negatively in their groups (Boaler, 2005; Zevenbergen, 2005). This positioning impacted negatively on students' motivation and engagement (Banfield, 2005; Zevenbergen, 2003). Research over 30 years has posited that students in lower ability groups received less of almost all of the conditions associated with learning - instruction, time, curriculum, opportunity, and success. Murphy (1988) contended that the systematic discrimination of pupils in lower ability groups "is more attributable to teacher practices and behaviours than to student characteristics or ability" (p. 148).

The practice of ability grouping is common in numeracy classrooms in New Zealand and the student participants in my study were grouped for instruction by ability. Therefore, it is important to consider the use of ability grouping and its possible impact on teachers' positioning and students' achievement in my study. Grouping students by ability was also relevant to the background of this study as it is recommended within the NDP as an option for grouping students for instruction.

The NDP (MoE, 2007b) and the NZC (MoE, 2007a) guided the teaching and learning in this study. These two documents are described in the following sections and the emphasis they give to effective pedagogies and student development and learning are highlighted.

2.3 The New Zealand Numeracy Development Project

The NDP underpins the teaching and learning of numeracy in approximately 95% of New Zealand's primary schools (Higgins & Parsons, 2009). It is to be expected then, that mathematics research situated within New Zealand schools would consider aspects of the NDP. The NDP was developed in response to the poor performance of New Zealand students in the 1995 Third International Mathematics and Science Study (TIMSS, Garden, 1996, 1997). Results pertaining to TIMSS indicated below average standards for New Zealand students in relation to number, algebra, and measurement concepts when compared with international averages (Garden, 1996, 1997; Higgins, 2003).

In 1999, the New Zealand Government announced the goal that "By 2005, every child turning nine will be able to read, write, and do maths for success" (MoE, 1999, p. 1). Three key themes underpinning this goal were: raising expectations for students' progress and achievement; lifting professional capability to enhance the interactions between teachers and students; and developing community capability by encouraging and assisting family, and others to support students (MoE, 1999, 2002). An educational reform in mathematics designed to enhance teachers' content, pedagogical, and pedagogical content knowledge, and increase student achievement was implemented.

The Count Me in Too (CMIT) project in New South Wales (Bobis, 2003; Bobis et al., 2005; Department of Education and Training, New South Wales, 1998) significantly influenced the development of the NDP. CMIT was adapted from Wright and colleagues' Mathematics Recovery Programme (Wright, 1991, 1998, 2000; Wright, Martland, & Stafford, 2000; Wright, Stanger, Cowper, & Dyson, 1996). The aim was to provide "teachers with better understanding of young children's mathematical thinking and ways of developing more sophisticated mathematical thinking in their students" (Wright, 2000, p. 146). The success of the New Zealand based CMIT project led to a nation-wide pilot project in 2000 which informed the development of the NDP. The NDP development and review since 2001 has been iterative (Higgins & Parsons, 2009). Quantitative and qualitative research findings have led to improvements in the NDP structure, resources, and expectations.

The NDP (MoE, 2007b) accentuates the need for students to know how to communicate their mathematical thinking. It proposes that through written, modelled, and verbal explanations students are expected to share their mathematical strategies, listen to the strategies of others, and discuss what they know and are learning (MoE, 2007d). Teachers can prompt students to share what they know by asking them to explain, record, or model their mathematical thinking, justify their thinking, and challenge the thinking of others (MoE, 2007c). Further student involvement can be initiated by teachers appropriating their explanations, highlighting connections between mathematical concepts, and adjusting or extending the tasks as required (Smith & Stein, 2011; Walshaw & Anthony, 2008).

Being numerate is defined by the MoE (2007b) as "the ability and inclination to use mathematics effectively – at home, at work and in the community" (back cover). Initial stages of the NDP involved the development of a comprehensive numeracy policy and strategy and several pilot projects focusing on the professional development of teachers (Higgins & Parsons, 2009). Aspects of the professional development included: enhancing teaching quality and confidence; providing teacher support material; increasing the availability and accessibility of research information; aligning professional development with support material and research; providing support for teaching Māori and Pasifika students; emphasising the importance of mathematics education prior to school entry; giving greater emphasis to pre-service training; expecting greater involvement of parents and the community; and raising expectations of students' mathematic achievement (Higgins, Parsons, & Hyland, 2003; MoE, 2001).

The implementation of the NDP included the Early (Years 1-3), Advanced (Years 4-6), Intermediate (Years 7-8), and Senior (Years 9-10) Numeracy Projects. More than 25,000 teachers and 690,000 students have participated in the NDP since its inception in 2000 (Higgins & Parsons, 2009). Principals, teacher-aides, teacher educators, researchers, university lecturers, and pre-service student teachers have been involved in the NDP professional development, research, and its on-going development. The following section explores the pedagogical tools of the NDP that underpin the teaching in my study.

2.4 Numeracy Development Project Teaching Tools

The NDP is designed around three pedagogical tools: the Number Framework (MoE, 2007b), the Diagnostic Interview (MoE, 2007c), and the Strategy Teaching Model (STM, MoE, 2007d). The Number Framework, a progression of mathematical ideas, provides the link between the NDP and the NZC (MoE, 2007a). The Number Framework embodies the level one to five achievement aims and objectives of the mathematics and statistics curriculum number and algebra strand. The Diagnostic Interview (MoE, 2007c) provides an insight into what students know and how they strategise. The STM (MoE, 2007d) proposes three phases that students' work through as they master new learning. Each teaching tool gains "power from their interconnectedness, with each tool informing and supporting the other tools" (Higgins & Parsons, 2009, p. 235). The three pedagogical tools of the NDP are outlined in the following sections.

2.4.1 The Number Framework

Through the Number Framework (MoE, 2007b) teachers are provided with an understanding of the key mathematical ideas associated with learning number strategies and knowledge, the means to assess students' current levels of thinking and to measure their progress, guidance for planning and instruction, and an increased awareness of how to assist students to progress (Higgins & Parsons, 2009; Johnston, Thomas, & Ward, 2010). Global progressions in number knowledge and number strategies are proposed within the framework. Stage zero, the first stage, is considered an emergent stage where a student is learning one-to-one counting. The next four strategy stages represent the counting-all strategies: one-to-one counting (stage 1), counting from one on materials (stage 2), counting from one by imaging (stage 3), and advanced counting (stage 4). The four higher strategy stages represent additive, multiplicative, and proportional part-whole thinking: early additive (stage 5), advanced additive (stage 6), early multiplicative (stage 7), and advanced proportional (stage 8). The stages are presented as an inverted triangle to illustrate how the knowledge, range of strategies, and mathematical thinking increases at each stage (Johnston et al., 2010). The eight stages in a students' development occur across three strategy domains (addition and subtraction, multiplication and division, and proportions and ratios) and five knowledge domains (number identification, number sequence and order, grouping and place value, basic facts, and written recording). Strategies are the mental processes students use to solve number problems and knowledge is key information students need in order to apply strategies (Young-Loveridge, 2001).

Hughes (2002), a member of the Numeracy Reference Group and one of the original writers and reviewers of the NDP (MoE, 2007b), contended that although the distinction between knowledge and strategy was somewhat artificial, it was made for pedagogical reasons because teaching for knowledge and strategy development warranted different teaching approaches. Students learn knowledge for automatic recall and strategies as the means to reason with numbers (Cobb, 2012). Though taught differently, knowledge and strategy are interrelated. Existing knowledge provides the platform for new strategies to develop. Once mastered, strategies become accessible as new knowledge and over time new knowledge becomes existing knowledge. Thomas and Ward (2002) identified a strong correlation between achievements in knowledge and strategy whereby "students who demonstrate more complex number strategies are almost without exception those who have a stronger understanding of numeral identification and number sequences" (p. iii). Therefore, it is important that students make progress with strategy and knowledge concurrently (MoE, 2007b).

2.4.2 The Diagnostic Interview

The second pedagogical tool, the Diagnostic Interview (MoE, 2007c) has three main purposes. The first is to ascertain students' current knowledge and strategy stages, the second to recognise how their understandings have developed, and the third to identify gaps. The Diagnostic Interview is based on a verbal question and answer format comprising an individual, task-based, oral assessment. This format provides teachers and students with access to the students' mathematical thinking and reasoning, without their potential proficiency being constrained by literacy barriers (Young-Loveridge, 2006). Emphasised within the Diagnostic Interview is the need for teachers to understand students' strategic thinking and knowledge. As von Glasersfeld (1992) explained, "teachers, who have the goal of changing something in students' heads, must have some notion of what goes on in those other heads" (p. 3).

Assessment questions in the Diagnostic Interview (MoE, 2007c) are aligned with the strategy and knowledge stages of the Number Framework (MoE, 2007b) and are organised into three overlapping interviews at different difficulty levels. The strategy questions require students to explain how they derived their answer and the knowledge questions seek fluent responses (MoE, 2007c). The interview enhances teachers' and students' understandings of the learning progressions required in numeracy (Higgins & Parsons, 2009). Results from the interview are used to inform teaching and learning, such as determining the whole class knowledge focus and grouping students for strategy teaching.

The Diagnostic Interview has three embedded design elements (Higgins & Parsons, 2009). The interview models the types of open-ended questions teachers should be using when asking students to describe their mathematical thinking. By asking and listening, teachers are deepening their understandings of the kind of open-ended questions that elicit thoughtful answers and the thinking that occurs at each stage of the framework. Teachers are provided with summative (where is this student at?) and formative (where are they headed next?) assessment data. With this information teachers are able to plan and teach in more targeted ways.

The NDP calls for an explicit link between the data obtained from the Diagnostic Interview and the learning experiences teachers provide for their students (MoE, 2007d). The STM, the third pedagogical tool of the NDP, provides this link as a guide for the explicit teaching of strategies. The STM is discussed in the following section.

2.4.3 The Strategy Teaching Model

The STM guides the explicit teaching of number strategies (Hughes, 2002). Three ways students need to interact with new mathematical concepts are recommended. These are using materials, using imaging, and the abstract stage of using number properties to represent ideas (MoE, 2007d). The STM begins by acknowledging the existing knowledge and strategies that students bring to their learning.

The STM was influenced by, and appropriated from the Pirie-Kieren (P-K) Theory, the seminal work of Pirie and Kieren (1989, 1992, 1994) from Canada. The power of the P-K Theory is that it interprets the growth of mathematical understanding, not the understanding of mathematical growth (Martin, Towers, & Pirie, 2000). The dynamic relationship between the phases of using materials, using imaging, and using number properties is illustrated by the double-ended arrows in Figure 2.1.

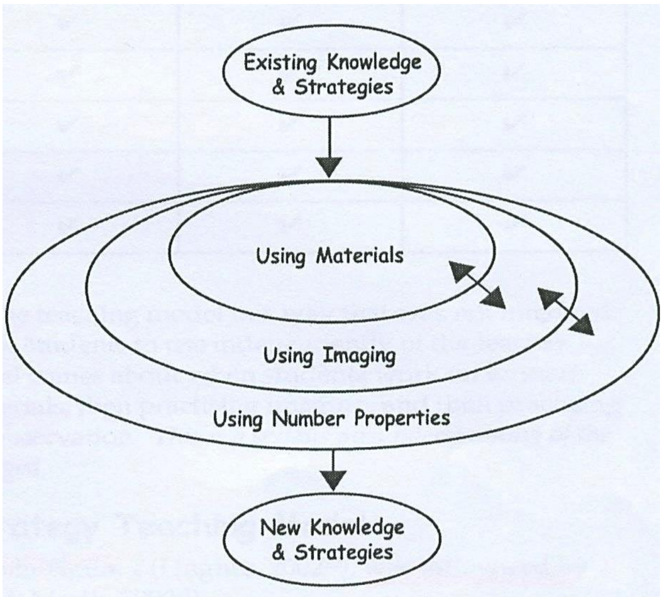

***Figure 2.1*: The Strategy Teaching Model (MoE, 2007d, p. 5).**

Using materials enables students to see and manipulate representations, equipment or diagrams. The use of materials in the STM differs from the more experiential or "hands-on" orientation where equipment was used to "keep students actively engaged" (Higgins, 2005, p. 89). In New Zealand, teachers of students in their early years of schooling have traditionally used materials to teach mathematics, and there has been an expectation that older students would experience more book-based studies (Higgins, 2005; Hughes, 2002). The STM anticipates that students of all ages would be accessing materials, thus reflecting the sociocultural influence cultural tools and artefacts have in mediating learning.

The using imaging phase is an attempt to bridge students' conceptual construction from materials to abstraction and assist them to make the connection between materials and generalisations or concrete and abstract cognition (Hughes, 2007). Teachers can provoke the use of imaging by moving between materials and imaging and imaging and abstracting, shielding materials from students, allowing students to see but not manipulate materials, and asking them to imagine the materials (Higgins & Parsons, 2009; Hughes & Peterson, 2003; Wright, 1991; Wright et al., 2000).

According to Hughes (2002), if children are having trouble imaging it can be assumed that manipulation of materials has not led to successful learning so the teacher should provide the materials again and fold-back to the using materials phase of the STM. Folding-back means returning to a previous phase of the STM (Pirie & Kieren, 1992). For example, if students are experiencing difficulty imaging addition problems to 10, the teacher may re-introduce materials to support students. A return to a previous phase does not indicate a return to the original activity but rather prompts a new activity stimulated and influenced by outer level knowing. By folding-back, a deeper understanding is achieved because the student has the opportunity to extend, reflect on, and reorganise their thinking before returning to the outer layer (Pirie, 2002; Pirie & Kieren, 1994).

Success at the using imaging phase indicates readiness for the final phase of using number properties. Students at the using number properties phase reason directly with the numbers, make generalisations, and do not need to use materials or imaging. Progression to using number properties is promoted by increasing the complexity or size of the numbers involved (MoE, 2007d). With larger and more complex numbers a reliance on materials or imaging becomes too onerous. At the using number properties phase students are also expected to look at the numbers they are working with and to apply the most efficient strategy for those numbers (Cobb, 2012).

Hughes (2007) identified a problem with the way some teachers were using the STM. He noted they were “reducing the model to a step-by-step set of rules that they delivered … not listening, observing, understanding, and acting in response to students’ actions and words” (p. 2). The MoE (2007d) also noted a “serious misunderstanding of the teaching model [that] should never be encouraged” (p. 6). Students should not be practising on materials, imaging, and/or number properties through teacher-provided worksheets, independent of guidance and observation from the teacher. It was never intended to be the teacher’s responsibility to lead the students through each phase of the STM and to the solution (Hughes, 2007). The teacher’s duty is to provide tasks at suitable phases and stages, observe and appropriate students' actions and discussions about the tasks, and ask questions that support students to derive their own mathematical understandings at each phase. New knowledge and strategy learning occurs when students shift from “an externalised representation to a visualised idea and then to an internalised representation” (Higgins, 2005, p. 89).

An essential component of the STM is the expectation that students are able to illustrate and articulate their strategies at each phase (MoE, 2007d). Students must be able to clearly explain their thinking before moving to the next phase. If the thinking is not clear then more experiences are required at the same or an earlier phase. There is no designated time frame in which students move through the three phases. With some concepts students may move through all three phases in one lesson or they may spend several lessons exploring thinking at any phase of the model.

Each pedagogical tool of the NDP - the Number Framework, Diagnostic Interview, and STM, emphasises the need for students to be able to illustrate and articulate their thinking and listen to others’ explanations. More important than the answer is the mathematical thinking and reflection that led to it (Anthony & Walshaw, 2009). The teacher’s guide to the Number Framework (MoE, 2007b) provides illustrations of students verbalising the strategies they could use at each stage. Opportunities for students to think, communicate, make connections, and reflect through pictures, diagrams, words, and symbols are emphasised. The Diagnostic Interview (MoE, 2007c) requires students to share what they know and explain the strategies they used on specific tasks. As students’ progress through the phases of the STM they are expected to explain, reason, and justify their mathematical thinking when using materials, imaging, and number properties. The ideas and values of the Number Framework, Diagnostic Interview, and STM are encapsulated in the NDP teacher resource books, which are examined in the following section.

2.4.4 Teacher resource books

The nine NDP teacher resource books can be thought of as curriculum materials because they “provide teachers with guidance for classroom instruction” and “foster teachers’ learning as they use them” (Remillard & Bryans, 2004, p. 356). Books 1, 2, and 3 (MoE, 2007b, 2007c, 2007d) contain the Number Framework, Diagnostic Interview, and STM respectively. Book 4 (MoE, 2007e) provides teaching ideas for developing students’ number knowledge. Books 5 to 8 (MoE, 2007f, 2007g, 2007h, 2007i) focus on the strategy teaching and learning of addition, subtraction and place value, multiplication and division, proportions and ratios, and teaching number sense and algebraic reasoning. The direct instruction of the explicit mathematical representations and procedures students are expected to acquire are promoted within books 5 to 8 (Ewing, 2011; Murphy, 2013).

The books are a core component of the NDP professional development and provide guides for the teaching and learning of number strategies (MoE, 2007b, 2007d). Each book contains model lessons that increase in difficulty from the earlier to the higher stages of the Number Strategy Framework. The model lessons can be viewed as cultural tools because they have the potential to mediate and influence teachers’ actions through suggested questions and responses. Each lesson focusses on a specific number strategy which is introduced as a learning intention: *I am learning to*. Learning intentions describe the knowledge, skill, understanding, and/or attitudes and values that are needed to develop the particular mathematical strategy (MoE, 2014). The MoE (2014) recommends that learning intentions should support students to understand both what they are learning and why. Connections with other strategies are explained and materials to guide each strategy are recommended. Most lessons are divided into the three phases of the STM: using materials, using imaging, and using number properties. Teaching notes and word and number story examples are provided for each phase as a basis for teaching the strategy.

The NDP claims that “the project is not about students learning a sequence of narrow, pre-described mental strategies” (MoE, 2007d, p. 2). It is implicit that the NDP never intended strategies to be taught as rules to be followed. However, research has shown that strategies may have been taught (unintentionally) in less desirable ways. International research has shown that some teachers mechanically follow the content and sequence of the lessons and do not always enact curriculum materials in ways that engage students with the complexity of the tasks (Choppin, 2011). New Zealand-based research found that teachers were implicitly expected to use and rely on the teaching, planning, and assessment resources provided and that inexperienced or unconfident teachers could become overly reliant on the books (Cobb, 2012; Scouller, 2009; Young-Loveridge, 2010). Teachers could be using the resource books as ready-made mathematics to be followed rather than as suggested ideas to be built upon, and the problems could be presented as routine problems (Ewing, 2011; Murphy, 2013). Routine problems are scripted, performance oriented, well defined, and previously demonstrated methods that can lead to rule-following (Askew, 2011; Choppin, 2011). They tend to be predictable with an obvious solution method that has been predetermined, and is promoted by the teacher or textbook (Holster, 2006). Students are guided toward solving a mathematical problem in a certain way and there is limited opportunity for them to share their own know-how or develop their own strategies (Murphy, 2013). Such teaching practices were seen to constrain students’ learning opportunities (Choppin, 2011).

Cirillo (2013a) advised that “if we want students to have interesting discussions, we need to give them something interesting to discuss” (p. 2). Non-routine problems are unscripted, unfamiliar, unpredictable, and require improvisation (Askew, 2011; Mullis et al., 2003). Improvisation is not about being able to think quickly on the spot but rather co-ordinating what is known in new ways. A higher level of interpretation, organisation, flexibility, conjecture, and review is required by students when solving non-routine problems.

The word and number problems in the NDP model lessons complement the strategy being learned and are most effectively solved using that same strategy. As such, they could be described as routine problems. A predetermined solution method could inadvertently promote the misnomer that there is one correct way to solve a problem and students could be learning that mathematics is about plugging the right numbers into a fixed strategy (Askew, 2011). Following recommended strategies could require compliance and not the active learning, creativity, and connectivity promoted as a vision of the NDP (MoE, 2007b). Students' independence of thought, conjecture, and creativity may not be drawn-out by routine problems and they could be implementing a strategy rather than interpreting a problem (Higgins & Parsons, 2009). The literature regarding the arrangement of the NDP model lessons focusses on the progression through the phases of the STM. There does not appear to be a rationale behind the use of routine problems in the model strategy lessons.

Mamona-Downs and Downs (2005) remind us that routine and non-routine problems should not be considered as a dichotomy. Instead, any mathematical task could have routine and non-routine aspects. Which aspects are highlighted and appropriated is dependent on the actions and interactions of those involved. If teachers are presenting the routine learning of number strategies in a transmission style they may be limiting the opportunities students in their classes have to engage with mathematics and each other (Franke, Kazemi, & Battey, 2007; Holmes & Tozer, 2004; Scouller, 2009).

The following section examines the second context in which teaching and learning mathematics in New Zealand is embedded and influenced. This context is the NZC *(*MoE, 2007a).

2.5 The New Zealand Curriculum

The NZC (MoE, 2007a) provides a distinct statement of the knowledge, competencies, and values deemed to be important for citizens in the 21st century. Students are viewed as "lifelong learners who are confident and creative, connected, and actively involved" (MoE, 2007a, p. 4). The previous curriculum implemented in 1992 was the first outcomes-based curriculum that set the expectation for student know-how, performance, and achievement. The revised 2007 NZC was in response to growing social change, population diversity, technological advancements, and vocational complexity in New Zealand (MoE, 2007a).

There are differences in the ways the learning area of mathematics have been represented in the 1992 and 2007 curriculum documents. The 1992 mathematics curriculum document articulated a constructivist approach to teaching and learning (Ell, 2001). The teaching and learning of mathematics was presented in an individual curriculum document comprising five strands: number, algebra, measure, geometry, and statistics (MoE, 1992). In 2007, the learning area was renamed Mathematics and Statistics. Mathematics and statistics are interrelated disciplines but they require different ways of thinking and problem solving. “Mathematics is the exploration and use of patterns and relationships in quantities, space, and time. Statistics is the exploration and use of patterns and relationships in data” (MoE, 2007a, p. 26). The five strands of the 1992 mathematics curriculum document were reduced to three in 2007: number and algebra, measurement and geometry, and statistics.

The Mathematics and Statistics learning area (MoE, 2007a) emphasises the need for students to be equipped with effective mathematical abilities, skills, and dispositions. Learning mathematics should prepare students: to investigate, discover, interpret, and clarify; to create, critique, strategise, and reason; to plan, organise, and act with flexibility and accuracy; to predict, conjecture, justify, verify, and generalise; to estimate and calculate; and to reflect. These abilities, skills, and dispositions are clearly important for any students’ mathematical progress and achievement. The NZC (MoE, 2007a) also stresses the need for students to know how to communicate their mathematical thinking through their models, representations, and explanations.

The NZC (MoE, 2007a) provides subject specific directions for teaching and learning through the nine learning areas and related achievement objectives. General directions for teaching and learning are provided through the vision, principles, values, and key competencies. The vision of the NZC is the desire for New Zealand’s young people to be “confident, connected, actively involved, lifelong learners” (p. 7). The principles of the NZC are: “high expectations; Treaty of Waitangi; cultural diversity; inclusion; learning to learn; community engagement; coherence; and future focus" (p. 7). Each principle provides the foundations for planning, prioritising, formalising, and reviewing the curriculum and places "students at the centre of teaching and learning" (MoE, 2007a, p. 9).

Values and key competencies provide the connections between the vision and principles. Values include: excellence; innovation, inquiry, and curiosity; diversity; equity; community and participation; ecological sustainability; integrity; and respect. The NZC (MoE, 2007a) recommends that values should be "encouraged, modelled, and explored" by teachers and students (p. 4). Students should be positioned to express, develop, and refine their values through their learning experiences and interactions with others on a daily basis.

The five interconnected key competencies are the capabilities students have for "living and learning" and include: "thinking, using language symbols and texts, managing self, relating to others, and participating and contributing" (MoE, 2007a, p. 12). The key competencies develop over time and "contribute to the realisation of a vision of young people who will be confident, connected, actively involved, lifelong learners" (MoE, 2007b, p. 37). Each key competency and its significance within the mathematics curriculum are explored below.

The key competency of thinking is described as the "creative, critical, and metacognitive processes" students use to "make sense of information, experiences, and ideas" (MoE, 2007a, p. 12). It is proposed that creativity, criticality, and metacognition assist students with perceptions, comprehension, decision making, determining next steps, and knowledge construction. Intellectual curiosity is seen to be at the heart of thinking. The mathematics and statistics learning area aims to develop students' abilities to think creatively, critically, strategically, logically, and flexibly with reasonableness (MoE, 2007a, p. 26). Students are expected to estimate structure, organise, predict, connect and carry out mathematical and statistical procedures with accuracy and confidence. When students work with and make meaning from the codes and representations through which knowledge is expressed and communicated, they are using language, and symbols and texts. Included in this key competency is an explicit link to mathematical language, and symbols and texts. Students are continuously working with language, symbols, and texts as they learn to conjecture, argue, and justify their mathematical and statistical thinking (MoE, 2007a).

Managing self, another key competency, emphasises students knowing “when and how to act independently” (MoE, 2007a, p. 12). The ability to establish goals, plan, manage, challenge, and self-assess are integral to students’ self-management and independence. Students need to be enterprising, resourceful, reliable, resilient, and persistent. They need to be able to interact with diverse groups of people in a variety of contexts. Students manage themselves in mathematics and statistics when they are self-aware of, and strategic, about their learning. Self-managing students know “when their results are precise and when they must be interpreted with uncertainty” (MoE, 2007a, p. 26). Relating to others requires an awareness of personal influence and influences, willingness to actively listen, respect and consider different points of view, negotiate, and share ideas. The fifth key competency, participating and contributing, is about students having an active involvement in their school, home, social, cultural, or physical environments. Students are expected to contribute to the group and make connections in ways that benefit themselves and others. Participating and contributing enhances students’ sense of belonging, their confidence to participate in new contexts, and their pride in their community. The mathematics and statistics learning area emphasises the need for students to be able to communicate their thoughts, strategies, and findings (MoE, 2007a).

2.6 Student Achievement

The success of the NDP has been qualitatively and quantitatively researched and evaluated since its inception (see Higgins & Parsons, 2009, 2011; Thomas & Tagg, 2004, 2005, 2006; Thomas, Tagg, & Ward, 2002, 2003; Thomas & Ward, 2001, 2002; Young-Loveridge, 2005, 2006, 2010). Evaluations have shown that teacher knowledge and practice and student outcomes are improving and that “teachers have a lot to be proud of” (Young-Loveridge, 2010, p. 28). Successes have been attributed to the soundness of the Number Framework, the strength of the Teaching Model, the ability of the numeracy facilitators, and the professional development programme (Bobis et al., 2005; Higgins & Parsons, 2009).

Despite large-scale numeracy initiatives across primary and secondary schools and systemic attempts to reform primary mathematics programmes (Higgins & Parsons, 2009), proposed acceptable levels of achievement for students to attain by Year 12 are not being met (Young-Loveridge, 2010). Findings from New Zealand's Year 5 (aged 10 and 11) students' participation in the 2010/2011 *Trends in International Mathematics and Science Study* (TIMSS) showed that the mean for New Zealand students was lower than the international mean; only four percent reached the Advanced International Benchmark, and students were slightly over-represented in the lower benchmarks (Chamberlain & Caygill, 2012). Specific results from TIMSS showed that whilst there was no significant difference between boys' and girls' achievement, Asian and Pākehā students tended to have higher achievement than Māori and Pasifika students. The tracked data from TIMSS show that since 2002/2003 the mean achievement of Year 5 students in New Zealand has declined (Chamberlain & Caygill, 2012). *The 2009 Program for International Student Assessment* (Lee, 2009) and the *2010 National Education Monitoring Programme* results (Crooks, Smith, & Flockton, 2010) revealed students had positive attitudes towards doing maths in schools. In contrast, the 2010/2011 TIMSS analysis noted that compared to their international contemporaries, New Zealand Year 5 students were indifferent toward mathematics, less confident, and less engaged.

New Zealand teachers and researchers recognise that a persistent issue of underachievement exists for some students (Holmes & Tait-McCutcheon, 2009; Neill, Fisher, & Dingle, 2010; Young-Loveridge, 2010). Underachievement is defined as those students whose achievement is below the national expectations by such a degree that their future learning in mathematics is perceived to be in jeopardy (MoE, 2012). Reasons for the lack of mathematical success for some students have varied but few have explained "why achievement comes to some learners through a hard and painful route" (Anthony & Walshaw, 2007, p. 9). This study explored the influence teachers' positioning of themselves and their students has on the opportunities students have to share their mathematical reasoning.

2.7 Summary

This chapter has provided the background to my study. It began by explaining the social constructivist theoretical framework that underpins teaching and learning mathematics in New Zealand. The importance of contextual and environmental aspects of students' individual and shared mathematical knowledge within this framework was highlighted. The teaching and learning analysed and reported on in this study is embedded within the NDP (MoE, 2007b) and the NZC (MoE, 2007a). The features of the NZC that correlate directly with the teaching and learning of mathematics were examined. The development and interconnectedness of the NDP teaching tools were described. Teaching resources are a core component of the NDP and the use of routine problems within the model lessons was explored. The hypothesis was made that the use of routine problems could prompt some teachers to present strategies as rules to follow.

This study examined the ways teachers position themselves and students in their lowest and highest strategy groups to share their mathematical thinking and to listen to the thinking of others. Therefore, it was important to consider, as part of the background to the study, the trends in mathematics achievement for New Zealand students. The claim was made that for some students achievement does not come as readily as it does for others. By analysing the positioning of teachers and students, the opportunities both have to share what they know, and what they can do with what they know, we may gain insights into why some student's mathematics achievement remains at risk.

Chapter Three examines the notion of student mathematical know-how. The three specific constructs of student mathematical know-how used in this study - independence, judgment, and creativity - are explored. Historical and more recent literature is used to describe each construct and to consider the affording and limiting influences teachers and environmental contexts have on students' developing know-how. The constructs of student mathematical know-how are also examined in regards to how teachers can use them as pedagogical tools to advance students' interest, commitment, and success in mathematics.

Chapter Three: Student Mathematical Know-how

> Our knowledge about any subject consists of information and of know-how. If you have genuine bona fide experience of mathematical work on any level, elementary or advanced, there will be no doubt in your mind that, in mathematics, know-how is much more important than mere possession of information.... What is know-how in mathematics? The ability to solve problems — not merely routine problems but problems requiring some degree of independence, judgement, originality, creativity. *(*Pólya, 1965, p. 191)

3.1 Introduction

At the core of any student's ability to solve problems in mathematics are the opportunities they have to use and explain what they know and listen to and understand what others know. Chapter Two examined the structure of teaching and learning mathematics in New Zealand as defined through the New Zealand NDP (MoE, 2007b) and the NZC (MoE, 2007a). The emphasis given by both documents to students having opportunities to share their mathematical thinking and to engage with others' explanations was explored. Attention was drawn to the concern that some students are not achieving at a nationally expected level which could place their future learning and achievement as tenuous (MoE, 2012).

This chapter reviews three constructs of mathematical know-how adapted from Pólya's (1963, 1965) seminal work: independence, judgement, and creativity. In 1965, Pólya contended that mathematical knowledge comprised both information and know-how. A connection is evident between Pólya's claim and the knowledge and strategy domains of the NDP Number Framework. Knowledge is similar to Pólya's definition of information in the quote above as it is what you know. Strategy is similar to know-how as it is what you can do with what you know. Pólya (1965) promoted mathematical thinking beyond what you know to knowing-how with "independence, judgement, creativity and originality" (p. 191).

Pólya originally suggested four constructs of mathematical know-how with both creativity and originality included. More recently, creativity and originality have been used interchangeably (Aljughaiman & Mowrer-Reynolds, 2005; Leikin, 2009) or one has been described as a characteristic of the other (Lev-Zamir & Leikin, 2011). For this reason, creativity and originality are combined as creativity in my study.

Section 3.2 introduces mathematical know-how as being much more than the possession of mathematical knowledge or strategies. Whilst knowledge and strategies are important, knowing which knowledge and strategies to use, when to select them, and how to apply them is key to the development of mathematical know-how. Student independence, judgement, and creativity are outlined in Sections 3.3, 3.4, and 3.5. For each construct, emphasis is given to literature that is both historical and contemporary in order to review its development significance to this study.

Student independence in this study is explored through students' self-regulated learning (SRL) strategies. Social and sociomathematical norms and students' self-efficacy are explored as part of SRL strategies. The advantages and limitations of students' effective and limited SRL strategy use are compared and possible reasons for differing SRL use are explored. The contexts influencing different groups of students are considered as well as the potential of positive and negative outcomes of those contextual influences. Teachers' potentially positive and adverse influences on students' SRL strategies are explained.

Section 3.4 examines judgement as a construct of students' mathematical know-how. Judgement starts with a reasoned guess, which is then tested and justified. Guessing is explored as a pedagogical tool for teachers and as a means of enhancing students' willingness to take risks and their commitment to their mathematics. Examples of how teachers could use guessing as a tool for teaching and learning are explored. The importance of teachers providing opportunities for students to guess, test, and justify with non-routine problems is highlighted. The effect of plausible guesses on students' mathematical commitment, risk-taking, and progress is outlined.

The third construct of students' mathematical know-how, creativity, is examined in Section 3.5. Creativity begins with curiosity and encompasses students' mathematical explanations and actions that are intuitive, unique, or novel. The use of non-routine problems to stimulate and foster creativity is explored. Teachers' conceptions of creativity are considered, as are the influences their conceptions have on students' opportunities for mathematical creativity. The relationship between student independence, judgement, and creativity is described in Section 3.6. Included in this section is a précis of teacher influences on students' opportunities to develop and share individually and socially constructed mathematical know-how.

3.2 Mathematical Know-how

Pólya (1965) coined the term "mathematical know-how" for knowing about mathematics that required some degree of "independence, judgement, originality, and creativity" (p. 191). This is the definition used in this study. Pólya was a respected mathematician and teacher, particularly well regarded for his work in teaching mathematics problem solving to undergraduate students at Stanford University, California (Leinhardt & Schwarz, 1997; Truxaw & DeFranco, 2007). According to Pólya (1965), students needed to be taught to think and teachers needed to be taught how to teach students to think. Students needed to understand what any mathematical problem was asking of them so that they would also know how to confidently and competently approach and engage with the problem. Within any mathematical experience information is what we know and know-how is what we can do with what we know. However, mathematical know-how is more than knowing how or knowing what to do. As Schoenfeld (1992) advised, "it's not just what you know; it's how, when, and whether you use it" (p. 60).

Mathematical know-how could be seen as the tool-kit that contains the practices used by proficient mathematics learners (Anderson, 2003, Hunter, 2007a), that is, the what, which, why, when, and how of applying mathematical knowledge (Darr & Fisher, 2005; Mason & Spence, 1999; National Council of Teachers of Mathematics, 2000; Pape & Smith, 2002; Willoughby, 2000). For example, students with mathematical know-how know *which* strategy to use and *why* it would be the most efficient. They know *when* to persevere with problem solving and *when* to start again. They know *how* to attend to errors or misconceptions and *how* to ask for, and provide, assistance. They know *how* to think and *how* to monitor thinking. Leinhardt and Schwarz (1997) contended that students who know how, when, and why are thoughtfully doing mathematics, not prescriptively solving problems. Proficiency with the what, which, why, when, and how of mathematics empowers students to “cultivate an awareness of themselves as legitimate creators of mathematical knowledge” (Anthony & Walshaw, 2007, p. 61). For students to enhance their mathematical know-how they must be positioned to experience and engage with mathematics with independence, judgement, and creativity (Pólya, 1965; Schoenfeld, 1992).

3.3 Independence

Independence, as a construct of mathematical know-how, can be thought of as students’ capacities to think, act, reflect, and make decisions that maximise their opportunities for learning (Pólya, 1963). In this study independence does not mean to be removed or disconnected from others. Instead, independence is viewed through a social constructivist lens whereby relationships and interactions with others are important. The NDP described effective teachers as those who encourage students to regulate their learning by providing motivation, acknowledgment, and support (MoE, 2007d). Students could decide who to work with, how much time to give to a task, which strategy or materials to use, what questions to ask and of whom, whether to persevere or start again, and when a solution has been reached.

The decisions students make about mathematics are influenced by the social and sociomathematical norms of the classroom. Social norms are the general ways students participate in classroom activities. Examples include explaining and justifying thinking, listening to, making sense of, then appropriating or challenging others’ thinking, questioning, and persevering (Cobb, Yackel, & Wood, 1989; Yackel & Cobb, 1996). Social norms in teacher-student interactions are important as they can promote and foster student independence (Kazemi & Stipek, 2001; Lockhurst, Wubbels, & Van Oers, 2010; Weber, Radu, Mueller, Powell, & Maher, 2010). Co-constructed social norms can increase students’ willingness to solve challenging problems, present justifications, question each other, and contest each other’s thinking. Social norms can increase opportunities for students to advance their mathematical understanding and provide the foundation on which sociomathematical norms could be built (Yackel & Cobb, 1996).

Sociomathematical norms are the “normative aspects of mathematical discussions that are specific to students’ mathematical activity” (Yackel & Cobb, 1996, p. 458). They differ to social norms by having a mathematical foundation and grounding in students’ mathematical activity. The interactively constructed, negotiated, and agreed to understandings of what signifies a mathematically acceptable, different, sophisticated, or efficient explanation are sociomathematical norms (Bowers, Cobb, & McClain, 1999; Yackel & Cobb, 1996). The differences between social and sociomathematical norms are highlighted in the following examples from Kazemi and Stipek (2001). In the context of mathematics, encouraging students to share different strategies is a social norm; exploring the efficiency of the shared strategies is a sociomathematical norm. Expecting students to justify their answer is a social norm; expecting students to interactively constitute and socially construct what constitutes mathematical justification is a sociomathematical norm. Students working collaboratively to solve problems is a social norm; students reaching consensus through mathematical argumentation (explanation and justification) is a sociomathematical norm.

Independent decision-making requires students to capitalise on their learning opportunities as “masters of their own learning processes” (Zimmerman, 1989, p. 1). Each decision made by students is grounded in their desire to enhance their learning opportunities and requires them to actively and constructively direct their own efforts to acquire mathematical know-how (Darr & Fisher, 2005). With independence, students become increasingly self-reliant and less reliant on teachers and textbooks. Students’ self-reliance requires a level of self-regulation whereby they can analyse tasks, set goals, monitor progress, reflect on development, modify actions and goals as required, and seek out information or assistance when needed. Self-regulating learning strategies are an important aspect of independence and are explained in the next section.

3.3.1 Self-regulating learning strategies

Students who act and interact with and from a stance of independence are identified in the literature as self-regulating (Pape & Smith, 2002; Zimmerman, 2002; Zimmerman & Labuhn, 2012; Zimmerman & Martinez-Pons, 1990). Self-regulating students make decisions that optimise their learning opportunities and accept cognitive, metacognitive, volitional, and emotional control of their learning (Zimmerman, 1994, 2000). Cognition requires thinking and metacognition requires thinking about thinking, volition entails monitoring and controlling thinking, and emotion involves efficacious behaviours such as initiative, motivation, and perseverance. A strong mathematics self-efficacy was found to correlate with students who were effective self-regulators (Darr & Fisher, 2005; Pape & Smith, 2002; Zimmerman, 2002; Zimmerman & Labuhn, 2012; Zimmerman & Martinez-Pons, 1990). De Corte, Vershaffel, and Op' T Eynde (2000) contended that self-regulation was "a major objective of mathematics education …. and …. a crucial characteristic of effective mathematics learning" (p. 721). The collective nature of SRL acknowledges that both individual and social forms of learning, such as seeking help and collaborating with others, are important (Zimmerman & Labuhn, 2012).

Research and theory regarding SRL have grown substantially over the last 30 years. Emphasis has been given to the cognitive and environmental development of students (for example Boekaerts, 2002; Pintrich, 1999) and interactions between students and teachers in social environments (for example Schunk, 2001; Zimmerman, 1989). The material that follows focuses on the interactional aspects of SRL within a context of teaching and learning mathematics. Interactional empirical research has highlighted the positive effect SRL has on mathematical achievement for some students (Darr & Fisher, 2005; Pape, Bell, & Yetkin, 2003; Pape & Wang, 2003; Schoenfeld, 1985, 1987, 1988, 1992, 1994; Zimmerman & Martinez-Pons, 1986, 1988, 1990). This same body of literature has also identified that for others, SRL strategies are not as accessible, readily understood, applied, or valued.

Self-regulating students work through interrelated phases of thought, action, and reflection when they commit to solving a mathematics problem, respond to their progress, and anticipate revisions or solutions. Zimmerman (2000, 2002) categorised these phases as forethought, performance, and self-reflection. In the forethought phase students analyse what the mathematical problem is asking of them, what goals are set, and what strategic plans are determined before the problem solving begins. The performance phase occurs during the learning when students observe and monitor the progress of their plans and either persevere or make changes depending on what their observations reveal. At this stage students could analyse errors or misconceptions, review their plan, or determine whether they are on the right track. At the third phase, self-reflection, a solution has been reached and students reflect on the mathematics processes they used. They could decide the best process was employed, or start again with a different plan. If the answer is accepted as correct, learning and SRL strategies acquired during the three phases can now be transferred to new or different problems. Pape and Smith (2002) described the process of working though each SRL phase as “learn[ing] how to learn mathematics” (p. 97).

Students' use of SRL strategies is influenced by the strategies they know, the strategies they are developing, their metacognitive decision-making processes, and the effect these have on achievement outcomes (Zimmerman, 1989). As such, not all students are going to have the same knowledge of, or access to, SRL strategies. The following section reviews the characteristics of students who are less effective self-regulators.

3.3.2 Reactive or novice learners

Students who lack independence and SRL skills are described as reactive or novice (Darr & Fisher, 2005; Zimmerman & Labuhn, 2012). Schoenfeld (1992) defined such students as having a ‘hit or miss’ approach to their learning. Non-SRL students lack the ability to transfer knowledge to new learning, struggle to recognise the usefulness of their know-how, are hesitant to monitor the success of their mathematical decisions, and may experience difficulty retaining learning over time (Darr & Fisher, 2005). Unlearning inappropriate SRL strategies, or the inconsistent application of strategies, could prove challenging for many students.

When considering the SRL strategies of 16-18 year old students, Schoenfeld (1992) hypothesised they would be able to keep “tabs on how well things are going” (p. 58) by monitoring their SRL strategy use and reflecting on the effectiveness of their strategies. Evidence from over 100 videoed lessons and over 200 interviews of high school and college mathematics students across the United States proved otherwise (Schoenfeld, 1983, 1985, 1988). Analysis of the classroom observations and interviews showed that students were not monitoring or regulating their understandings; instead, they quickly selected a solution strategy and stuck with it even when they were not making progress. Schoenfeld (1992) described this approach as “read, make a decision quickly, and pursue that direction come hell or high water” (p. 61). Students were not reconsidering their strategies so if their first choice was incorrect, failure was virtually guaranteed. Any perceived or actual failure may have been due to a lack of effective self-regulation rather than inadequate mathematical knowledge.

Underpinning these students’ hesitancy to use SRL strategies was their belief that mathematics problems should be able to be solved in less than 10 minutes, using one stipulated strategy (Schoenfeld, 1985, 1988). Problems that could not be solved this way, according to the students, were either unsolvable or required the work of a genius. Schoenfeld’s (1985, 1988) research provided an insight into the beliefs and self-efficacies held by students that could negatively influence their decisions to use SRL strategies. It is important to consider contextual influences beyond students’ beliefs, such as their levels of achievement, age, and gender. In the following section the work of Zimmerman and Martinez-Pons (1986, 1988, 1990), and Pape and Wang (2003), highlights the different experiences cohorts of students have had with knowing about the strategies of SRL and knowing how to effectively use them.

3.3.3 Contextual influences on self-regulating behaviours

Zimmerman and Martinez-Pons (1986, 1988, 1990) used a social cognitive framework to explore the behaviours students exhibited when self-regulating their learning. The researchers’ emphasis in all three studies was understanding the influences on elementary and secondary school students’ social development of SRL strategies. Over 200 students from New York, with high and low mathematical ability, from gifted and regular schools, were selected to enable comparisons of the different SRL strategies students employed and the effect the use or non-use of the strategies had on their academic achievement. In each research project students were interviewed in clinical settings, rather than their classroom, and this could be seen as a limitation to their findings. The findings from the three research projects published in 1986, 1988, and 1990 are examined in the following sections.

In their first study, Zimmerman and Martinez-Pons (1986) interviewed 80, 10th grade (aged 16) students from high and low achievement groupings and asked them to reflect on and explain the SRL strategies they used. Students were asked to consider their SRL strategy use in different learning settings such as classrooms, homes, and libraries and during class, homework, and study times. From the students' responses a list of SRL categories was compiled. These categories were: "self-evaluating, organising and transforming; goal-setting and planning; seeking information; keeping records and monitoring; environmental structuring; self-consequences; rehearsing and memorising; seeking social assistance (from teachers, peers, or others); and reviewing texts, notes, or textbooks" (Zimmerman & Martinez-Pons, 1986, p. 618). The category of "other" was included to provide the opportunity for non-SRL strategies to be noted.

Zimmerman and Martinez-Pons (1986) identified that students regarded by their teacher as having higher abilities in mathematics used different SRL strategies more effectively than their less able counterparts. More able students made greater, and more effective, use of most SRL strategies and they were more willing and competent at asking for help from others to support their learning. In results similar to those of Schoenfeld (1992), less able students tended to rely on one or two SRL strategies and used them even in inappropriate situations. Less able students also reported using substantially more non-SRL strategies such as asking someone else what they should do, applying an inappropriate strategy, or using the same strategy every time (Zimmerman & Martinez-Pons, 1986).

In their second study, Zimmerman and Martinez-Pons (1988) interviewed 80 high school students and asked them to describe their use of the SRL strategies identified in their 1986 research. For this study, teacher ratings of students' SRL strategy use during class time and students' achievement results were included to triangulate their data sources. Analysis of the three data sets, students' SRL strategy observations, their achievement outcomes, and teacher ratings showed two correlations. The first was between students' use of SRL strategies and teachers' ratings of their prediction of students' test results. The second correlation was between students' use of SRL strategies and their achievement outcomes. The researchers noted that whilst these correlations enhanced their understandings of students' SRL strategy use, other measures, such as observations of students in classroom settings and evaluations from peers or parents, could broaden their interpretations.

Results from Zimmerman and Martinez-Pons' 1986 and 1988 research were repeated and extended in 1990. Participants in the 1990 research included 90 5th, 8th, and 11th grade students (aged 11, 14, & 17) from a school for the academically gifted and from a regular school. Students were asked to describe their use of the SRL strategies and to estimate their verbal and mathematical efficacy. Again, more able students used SRL strategies more confidently and competently and the application of SRL strategies tended to be indicative of student achievement outcomes. This latest research found differences in gender and age. Girls were more likely to use SRL strategies than boys, but boys were more likely to report they used them. As students aged, they tended to seek help more from teachers and peers and less from parents. This may have been due to the increased difficulty of the mathematics or indicative of a time where students were generally becoming less dependent on their families (Zimmerman & Martinez-Pons, 1990).

In each research project Zimmerman and Martinez-Pons (1986, 1988, 1990) acknowledged that students' use of self-regulating strategies was subject to change because of influences such as personal contexts and classroom norms. Diverse contexts and norms meant that not all more able, less able, older, or younger students described or used SRL strategies in the same way or to the same degree. However, their findings did indicate that SRL strategy use was "highly predictive of students' performance in class" (Zimmerman & Martinez-Pons, 1988, p. 336).

Zimmerman and Martinez-Pons (1986, 1988, 1990) identified the different SRL strategies cohorts of students used and the effect their use had on academic achievement. Missing from their research was an awareness of the different opportunities cohorts may have had to use and develop their SRL strategies. It is possible that teachers in Zimmerman and Martinez-Pons' three research projects may have (intentionally or unintentionally) positioned the more able or older students as being more capable, and the less able or younger students as not being capable of effectively using SRL strategies to advance their learning. Teachers of the less able or younger students may have believed they needed more teacher time or direction and less independence. Such positioning could influence how often and which SRL strategies students used and adversely affected students' self-efficacy regarding their ability to self-regulate.

An iterative and positive feedback loop was identified by Zimmerman and Martinez-Pons (1990) for some students. Those with a strong sense of self-efficacy who effectively self-regulated their learning made greater gains in academic achievement. Increased academic achievement enabled students to use additional SRL strategies more effectively. Effective SRL strategy use confirmed and enhanced students' confidence in themselves to succeed and their willingness to trial different strategies. Students who knew how and when to self-regulate were better positioned for independence and future success.

The presence of a negative feedback loop was not investigated by Zimmerman and Martinez-Pons (1986, 1988, 1990) but could be inferred as a possible finding of their research. Students not succeeding mathematically could be in a negative feedback loop whereby they attribute their lack of achievement to their own efforts. A lack of success could limit students' willingness to try new SRL strategies. The outcome could be a group of students becoming increasingly reticent about attempting to use SRL strategies for their future achievement. The potential of a negative feedback loop, and its likely effects, was explored by Pape and Wang (2003) and is discussed next.

Pape and Wang (2003) used Zimmerman and Martinez-Pons' (1986) categories of SRL strategies in their classroom-based research. Throughout one year, 80 6th and 7th grade students (aged 12 & 13) from low and high achievement groups were videoed as they completed mathematics tasks and recorded and articulated the mathematical and SRL strategies they used. Pape and Wang (2003) aimed to identify and understand which of Zimmerman and Martinez-Pons' (1986) SRL strategies students reported using during mathematics problem solving tasks. The frequency with which each SRL strategy was used and how the students felt about the usefulness of each strategy was examined. Students were encouraged to think aloud and share all their thoughts including the SRL strategies they used when experiencing difficulty (Pape & Wang, 2003).

Analysis of students' responses showed that those who were more and less able used similar SRL strategies (Pape & Wang, 2003). The frequency of the strategies students in high and low achievement groups selected was also similar and students showed corresponding levels of confidence in the strategies they selected. The following SRL strategies were reported by 90% of students as those used most frequently: seeking information, seeking social assistance (from teachers and other adults), goal setting and planning, and organising and transforming. Fewer than 50% of the students noted their use of seeking social assistance (from peers), self-evaluation, rehearsing and memorising, keeping records and monitoring, environmental structuring, and self-consequences.

The difference highlighted by Pape and Wang (2003) was that students in the more able cohort trialled and applied more SRL strategies than those in the less able cohort. The additional strategies used by the more able students included self-evaluation, organising and transforming, and goal setting and monitoring. These students were able to discern the appropriateness of possible SRL strategies and decide on the relevant ones to apply. The less able cohort tended to report using the same strategies. They sought social assistance because they did not know what to do next rather than as a strategy to self-regulate their problem solving. It appeared the less able students were overly reliant on one or two strategies and had a different understanding of how SRL strategies could be applied to benefit their learning.

Pape and Wang (2003) acknowledged that not all mathematical scenarios may have elicited the same SRL strategies to the same degree. However, they highlighted the variance in SRL strategy use by different groups of students and found evidence of a negative feedback loop for some students. Similar to Zimmerman and Martinez-Pons' (1986, 1988, 1990) research, Pape and Wang did not explore how the teachers in their research taught SRL strategies or provided opportunities for students to use their strategies. Students may have been applying SRL strategies inappropriately because that was how it was taught. Teachers may have given students a chance to solve a problem but may have offered assistance too quickly or directed the student to seek help from another. Pape and Wang (2003) concluded that knowing about the existence of a SRL strategy was not enough. Students needed the know-how to use and review the SRL strategies in ways that advanced their learning. Teachers needed to teach students SRL strategies and when and how to apply them.

The research of Zimmerman and Martinez-Pons (1986, 1988, 1990) and Pape and Wang (2003) highlighted the possible contexts influencing students' use of SRL strategies and students' positioning as independent learners. An emphasis on the teacher's positioning as an influence on students' SRL use was absent from their research but one that was explored in my study. Pape et al. (2003) and Darr and Fisher (2005) did seek to understand the positions teachers take when supporting their mathematics students to use and develop SRL strategies, and this is the focus of the following section.

3.3.4 Teacher influences on SRL behaviours

Pape and colleagues (2003) drew on the work of Zimmerman and Martinez-Pons' (1986, 1988, 1990) in an attempt to "account for the diversity of learners' experiences, motivations, and dispositions, and their relationship to mathematics learning" (p. 183). Throughout a year-long teaching experiment, Pape et al. (2003) worked with a teacher of mathematics and her 29, 7th grade (aged 13) students in mid-western United States. Data included classroom observations, interviews, and students' work samples.

Using a socio-cultural lens, Pape et al. (2003) inquired into the explicit acts of SRL strategy teaching. They stressed the need for explicit acts of teaching because "although explicit instruction is not contradictory to sociocultural theories of teaching and learning, often more implicit instruction predominates" (Pape et al., 2003, p. 180). The teacher was observed providing opportunities for students to think about and articulate their mathematics learning and observations of their learning. Examples included the teacher positioning students to analyse mathematical situations, explore multiple representations, critically examine, explain, and justify their own and others' mathematical thinking, and extend their use of mathematical talk. Whilst the teacher's positioning of students was seen as beneficial, the researchers hypothesised that implicit teaching of SRL strategies would not be enough for some students and advocated explicit teaching and learning opportunities for SRL strategy use, review, and development.

To test their hypothesis, Pape et al. (2003) designed and implemented a Strategy Observation Tool (SOT). Students were asked to record daily observations of their learning and the SRL strategies they used as they completed mathematics tasks. The aim of the SOT was twofold. First, it assisted students to develop a sense of control of their own learning and second, it provided researchers with the means to evaluate the control students had or were given. Students' recordings and their discussions and reflections about the tool and its uses provided the sources of data for examining the effectiveness of the SOT.

On the first SOT version students noted the strategies they utilised to learn mathematics both at school and home, what modifications they could make to those strategies, how they prepared for assessments, how well they achieved, and how they felt about mathematics. Initial analysis indicated few students were recording their mathematical or SRL strategies, suggesting that students did not recognise what they did to assist or regulate their learning. The SOT did not elicit comments from students that Pape et al. (2003) felt would provide helpful data. The first SOT was modified by adding the categories of SRL strategies from Zimmerman and Martinez-Pons' (1986) work to prompt students to contemplate and articulate their SRL strategy use. By providing categories for reflection and discussion it was hoped students would start to see the relationship between their behaviours whilst learning mathematics and the effect those behaviours had on their achievement.

Analysis of students' use of the second SOT revealed three results — students whose learning was shown to benefit from the tool, those who thought they benefitted, and those who described the tool as bothersome and unnecessary. The SOT was reported by one group of students as assisting their organisation, help-seeking, study modifications, and motivation. These students were applying SRL strategies to scaffold their mathematics learning (Pape et al., 2003). The objective of scaffolding was to fine-tune the task difficulty to match a student's level of performance and eventually remove all support systems when they were ready to think on their own (Wood, Bruner, & Ross, 1976). However, when self-reports were compared with achievement outcomes it was revealed that some students were unable to monitor and control their thinking to sustain the SRL strategies to a point where they made a difference to their achievement outcomes. Students were going through the motions of applying SRL strategies but not thinking about or engaging with them. As the application of strategies was almost rule based, the effect on achievement for these students was minimal.

According to Pape et al. (2003), students who did not value the tool were either struggling or highly proficient. Struggling students recorded their observations and cognitive thinking; they did not act on their observations or reflect on their thinking. They required additional support to know how to engage with and use observations of their learning and how to operate within a metacognitive or self-reflective phase.

Students categorised as proficient on the other hand did not use their observations because they knew what to do next and did not need to plan. They approached the SOT as a set task, not as a potential learning tool. Pape and colleagues (2003) concluded that teachers needed to support students differently to become self-regulating learners. Students need to understand what they know; they also need to appreciate what they can do with what they know, and it is the teacher's responsibility to position students as having and being able to acquire mathematical understanding and awareness.

In a New Zealand study, Darr and Fisher (2005) also examined the acts of teaching that provided students with opportunities to explore and reflect on their SRL strategy use as they shared their mathematical understandings. Within a sociocultural framework, Darr and Fisher conducted a four-week teaching experiment with a classroom teacher and her Year 7 students (aged 11 and 12). Data included videoing of whole class, group, pair, and individual teaching and learning, as well as field notes, the teacher's planning, and student interviews, pre-tests, work samples, and the teacher's reflective journal entries. In findings similar to those of Schoenfeld (1985, 1988, 1992), students in Darr and Fishers' study (2005) who lacked self-regulatory skills tended to have one plan, and if that proved unsuccessful they reverted to inappropriate calculations and implausible guesses. They found that the majority of the students would regulate their learning only when prompted by the teacher. The regulation of learning was more other-directed than self-directed. According to Darr and Fisher (2005), teachers needed to provide learning experiences through which students could “make their thinking visible to themselves and others” (p. 47). Both Darr and Fisher (2005) and Pape and colleagues (2003) concluded that teachers and students should be co-constructing classroom practices that foster self-regulation strategies and mathematical independence.

Classroom practices that assist students’ mathematical know-how to become more visible could include teachers or more knowledgeable peers modelling SRL strategies, providing models for discussion and reflection, making conceptual connections, providing time for ideas and conjectures to be explored, and pressing for explanation, meaning, and understanding (Fraivillig et al., 1999; Pape et al., 2003). Students could be expected to explain the problem and their plan, they could be asked to check and correct any errors, or to review their strategies for efficiency. Through each practice, responsibility for and regulation of the learning should be systematically and increasingly transferred to the student (Zimmerman, 2000). As students’ regulated learning increased, so too could their self-efficacy and awareness that their mathematical successes and failures were attributable to their actions (Pape & Smith, 2002). Students would be expected to continue to develop their ability to regulate their own learning and take increasing responsibility for making mathematical meaning (Anthony & Hunter, 2005; Pape et al., 2003).

3.3.5 Summary of independence

This section of the literature review has shown that students with mathematical independence are able to capitalise on their own and others’ mathematical knowledge and strategies in ways that optimise and maximise their opportunities for learning. Students with mathematical independence expect to become increasingly self-reliant. Mathematical independence in this study refers to the SRL strategies students use.

The access students have to SRL strategies and norms and the opportunities they have to further develop both are influenced by contexts such as ability, age, gender, and task content and by teachers’ positioning of them according to those contexts. Some teachers tended to position more able and older students as having and being more able to apply SRL strategies (Zimmerman & Martinez

Pons, 1990). The position of the teacher was found to be instrumental in determining the accessibility, use, and value of SRL strategies as aspects of students' independence. The literature recommended explicit teaching of differentiated SRL strategies (Pape & Smith, 2002; Zimmerman, 2000).

3.4 Judgement

Judgement as an aspect of students' mathematical know-how has been characterised as the ability to distinguish between "facts and fancy, facts and impressions, facts and suspicion, facts and theory, facts and guesses, proofs and guesses … and guesses and guesses" (Pólya, 1979, p. 256). Perhaps the most important of these is the ability to know the difference between wild and plausible guesses (Pólya, 1958). Wild guesses tend to lack forethought and mathematical reasoning (Leinhardt & Schwarz, 1997). They are often posed as a question, given with a shoulder shrug, not based on sound mathematical thinking, and not readily justified with a mathematical explanation. A wild guess that is correct is more likely to be the result of good luck than good thinking. A plausible guess is more thought out and mathematically sound and can be explained with mathematical reasoning (Wong, Marton, Wong, & Lamb, 2002). It is important that students develop the right attitude for determining what makes a guess plausible and sound (Pólya, 1963, 1979).

The New Zealand MoE (2013) noted that the difference between a wild and plausible guess was justification. Justification "is what sets mathematics apart from every other discipline" (MoE, 2013) and a guess can only become plausible once it has been justified. Plausible guesses are based on intuition and confirmed through an investigation that tests the usefulness or correctness of the guess. Students might guess what the mathematical problem was asking of them, where to start their solution strategy, what strategies were available and what strategy to use, what next step to take, or what the answer might be. In each situation students should be expected to conjecture, explain and justify the thinking behind their guess, and to monitor, regulate, and generalise. Ultimately all guesses should progress students' mathematical thinking toward solution strategies (Johanning, 2007; Lampert, 1990).

Teachers can assist students to justify their guesses by asking questions such as how can we test our guess? How will we know our guess is helpful or correct? What evidence do we have to support our guess? What adaptations are needed to our guess? Is the guess generalisable? Would models or diagrams justify our guess? And is it transferrable to other problems? (Pólya, 1978, 1979). These questions also support students to improve the plausibility of their guesses and by defending them students become better acquainted with the notion of mathematical argumentation and justification (Maher & Martino, 1996)..

Pólya (1958, 1963) proposed guessing as a pedagogical tool for advertising and marketing mathematics to students. When students expressed their mathematical opinion by guessing, they were drawn into the mathematical conversation and made a commitment to finding the solution. Students' responsibility for, and loyalty to, the mathematics increases because they give something of themselves to solving the problem (Pólya, 1958). The guess provides an opening for further discussion and the student who provided the guess had a vested interest in finding out whether or not their guess was correct (Pólya, 1963).

The contribution Pólya has made to understanding students' judgements and justifications has been highlighted in the previous paragraphs. As well as making a personal contribution through his research and publications, Pólya's mathematics teaching has been analysed by others to increase this body of knowledge (Leinhardt & Schwarz, 1997; Truxaw & DeFranco, 2007). This literature is explored in the following section.

3.4.1 Making mathematical discoveries

Leinhardt and Schwarz (1997) analysed Pólya's (1966a) Five Planes Problem lesson plan, video, and transcript. Analysis included a model of instructional explanation used to unpack and schematise the lesson and examination of student voice within the lesson. The analytical process was described as using "an early 20th-century master of mathematical pedagogy in light of current (late 20th century) ideas, especially constructivist ones, of mathematics teaching and learning" (Leinhardt & Schwarz, 1997, p. 397). The five planes problem given to a class of Stanford University undergraduates was: *Into how many parts will 5* *random planes divide space?* The students were purposefully selected because they were not mathematics majors and had not previously been taught by Pólya. The lesson provided a vehicle for Pólya to demonstrate his theories about teaching and learning mathematics, and in particular, guessing (Leinhardt & Schwarz, 1997). Of specific interest to the researchers was the constructivist pedagogies Pólya employed to teach guessing as a strategy for problem solving.

Leinhardt and Schwarz (1997) observed that Pólya illustrated and explained guessing by assisting students to simplify the problem, ground it in their own experiences, visualise and represent the problem in different ways and through different media (talk and written recordings), look for patterns, draw on their intuition, generalise, and argue. Particularly prominent in Pólya's teaching practice was his use of students' arguments as the catalyst for guesses to be checked and justified. The pedagogical decisions Pólya made and the actions he took are examples of how he drew students into the problem solving by making the mathematics about their experiences, existing knowledge, and insights. Leinhardt and Schwarz (1997) noted that as a result of Pólya's practices students showed a high commitment to their learning by carefully keeping track of the problem solving steps and actively constructing good guesses. Students were able to improve their skills of plausible guessing and solve the problem.

Truxaw and DeFranco (2007) extended the work of Leinhardt and Schwarz (1997) to examine Pólya's pedagogical approaches to "mathematics in the making which consists of guesses" (p. 96). The mathematical talk within Pólya's lesson formed the basis of analysis for Truxaw and DeFranco's (2007) research. Throughout the lesson Pólya talked students through recursive cycles rather than linear steps of evidence-based guesses, investigations, and explanations. The result of these cycles was shared mathematical meaning about the problem and about the guess.

By participating in the dialogue of guessing, investigating, and explaining, students were able to confront misconceptions, revise conjectures, generalise, and generate mathematical know-how. Leinhardt and Schwarz (1997) and Truxaw and DeFranco (2007) concluded that Pólya went beyond merely imparting information. Instead, he positioned students to make their own mathematical discoveries. Pólya himself was not inactive within the learning but the responsibility he gave himself was to position the students to do the mathematics. To enhance students' judgements Pólya highlighted students' guessing actions and made clear the different tools they had available to them for guessing (Leinhardt & Schwarz, 1997).

Pólya took both an ethical and metacognitive approach to student judgement and guessing. Ethically, guessing was seen as both a means for increasing students' commitment and duty to their mathematics. Metacognitively it provided the means for students to reflect on their learning. The ways in which different researchers have interpreted the ethical and metacognitive aspects of students' guesses are explored next.

3.4.2 Conscious guessing with courage and modesty

Conscious guessing was said to come "from the best human qualities: courage and modesty" (Lakatos, 1976, p. 30). When engaged in conscious guessing, students zig-zag between guesses, explorations, counter-examples, negations, and justifications. The processes of coming to know represent the students' mathematical understandings not the justification. Finding a justification should not signify an end to the mathematical thinking; the justification should remain open to further examination, revision, and development (Lakatos, 1976). Conscious guessing could be interpreted as a form of risk-taking and risk-taking necessitates courage and modesty. Sharing your guess could make you vulnerable to disagreement, criticism, or indifference. According to Lakatos (1976), listening to others involves modesty to ensure ideas are treated with respect and given due consideration.

Lakatos' (1976) ideas about conscious guesses, courage, and modesty were put into practice by Lampert (1990). Lampert proposed a new way of knowing as "beginning with a guess and exploring it with courage and modesty" (p. 53). As the teacher of an American 5th grade (aged 11) mathematics class, Lampert undertook an action research project to understand how she could create and maintain a culture in which students experienced mathematics as a discipline. She presented mathematics problems that were messy and raw, ones that required students to guess, argue, justify, challenge, and defend. The classroom social and sociomathematical norms regarding what it means to know and do mathematics were challenged and students were expected to explore the thinking behind their guesses as well as their justifications. Knowing mathematics in the classroom became more like knowing mathematics as a discipline (Boerst, Sleep, Ball, & Bass, 2011), that is, the mathematics became the focus and authority in the lesson more than the teacher or textbook.

Lampert (1990) recommended that classroom discussions should include words such as ‘maybe’ and ‘perhaps’ and students should be able to try out their ideas without having to commit to a final answer. As different ideas are trialled, students have the opportunity to test, then justify, their own and others’ know-how. If students are to experience a new way of knowing mathematics, teachers and students need to be collaboratively engaged in different activities and accept different positions and responsibilities (Lampert, 1990).

3.4.3 Guess and check

Maher and Martino (1996) defined guess and check as a metacognitive process for encouraging students to assess the development and progress of their mathematical reasoning and understanding. Guess and check is metacognitive because students need to know how to monitor and regulate their guesses and justifications and to reflect on these (Leinhardt & Schwarz, 1997). Strategies for guessing then checking include solving a simpler problem, working backwards, and looking for patterns. Guess and check is often illustrated with a problem involving animals and legs. For example: *Jamie went to her grandfather's farm. Her grandfather has pigs and chickens on his farm. She noticed that there was a total of 26 heads and 68 feet among them. How many chickens and how many pigs did her grandfather have?*

The mathematical thinking and progress of one student from New Jersey in the United States were analysed over five years from age six to 11. Maher and Martino (1996) focussed on Stephanie’s (the student’s) judgements using data from classroom activities and discussions, interviews, and assessments. As time progressed, Stephanie’s awareness of the complexity of judgements increased and she was able to articulate mathematical argumentations and justifications, explained and defended through her mathematical understandings. This was more than simply proving her answer was correct; she was able to defend the thinking behind the justification as well. Stephanie developed her guess and check method into what Maher and Martino (1996) called “proof by cases” (p. 194) whereby she systematically invented a justification and then set about proving it.

Maher and Martino concluded that one condition that needed to be in place before students could engage in mathematical argumentation and justification was the opportunity to guess. Without guess work students were positioned to apply what they knew rather than trial what they knew. The latter, according to Maher and Martino, enabled students to develop mathematical reasoning and judgement. A limitation of Maher and Martino’s (1996) research is that their findings may not be generalisable to other students. Their work also assumed all students have the same opportunities as Stephanie to apply and enhance their judgement.

Guess and check has been labelled by some as an unproductive approach. Mason (1998) found that students tended to apply what he termed “local tactics” by attempting to find a rule that fits rather than understanding what the problem was asking of them. Students tended to apply any strategy until they found one the numbers in the problem worked with (Healy & Hoyles, 1998). Lannin (2005) agreed that students tended to guess strategies but did not consider if they were the most appropriate to apply and why. In these examples the rule was validated through empirical results, not through the efficiency or effectiveness of the strategy.

To overcome the random use of guessing and checking, Johanning (2007) proposed “systematic guess and check” (p. 123). Systematic guess and check highlighted the connections between previous guesses and next steps and required students to use the information gained from initial guesses to move toward a solution. Participants included 31, 6th to 8th grade students (aged 12 to 14) from three schools in Ohio. Data included field notes, students’ written and narrative solutions, and comments and questions, and interviews with students whose solutions were identified as unclear or interesting. Johanning (2007) noticed that when students engaged in systematic guess and check they focussed on the situational context of the problem, identified relationships between their guesses and their next steps, generalised from their guess to their next step, and applied relational reasoning to their guesses and checks. The guess itself became less significant and the thinking behind the guess was emphasised. Johanning contended that when students had to “articulate their guess and check thinking and share it with others” (p. 132) they had to first make sense of their thinking for themselves.

In another study, Guerrero (2010) proposed that guessing and checking had to engage students in “the analysis of quantitative relationships” and “emphasise sense-making over merely applying rote computational strategies” (p. 393). Without such an approach students become quickly frustrated with their lack of success from wild guesses. In her experientially-based research, Guerrero (2010) proposed that guessing and checking was as much about developing mathematical reasoning, logic, and representation as it was about determining the correct answer. The check part of guess and check was essential and when checking students should be asking questions of themselves such as: “How can I use the results from my previous guess to make a better guess?” and “How do I know if my next guess should be greater or less than my previous guess?” (p. 395). If students are to check their guesses and improve the plausibility of their guesses and justifications, they need to be given the chance to guess, justify, and reflect.

3.4.4 Teacher influences on student judgement

Teachers are instrumental in providing opportunities for students to guess and check. Students should be asked to provide a guess, then expected to test, justify, and defend their guess. They should be able to explain what makes their guess credible and generalise the mathematical concepts of their guess to new problems (Leinhardt & Schwarz, 1997). However, if the teacher or school culture does not sanction students’ guesses they are less likely to occur, or if they do occur are less likely to be appropriated or valued (Guerrero, 2010; Johanning, 2007; Wong et al., 2002).

Wong and colleagues (2002) asked 1216 students from Hong Kong in 3rd, 6th, 7th, and 9th grade (aged 9 to 15) to solve and explain three sets of non-routine computational, word, and open-ended mathematical problems. The students were not familiar with solving non-routine problems but they were specifically chosen in an attempt to make conceptions of classroom mathematics more visible. Guessing answers and solution strategies featured strongly with students in Wong et al's (2002) research. The guesses ranged from wild to plausible and whilst students admitted some guesses were wild, they did not make any attempt to improve the reasonableness of them. One student commented he made "guesses until I make the right guess" (Wong et al., 2002, p. 35). The student kept making arbitrary guesses until he stumbled upon the correct answer. With further questioning the student revealed he believed it was the teacher's responsibility to determine if he had made the right or wrong guess. A different student explained that rather than guess the answer he tried to guess the right strategy to use. He would look for a similar previously solved problem and guess that the same strategy would work for him.

Most students in Wong et al.'s study had an approach that entailed guessing, and positioning the teacher to check their guess. Few students positioned themselves to guess and check and to judge the reasonableness of their strategy and solution. Wong and colleagues (2002) contended that if teachers continue to present routine problems that can be solved through non-mathematical means such as wild guesses and relying on teachers, students will continue to approach problem solving in non-mathematical ways. The positioning of the teacher in Wong et al's (2002) research perhaps reflects a cultural belief of teachers as authority figures. However, it also provides insight into the students' beliefs about guesses and their opportunities to guess.

Teacher modelling of guesses has been suggested as one way of positively impacting on students' willingness to have a go and take risks (Lakatos, 1976; Lampert, 1990). Literature regarding teachers' modelling correct mathematical vocabulary, mathematical thinking, and explanations is prolific (Ball & Bass, 2003; Ball, Hill, & Bass, 2005; Fennema & Franke, 1992; Fennema, Franke, Carpenter, & Carey, 1993; Rowland, Huckstep, & Thwaites, 2005). Literature regarding teachers' modelling of genuine mathematical guesses is scarce. A possible reason for this is that teachers may seldom be mathematically challenged enough to need to guess. In this situation, Pólya (1963) advocated that teachers should pretend they need to guess or they should model a guess a student might make.

If teachers are not modelling plausible and reasoned guesses then students' developing skills for guessing may be impeded.

3.4.5 Summary of judgement

Judgement as a construct of students' mathematical know-how has been defined for my study as students' ability to make plausible guesses, justify their guesses, and reflect on both their guesses and their justifications. Guessing has been explored from teachers' and students' perspectives. Teachers can use guessing and proving as a pedagogical tool to engage students in, and commit them to, the mathematics (Pólya, 1958, 1963). With an ethical or metacognitive intent students can use their guesses and justifications and the resulting discussions and arguments to progress their own and others' mathematical know-how (Lakatos, 1976; Lampert, 1990; Maher & Martino, 1996).

The literature suggests students become adept at making plausible guesses when they are given the opportunity to guess then validate and defend their guess. Without the opportunity to guess, students may feel they have to be correct with their first answer and this could make participation precarious for some students. Without the opportunity to justify guesses, students may revert to wild or implausible guesses just to be heard, and shared understandings that progress learning could be at risk.

The influence teachers have on the presence and value of students' guesses and justifications has been explored. Teachers can encourage students to make plausible guesses and justify them by asking questions about the validity, adaptability, and generalisability of their guess. Teachers can also discourage student use of guesses and justifications if they expect the correct answer first or if they provide the justification for any student's guess (Wong et al., 2002). One research focus that appears to be omitted is that of teachers positioning themselves to model guesses and the potential benefits of this.

3.5 Creativity

Creativity is the third construct of student mathematical know-how in my study. As explained in the introduction to this chapter, creativity is a combination of Pólya's (1965) individual constructs of originality and creativity. Creativity is said to begin with curiosity (Barbeau & Taylor, 2005). When students become curious about mathematics they engage in reflection, deliberation, and investigation. They hypothesise and take risks when they ask questions, generate problems, and provide explanations, and they are able to tolerate and work through uncertainty.

Students with mathematical creativity are able "to make appropriate choices and decisions in unexpected situations" (Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012, p. 290). Choices and decisions could include using familiar strategies in unfamiliar ways, discovering unknown relationships, considering an old problem from a new angle, analysing a problem from different perspectives, presenting original work, or posing questions that extend the body of knowledge (Ervynck, 1991; Liljedahl & Sriraman, 2006; Sriraman, 2004, 2005). The processes and products of original, unusual, or intuitive ideas, approaches, actions, connections, explanations, questions, and discoveries represent mathematical creativity (Leikin, 2009; Lev-Zamir & Leikin, 2011; Liljedahl & Sriraman, 2006, Sriraman 2004, 2005). It is important to remember that mathematically creative ideas must be useful and broaden the teaching and learning opportunities of teachers and students (Nadjafikhah et al., 2012).

Creativity has been perceived by researchers and educators as a favourable attribute of mathematical knowledge and skill. It has been described as the essence of mathematics (Ervynck, 1991) and one of the most important characteristics of advanced mathematical thinking (Ginsberg, 1996).The National Council of Teachers of Mathematics (2000) recommended that developing students' creativity should be one of the primary goals of mathematics education in general. Whilst students' creativity in mathematics has been positively viewed, a shared definition of mathematical creativity has been less readily agreed (Haylock, 1987; Sriraman, 2005). Definitions offered have tended to be based on opinions and considered vague or elusive (Nadjafikhah et al., 2012).

The diversity of students' mathematical creativity, as understood through empirical studies, has featured only recently in mathematics education research (Haylock, 1987; Leikin, 2009; Lev-Zamir & Leikin, 2011; Lin, 2011). There are three possible reasons for this. First, student mathematical creativity has not always been viewed by teachers as a positive or valued attribute. Teachers have described creative students as disruptive, stubborn, rebellious, argumentative, selfish, and easily distracted (Davis, 1999; Torrance, 1963). The negative tone of these descriptors and findings may have deterred some researchers. The second possible reason is that creativity has been more aligned in literature with artistic talents rather than mathematical skills (Aljughaiman & Mowrer-Reynolds, 2005; Isaksen, Dorval, & Treffinger, 2000). Mathematical creativity has been perceived as a static gift reserved for some students and this narrow perception may have reduced researcher interest. The third reason is the assumption that creative students are also gifted students and vice versa (Davis, 1999). Creativity has tended to be viewed as a fixed personal attribute of bright students and as such could be taught only to students who exhibited creativity or giftedness (Treffinger, Young, Selby, & Shepardson, 2002). The understanding now shared is that at some level all students have a sense of creativity that can be developed and taught (Aljughaiman & Mowrer-Reynolds, 2005; Isaksen et al., 2000).

In his seminal work on creativity, Torrance (1974) proposed four components: fluency, flexibility, novelty, and elaboration. These components have been further explored within a specific mathematical context. Fluency in mathematics relates to "the continuity of ideas, flow of associations, and use of basic and universal knowledge" (Lev-Zamir & Leikin, 2011, p. 19). Mathematically fluent students are able to change their approach mid-process when generating a response (Silver, 1997). Flexibility, the second component, is the ability to consider one problem from a variety of solution perspectives. Mathematically flexible students understand that any problem could have multiple strategies leading to the correct answer (Bolden, Harries, & Newton, 2009; Sullivan, Warren, & White, 1999). Students are able to approach a problem from varying perspectives, monitor their solution processes, make changes to their methods mid-process, and produce a variety of solution strategies (Vale, Pimentel, Cabrita, Barbosa, & Fonseca, 2012). Holton, Ahmed, Williams, and Hill (2001) described flexibility as a kind of mathematical play where students use both "experimentation and creativity to generate ideas, and using the formal rules of mathematics to follow any ideas to some sort of a conclusion" (p. 403). The third component, novelty, is characterised by fresh, unique, unusual, and divergent ways of thinking that produce solution strategies not previously experienced by the group. Finally, elaboration is associated with the capacity to describe, illuminate, and generalise ideas (Lev-Zamir & Leikin, 2011).

Lithner (2008) proposed that the opposite of creative and original mathematical thinking was imitative thinking which emphasises memorising and mimicking before, or instead of, thinking. Teachers promote imitative thinking when they ask students to follow a particular strategy, rule, or procedure, when they stress speed or precision in problem solving, when they do the majority of the mathematical thinking and actions for the students, and when they protect students from mathematical challenges (Nadjafikhah et al., 2012). The pedagogies that both assist and inhibit developing students' creativity are explored in the following sections.

3.5.1 Non-routine problems for teachers and students

Pólya (1963, 1969) suggested that mathematical creativity could be awakened and enhanced when teachers and students were positioned as novice mathematicians. Novice mathematicians intuitively hypothesise and decipher what is going on in order to make mathematical sense. They formulate and contribute examples, questions, and explanations; conjecture and communicate possible solution strategies; and explain, justify, and evaluate their own and others' mathematical ideas. They describe connections with prior knowledge and between mathematical contexts; generalise between and from examples and representations; challenge, collaborate, and negotiate agreement; and change their mind, make mistakes, and persist (Barton, 2009; Civil, 2002). When teachers undertake these mathematical actions they are reminded of the significance and usefulness of struggling to solve a problem (Lampert, 1990). Students are learning that problems are not always solved quickly and that more than one solution strategy could exist (Schoenfeld, 1987, 1988). Haylock (1987) contended that such positioning of teachers and students was not common and a neglected aspect of some mathematics classrooms. He advocated a break from the stereotype of "rigid adherence to successful routines” and a move toward flexible, divergent, and quality mathematical thinking (pp. 59-60).

The suggestion has been made that for mathematical creativity to be present teachers needed to model and pose impromptu or unrehearsed non-routine problems (Pólya, 1963). As described in Chapter Two, non-routine problems are not pre-selected to match a rule or strategy. Non-routine problems would require teachers and students to draw on their mathematical creativity and originality and test different conjectures because no best-fit solution strategy would be immediately apparent (Pólya, 1963). In a similar way to the positioning of teachers guessing, not all problems modelled by teachers are going to be challenging or require their creativity. In this situation, Pólya (1963) suggested teachers draw on their acting skills and pretend they are approaching the problem for the first time. It is important that students observe teachers wrestling with mathematical ideas as this could encourage them to take risks, persevere, and experience mathematical creativity (Haylock, 1987).

3.5.2 Conceptions of creativity

Lev-Zamir and Leikin (2011) sought to describe and analyse Israeli teachers’ conceptions of creativity in teaching mathematics. Using a grounded theory approach, the researchers analysed 48 mathematics lessons for students in 4^{th} to 8^{th} grade (aged 10 to 14) taught by 11 teachers. Each teacher taught four or five lessons and they were interviewed before and after the lessons. In the interviews teachers were asked to describe what they planned for the lesson before it was taught and to reflect on what had happened after it was taught. From the classroom observations and teacher interviews, Lev-Zamir and Leikin determined that the creative actions and discourses of teachers and students tended to be similar. Teachers and students generated original mathematical tasks and ideas, applied multiple strategies, elaborated on their own and others’ understandings, asked questions that were unexpected, and suggested strategies that went beyond curriculum expectations and classroom sociomathematical norms. Such actions were deemed to flexibly move teachers and students in new mathematical directions and into “different mathematical territory’’ (Lampert, 2001, p. 44).

Further analysis of the data showed that conceptions of creativity were teacher-directed or student-directed. Teacher-directed conceptions of creativity were twofold. First, teachers' mathematics was creative, that is they modelled and explained the mathematics in creative ways. Secondly, the conception of creativity was pedagogical because the teacher taught in creative or innovative ways. Student-directed conceptions of creativity were collated as a framework that included students' mathematical fluency, flexibility, originality, and elaboration. Fluency related to the use, continuity, and flow of mathematical knowledge, ideas, and associations. Flexibility correlated with being able to reflect on and change mathematical strategies. Originality required a unique way of thinking, and applying thinking and elaboration is the ability to describe, illuminate, and generalise that thinking. Each component was reciprocally related but all components do not have to be present at the same time for creativity to occur (Lev-Zamir & Leikin, 2011). What does have to exist is a teacher who either provides or sanctions opportunities for students to approach their mathematics with creativity.

Lev-Zamir and Leikin (2011) argued that without creative teachers who were pedagogically flexible, attentive, and sensitive to students' needs, it was unlikely students would progress in their mathematics, creativity, or mathematical creativity. Extending Haylock's (1987) argument that ''any definition of mathematical creativity in school children must refer to both mathematics and creativity'' (p. 62), Lev-Zamir and Leikin (2011) proposed that "any definition of mathematical creativity in mathematics teaching must refer to mathematics, teaching, learning, and creativity" (p. 19). The responsibility for creative mathematics, teaching, and learning lies with the teacher teaching "with and for creativity" (Lev-Zamir & Leikin, 2011, p. 17).

Two limitations to the work of Lev-Zamir and Leikin (2011) are apparent. The first is that the teacher and student directed conceptions of creativity were developed by the researchers and not the teacher or student research participants. Secondly, the researchers did not consider the effect of contextual influences on levels of creativity. This second limitation was further investigated by Tabach and Friedlander (2013).

Tabach and Friedlander applied Lev-Zamir and Leikin's (2011) framework to investigate changes in creativity levels across Israeli student grade levels and the effects of student mathematical knowledge on levels of creativity. The problem solving skills of 76 students in 4^{th} to 9^{th} grade (aged 10 to 15) provided the data for their research. Students had 30 minutes to solve a mathematics problem such as: *There are chickens and cows on old McDonald's farm – altogether 70 heads and 186 feet. How many chickens and cows are on the farm?* . Students were asked to explain their solution and attempt different problem solving strategies. Analysis of the students' solution methods evidenced that the older more knowledgeable students in Tabach and Friedlander's study were more creative than their younger less knowledgeable peers. As students' familiarity and confidence with mathematics increased with knowledge and age, so too did their creativity. However, the researchers also noted a decrease in creativity toward the end of the 9^{th} grade (aged 15) because of an increased use of rule-based algebraic methods. The proposition was made that the more mathematical know-how students had, the more creative they were (Tabach & Friedlander, 2013). The researchers did not consider the different ways teachers positioned older or younger or differently able students to be creative. This is a limitation to Tabach and Friedlander's study because teachers may have provided older or more able students with more challenging tasks and more opportunities to be creative. The researchers did establish that the introduction of rule-based algebraic methods inhibited creativity so it is also possible that teacher or contextual influences could have the same effect. The following section describes teacher influences on students' creativity.

3.5.3 Teacher influences on student creativity

Through their decisions and actions teachers can both afford and limit the opportunities students have to be mathematically creative (Haylock, 1987). Teachers need to ensure students have the chance to interactively and collaboratively solve problems that have more than one solution strategy, consider problems in different ways, and create and trial explanations that are efficient or elegant (Leikin, 2009). Teachers also need to ensure that each opportunity is sanctioned as valuable and appreciated by all participants (Nadjafikhah et al., 2012). It is important that teachers promote and endorse an environment where it is acceptable to take a risk, change one's mind, and make mistakes.

As part of a wider project into teacher beliefs and actions, Stipek, Givvin, Salmon, and MacGyvers (2001) investigated the teacher beliefs and practices of 21 Californian 4th to 6th grade (aged 10 to 12) students in regard to mathematical creativity. Teachers were asked to complete a survey containing 57 statements by indicating how strongly they agreed or disagreed with the statement, for example, "Mathematics involves mostly facts and procedures that have to be learned" (Stipek et al., 2001, p. 217). Each teacher was videoed teaching at least two mathematics lessons, and their practices such as the emphasis they gave to performance outcomes, speed of task completion, or student effort were coded and analysed. Student data included the pre and post-achievement results of 437 students and students' answers to a questionnaire inquiring into their beliefs about their mathematical competence and enjoyment.

Results from Stipek and colleagues' (2001) research that related to teachers' beliefs and practices about students' creativity and originality are as follows. Teachers who believed they should and could control the classroom instruction were not likely to encourage or accept creativity in students' thinking. Teachers who valued extrinsic motivation were also less likely to emphasise creativity. Stipek and colleagues concluded that teachers who held more traditional beliefs about teaching and learning mathematics tended not to teach in ways that encouraged or promoted creativity and originality.

3.5.4 Summary of creativity

With creativity as a construct of their mathematical know-how, students are able to explore novel or innovative ways of solving problems and to make appropriate decisions to deal with unexpected problems. Teachers can support students to develop and apply their creativity by asking them to solve non-routine problems. This section has highlighted the need for teachers to value their own personal and pedagogical creativity if they are to position themselves and students as creative mathematicians. Teachers modelling creative thinking could influence students' willingness to take risks, and the reverse is also likely. Teachers modelling predetermined solution strategies may impede students' inclination to think creatively or beyond the teacher's example.

3.6 Concluding Comments

This chapter began by defining the importance of mathematical know-how as much more than knowing what to do. Know-how in this study encompasses the what, which, why, when, and how of applying mathematical knowledge and strategies. Knowing how to apply and regulate knowledge and strategies metacognitively, diversely, and creatively positions students as authentic creators of new knowledge and strategies (Anthony & Walshaw, 2007).

Independence, judgement, and creativity have been defined and reviewed as constructs of student mathematical know-how. With independence, students make decisions, think, act, and reflect in ways that enhance their own and others' opportunities for learning. Judgement enables students to draw distinctions between wild and plausible guesses and is based on their intuition, then substantiated and confirmed. As such, independence and judgement are interrelated because students' judgements are substantiated and confirmed through self-regulation. With creativity students are able to experiment with unusual strategies and apply their knowledge in novel ways. Experimentation and originality in mathematics may require judgement and guessing before creativity and checking for accuracy. Although examined as individual constructs of student mathematical know-how, independence, judgement, and creativity are also influenced by, and influence, the other.

The influences teachers have on students' opportunities to share their mathematical know-how have also been investigated in this chapter. One influential stance teachers can take is for them to personally and pedagogically engage with mathematical independence, judgement, and creativity. When teachers engage in creativity they are reminded that mathematics is challenging, problems are not always solved quickly, and more than one strategy solution is possible (Lampert, 1990, 2001). Teachers should be modelling and explaining mathematics as a discovery to be made rather than a rule to be followed. A second stance is that through their expectations and questions teachers position students to share their know-how. The teacher's and students' positioning can either afford or limit opportunities to develop and share independence, judgement, and creativity. It is timely then to consider positioning theory as a theoretical lens through which mathematical know-how can be analysed and understood. Positioning theory, as theorised by Bronwyn Davies and Rom Harré (1990) and Rom Harré and Luk van Langenhove (1991) is explored in Chapter Four.

Chapter Four: Positioning Theory

> The advent of Positioning Theory as a development of Vygotsky's conception of the person in an ocean of language, in intimate interaction with others in the construction of a flow of public and social cognition, opens up all sorts of insights and research opportunities. Moving beyond the overly restrictive frame of Role Theory it offers a conceptual system within which to follow the unfolding of episodes of everyday life in new and illuminating ways. (Harré, 2004, p. 11)

4.1 Introduction

This chapter presents the theoretical paradigm of positioning theory as first theorised by Davies and Harré (1990) and Harré and van Langenhove (1991). Section 4.2 introduces positioning theory and explains how dialogue and interactions can be understood by analysing people's talk, texts, and actions. Two significant influences on the development of positioning theory are discussed in this section. The first was the identified need to replace role theory with a more dynamic theory. The perceived limitations of role theory and the need for a theory that provided opportunities for understanding interactions from a flexible, changeable, and unpredictable perspective are explored. The second influence was the drive to move toward a more discursive approach to theoretical analysis and description. The influences of Wittgenstein (1953, 1969), Goffman (1959, 1974, 1981), Hollway (1984), Edwards and Potter (1992), and Edwards (1997) are examined. The conceptualisation of language as a public, social, customary practice determined by participants and their interactions, and the claim that dialogue, interactions, and historical contexts afford or limit people's opportunities to participate and the level to which they could participate, are described.

Social episodes, a construct of positioning theory analysed in this research, are introduced in Section 4.3. These comprise and are made comprehensible through the talk, text, and actions of those participating. The ways in which social episodes are created and sustained and their mutually determining nature are explained. Three features of social episodes relevant to this study - positions, storylines, and social acts - are examined within this section, as are their mutually determining relationships. Section 4.4 reviews literature that used positioning theory as the framework for analysing and understanding educational contexts such as teacher and student authority and rights and duties. Important research links between previous educational investigations and my study are outlined.

4.2 Positioning Theory

Positioning theory proposes that when people interactively engage in deliberate and authentic dialogues they do so from a position (Davies & Harré, 1990; Harré & van Langenhove, 1991). For dialogue to be considered deliberate and authentic it needs to occur naturally in the talk, text, and actions people use in their everyday lives (Harré & Secord, 1972). Scripted dialogue in a play would not be considered authentic but the actors' discussions about the play would. Once positioned, either reflexively by themselves or interactively by others, a person "sees the world from the vantage point of that position" (Davies & Harré, 1990, p. 46). From a vantage point people make and attempt to make theirs' and others' talk, texts, and actions meaningful (Davies & Harré, 1990; Harré & van Langenhove, 1991).

The meaningfulness of any contribution is influenced by past, present, and future contexts and, as such, positioning theory offers an appropriate framework through which to understand the complexities of teaching and learning. Any teaching and learning event is not an isolated incident; instead, it is affected by the histories of the teachers and students, how they wish to present themselves, and the future effect they desire to have. Positioning theory allows for the analysis of the changing, unpredictable, and interactive nature of classroom communities.

Positioning theory provided the framework for analysing and understanding social, political, medical, and corporate phenomena. For example, social phenomena include emotion, identity, inter-group relations, power, and guilt (Benson, 2003; Brinkmann, 2010; Harré & Slocum, 2003; Parrott, 2003). Political phenomena have included migration and immigration, international relations, deliberate democracy, and war (Menard-Warwick, 2007; Moghaddam & Harré, 2010; O'Doherty & Davidson, 2010; Slocum & van Langenhove, 2004). Within a medical setting, positioning theory has provided the theoretical framework for examining midwifery, people with Alzheimer's, home and hospice carers, and children born with cocaine addictions (Barone, 2000; Bisel & Barge, 2011; Phillips & Hayes, 2008; Sabat, 2008). The corporate world has also been analysed through positioning theory (Boxer 2003; Zelle, 2009). Not all research has been accounted for here; however, an overview of the phenomena being analysed and understood through positioning theory has been provided.

Two significant influences on the development of positioning theory were the need to move beyond the perceived confines of role theory and the relationship with discursive psychology. A replacement for role theory, the relationship with discursive psychology and the work of psychologists, social scientists, and sociologists who contributed to the development of positioning theory are discussed in the following sections.

4.2.1 A replacement for role theory

Harré and colleagues proposed positioning theory as a replacement to what they described as the more static, non-discursive, metaphorical notion of role, and the overly cognitive concept of role theory (Davies & Harré, 1990; Harré & Van Langenhove, 1991). Role theory, according to Harré and colleagues, focussed on revealing the rules that guide interactions rather than exploring the self-dynamics that gave rise to interactions. Goffman's (1959, 1974, 1981) metaphors of frame and footing for understanding interactions between people were seen by Harré and colleagues to focus more on what happened than why what transpired, happened. For example, a framework provided an analytical way of making sense of events by framing the ways participants communicate and understand an activity they are co-constructing (Goffman, 1974). People used frames to reference and make sense of their own and others' words and actions, but not to examine the reasons or motivation behind these words and actions. A person's footing or position could be spoken from, changed, gained, or lost in a conversation when their rights and duties increased or decreased (Goffman, 1981). Less understood, but viewed as equally important by Harré and colleagues, was an understanding of why rights and duties changed. Over time Harré and colleagues have argued that Goffman's (1981) notions of framing and footing were limited by the constraints of role theory (Davies & Harré, 1990; Harré & Moghaddam, 2003).

Three problems that highlighted the need to replace role theory with a more vigorous and flexible research frame were identified. The first problem was that roles were experienced and enacted by individual participants acting in isolation (Harré, 2004). For example, teacher roles tend to remain static whereas teacher positions are negotiated and renegotiated during interactions. Teachers may assume the same role within a lesson, but the position they hold may change from director, to questioner, to challenger, and to learner (Barnes, 2003, 2004). The second problem was that human interactions such as conversations tended to be analysed and explained through rules and conventions that existed independently, and sat outside the actual conversations (Räisänen & Stenberg, 2011). The third problem was that the words that were spoken were to some extent dictated by the role and interpreted in these terms. For example, the anticipated roles of the teacher and students could impede their interactions from being analysed as anything other than standard and customary. How teachers and students are is predetermined by the expected roles they hold. Each problem highlighted for Harré and colleagues (Davies & Harré, 1990; Harré & Van Langenhove, 1991) the decontextualised, internalised, and prescribed nature of role theory.

Redman and Fawns (2010) used the metaphor of the climate and the weather to illustrate the difference between role theory and positioning theory. Role theory, they suggested, was “more aligned to the idea of institutional climate”. Positioning theory was “more aligned to our understanding of the weather in that it is changeable, more reflective of the moment, and not as reliably predictable as climate” (p. 165). Clarke (2003) contended that roles indicated a more permanent classification such as teacher or student, and had institutionalised status whereas the status of positions was social and as such was constructed as a social artefact through the interactions of the group. Hence, positioning theory provides a more suitable analysis of interactions in classrooms that are dynamic.

4.2.2 A discursive approach

In response to the limitations identified within role theory, Harré and colleagues proposed a discursive approach to positioning theory. Dialogues in discursive psychology and positioning theory are regarded as having a social action or function; conversations and interactions are the means through which goals can be achieved in a socially meaningful world (Osbeck & Nerssesian, 2010). In this sense, discourse is “a social phenomenon” (Bakhtin, 1981, p. 259) that includes talk, text, and actions of any kind (Davies & Harré, 1990). Words and actions are contextualised to those that use them, shaped by how others have used them, and how others expect them to be used (Bakhtin, 1984; Davies & Harré, 1990). The contextualised nature of discourse means it is liable to change, able to be challenged, divergent, and transitory (Davies & Harré, 1990).

The degree to which people’s discourses can be socially persuasive is dependent on the history of their interactions (Barnes, 2004), how they wish to present themselves in the moment, and the future effect they wish their discourses to have (Harré & van Langenhove, 1991, 1999). What participants do and what they can do is afforded or constrained by the rights and duties they acquire, assume, or have imposed on them and the collaboratively established and agreed to norms of the group (Harré & van Langenhove, 1999; Varela & Harré, 1996). Different versions, outcomes, and social actions of the same interaction can be achieved because speakers design what they are going to say based on past, present, and possible future accounts of what happened, who acted how, and what was claimed (Edwards, 1997; Edwards & Potter, 1992). Similarities are often evident between present and past discourses. Davies and Harré (1999) contended that “actual conversations which have already occurred … are the archetypes of current conversations” (p. 43). Likenesses between past and present conversations are explained by reference to the personal and cultural resources participants draw upon to construct the present moment. Personal and cultural resources include what has actually happened before and participants’ memories of what happened before.

The discursive approach to positioning theory was influenced by Wittgenstein (1953, 1969), Hollway (1984), Edwards and Potter (1992), and Edwards (1997). Wittgenstein’s influence on Harré and colleagues’ work is found in their shared conceptions of discourse as a public, socially constructed, and normatively guided practice. Arguing against the prospect of language being private, Wittgenstein (1953) contended that for language to be meaningful it had to be an agreed to, publicly acquired, social activity with a shared grammar (Harré, 1989). The meaning of any word or action was collaboratively determined and dependent on the context in which it is used and agreed to (Howie & Peters, 1996). Claiming that the actual dialogue of particular individuals voiced in social contexts to particular audiences provided an analytical and clearly expressed representation rather than a theoretical one, Wittgenstein (1980) argued that “words are deeds” (p. 46).

Social scientist Hollway (1984) used the concepts of position and positioning to describe women’s and men’s subjectivities in her analysis of the inter-personal construction of bias in heterosexual relations. Describing the construction of men’s and women’s subjectivity in heterosexual relations as “the product of their history of positioning in discourses” (p. 228), Hollway contended that certain discourses afforded or constrained the availability of positions to men and women. Positioning was used to explain why women in a mixed gender group said less than men but when in a single sex group, women said more to each other than the men in their group. Hollway (1984) asserted that when participating in mixed gender groups women spoke less than when participating in single gender groups and they had fewer rights to speak than their male counterparts.

Connections can also be seen in the social function and force given to discourse, the contextualised nature of discourse, and the connectedness of past, present, and future discourses (Edwards, 1997; Edwards & Potter, 1992). Discursive psychology provides the framework through which to explore the motives, attitudes, and morals that underpin conversations and interactions (Edwards, 1997; Edwards & Potter, 1992). The focus of discursive psychology is on the external, the observations of how people deal with their own and others’ actions, express opinions, and position themselves and others within conversations. Discursive psychology does not deny the existence of inner thoughts and experiences but rather suggests that such units of analysis are “methodologically always just out of reach” (Billig, 2009, p. 7). Talk is conceptualised as the event of interest itself and language and variability in talk shapes reality (Harré, 1998a).

Close attention is paid within positioning theory to the social episodes of groups and the positions, positioning, storylines, and social acts created within each episode (Harré & van Langenhove, 1999). The following sections explore social episodes and because of their relevance to this study positions and positionings, storylines, and social acts are illustrated through examples using educational research contexts.

4.3 Social Episodes in Positioning Theory

A social episode is a naturally occurring event within an interaction that includes the thoughts, feelings, and intentions of all participants (Harré, 1998b; Harré & Secord, 1972; Harré & van Langenhove, 1999). Social episodes can be formal with explicit scripts and rules that need to be consciously followed or they can be informal (Harré & Secord, 1972). An example of a formal social episode is a traditional wedding ceremony in which the participants’ words and actions are significantly scripted and choreographed (Harré & van Langenhove, 1999). Informal social episodes are more impromptu and improvised. However, formal and informal social episodes still have aspects to them that are indeterminate, something which makes it impossible to confirm exactly what will happen next. Whilst a traditional wedding may have been substantially scripted and choreographed, it is still up to the participants in the episode to follow, deviate from, or ignore the scripted words and choreographed moves. In regard to this study, both formal and informal social episodes are apparent within classroom interactions and so both will be included in the analysis.

Any social episode is created and sustained by the talk, text, and actions of particular people, the positions from which those people say and do, the storylines the discourses develop, and the social force of their words and actions on that particular occasion (Davies & Harré, 1990; Harré, 1997). The personal stories of participants in a social episode become comprehensible to others through their words and actions and the positions from which they say and do. The shaping of a social episode is mutually determining because participants contributing to the episode create and shape it, and the episode itself affords and constrains people’s discourses within it (Harré & Secord, 1972; Harré & van Langenhove, 1999). Harré and Secord observed social episodes as being shaped by “things done by a person” and “things done to a person” (p. 148).

The structure of a social episode draws on Vygotsky’s (1978, 1986) views about the cultural embeddedness of thought and language (Howie & Peters, 1996; Moghaddam et al., 2008). According to Vygotsky (1978), all higher order mental processes exist twice, once in the relevant group, influenced by culture and history, and then in the mind of the individual: “Every function in the child’s cultural development appears twice: first, between people (interpsychological), and then inside the child (intrapsychological)” (p. 57). Therefore, the development of a participant is reliant on interpersonal relations and individual maturation, but for private language to become public the facilitation of meaning by another is essential. For Vygotsky and Harré, both private (intrapsychological) and public (interpsychological) displays of language and meanings result from social use and cultural embeddedness. Social use and cultural embeddedness are expressed as “institutional practices” (Harré & van Langenhove, 1999, p. 394).

Institutional practices are the way entities and events are taken as shared between groups of participants in the community in which they occur. They can be viewed similarly to Cobb and colleagues’ notions of social and sociomathematical norms (Cobb et al., 1989; Yackel & Cobb, 1996). Tacit and historical institutional practices embedded in daily practice influence participants’ “doings and sayings” (Redman, 2007, p. 7) by enhancing or impeding their rights and duties, and agency within those rights and duties. Therefore, an individual’s ability to influence the conception and development of a social episode relies on that person having some recognised rights and duties (Harré et al., 2009). It is important to examine tacit institutional practices and how these position teachers and students as having particular rights and duties in what they do and say when teaching and learning mathematics.

Social episodes provide a productive frame from which to discursively analyse, understand, and interpret people’s individual, collective, private and public activities (Edwards & Potter, 1992; Harré, 1998a). Davies and Harré (1990) posited that such a framework could assist researchers to describe and explain how “people do being a person” (p. 62). In my research I will analyse how teachers do being a teacher by exploring the positions teachers take and are given, the rights and duties afforded and constrained by the positions, and the storylines and social acts that are realised through the interactions (Davies &

Harré, 1999; Harré et al., 2009; Harré & Moghaddam, 2003; Slocum-Bradley, 2010). Positions, storylines, and social acts as features of a social episode are explored in the following sections.

4.3.1 Positions and positioning

Positions are the autobiographical parts being performed, and the patterns of belief being distributed by the participants in the social episode (Davies & Harré, 1990; Harré & Moghaddam, 2003). Positions can be assigned reflexively by an individual or interactively by another participant (van Langenhove & Harré, 1999). When a position is assigned interactively the person being positioned may "acquiesce in such an assignment, contest it or subvert it" (Harré & van Langenhove, 1999, p. 2). How people are positioned in any social episode depends on the background, values, personal characteristics, histories, predilections, and capabilities of all concerned (Barnes, 2004).

Participants can construct and shift between positions and occupy more than one position simultaneously during a social episode. As such, positions are dynamic and responsive to context. To position someone means to establish what their rights and duties are, and determine what they are allowed or obliged, and not allowed or obliged, to do (Harré & Moghaddam, 2003; Harré & Slocum, 2003). One person's position within a group can only be understood in relation to another person's (Harré & Moghaddam, 2003) and ascriptions of good or bad character can strengthen or weaken that position (Harré, 2004). It is important to note that the concept of position implies neither coherence nor consensus across individuals or groups. Instead, it is through the assignment of individual rights and duties that positions are sustained by the group (Harré & van Langenhove, 1999).

To position oneself, position others, or be positioned means to establish what participants have or do not have the right or duty to say and do, or not to say and do (Harré & Moghaddam, 2003; Harré & Slocum, 2003). Redman and Fawns (2010) described a right as "what a participant expects others may be reasonably held to be accountable for, and to provide and protect for them" and a duty as "what others can expect a person to be providing and to be accountable to and responsible for" (p. 166). Barnes (2004) contended that positions carry rights such as "the right to be heard, the right to be taken seriously, the right to be helped, or the right to be looked after" (p. 2). Reciprocal duties would include the duty to listen, to be respectful, and/or to help and care.

People have different rights and duties within any interaction. Differences are influenced by the positions people occupy, their words and actions, and the distribution and acknowledgement of their rights and duties within the storyline. Not all people are able to perform the same acts because their rights and duties may limit or extend the potential social acts (Moghaddam, Harré, & Lee, 2008). Issues of legitimacy and entitlement are intertwined with the allocation of rights and duties and the relationship between the rights and authority of participants is usually linear. Participants with more rights tend to have more authority and an increase in rights usually leads to an increase in authority. Being positioned as incompetent may limit a person's opportunities to contribute to the conversation and to have their contributions taken seriously, thus reinforcing the position of incompetence. In some cases the internal structure is that of "determinable to determinate" (Harré & van Langenhove, 1999, p. 2).

4.3.2 Storylines

Storylines develop when people make and attempt to make past, present, and future words and actions meaningful to themselves and others (Davies & Harré, 1990). As people tell a story about themselves, a storyline evolves. The meanings given to and taken from the evolving storyline are contextualised to how people want to present themselves and be seen by others (Slocum-Bradley, 2010: van Langenhove & Harré, 1999). The contextualised nature of a storyline means there are multiple commentaries, interpretations, and relationships in play (Harré & Secord, 1972), so the exact same words and actions in a conversation can convey a different storyline to different people (van Langenhove & Harré, 1999). For example, the words 'excuse me' used by a teacher could in one storyline be an apology and in a different storyline an exclamation of astonishment. Therefore, positions within a storyline can "co-exist in a complex weave" (Wagner & Herbel-Eisenmann, 2009, p. 10), with two or more people "living quite different narratives without realising they are doing so" (Davies & Harré, 1999, pp. 47-48). Accordingly, storylines should not be considered as correct, and all are revisable because perspectives within any storyline differ, so there is never any "perfect truth" to the storyline (Harré & Secord, 1972, p. 9).

A storyline can be implicit or explicit, actively constructed, contested, or taken for granted. People may choose to be complicit or resistant with the storyline and may not always choose to participate in the storyline. The creation and survival of any storyline is contingent on it being jointly constructed and sustained. Consequently, the structure of a storyline or how it is formed cannot be forced. Instead, storylines tend to follow already established patterns of development within a cluster of narrative principles and practices (Harré & Moghaddam, 2003). The principles and practices of individuals and groups act as a guide as to what are considered contextually appropriate discourses by participants.

Storylines can be initiated and altered by the participants creating them. The capacity and willingness of participants to initiate storylines that are taken up by others differ (Harré & van Langenhove, 1999). Differences may be underpinned by cultural factors, people’s learned ways of positioning themselves in different contexts, and the enactment of storylines that invite or discourage initiative. For instance, a teacher’s repetitive use of the evaluation component of the initiation-response-evaluation sequence can “reinforce an authority structure that strips initiative from students” (Wagner & Herbel-Eisenmann, 2009, p. 5). Storylines can be altered through the presence or absence of certain positions that enable or constrain certain social acts, people challenging the positioning of the first speaker, and by means of their own positioning giving the storyline a new twist. An altered storyline can affect the initial social force of a social act and therefore shape the conversation and its outcome in a different direction. The action-social act structure is examined in the following section.

4.3.3 The action — social act structure

The action — social act structure gives meaning to participants’ actions. An action is “the means through which social acts are performed” and a social act is “what an activity is taken to achieve socially” (Harré, 1979, p. 14). Actions are what participants say and do within a conversation and could be verbal, non-verbal, or written (Harré, 1979). When actions are appropriated and given meaning by others they take on a social force and become socially significant to the group as social acts (Davies & Harré, 1990; Harré & Secord, 1972). As the joint production of all those involved in the social episode, the action social act structure is reliant on the intention of the speaker and how the action is received (Harré, 1991). The way any social act is heard or interpreted can validate or make void the meaning and social force behind it (Muhlhausler & Harré, 1990). The meaning and force of a social act is reliant on interpretations of contextual factors such as the position, background, and identity of the speaker (Slocum-Bradley, 2010). As such, there can be multiple speech-acts accomplished or unrealised in any one social episode (Muhlhauser & Harré, 1990). The assumptions people make as to the integrity or duplicity of the social episode in which they are engaged can have a profound influence on what they say and do. Therefore, a sense of reciprocity exists between actions and social acts.

Harré and Davies (1991) proposed that social acts have an illocutionary force. The illocutionary force is the intended, then and there social force of saying or doing something. Any participants' illocutionary force is dependent on their position and their rights and duties afforded or constrained by that position: as such, illocutionary force and position are mutually determining (Harré & van Langenhove, 1999). Illocutionary force was later contrasted with perlocutionary force which is the effect or consequence of saying or doing something (van Langenhove & Harré, 1999). In social life, such as in a classroom, the same words, gestures, or symbols may have a variety of meanings and a differing social force depending on who is using them, where, what for, and why. For instance, in a classroom situation the illocutionary force of a teacher commending students for clear explanations of their answer could be to praise or compliment. The perlocutionary force depends on how the students interpret the compliment. Students could feel proud of their explanations or they could feel embarrassed at being singled out. The compliment takes on a force and becomes significant only when it is interpreted and given meaning by others and accepted as the performance of a specific social act (Harré & Moghaddam, 2003; van Langenhove & Harré, 1999). The assumptions people make as to the integrity or duplicity of the social episode in which they are engaged can have a profound influence on what they say and do.

Positioning theory promotes the analysis of talk, text, and actions to interpret and understand the changing positions people take and are given when attempting to make their talk, text, and actions meaningful to themselves and others. Positions, storylines and social acts form the positioning triangle, which is an analytic tool that can be flexibly used to characterise the "shifting responsibilities and interactive involvements of members in a community" (Linehan & McCarthy, 2004, p. 441). Section 4.4 reviews the findings from empirical educational research investigated through positioning theory.

4.4 Educational Contexts

Educational phenomena researched through a positioning theory have included the binary positions of powerful and powerless (Davies & Hunt, 1994), the positioning of students as the kinds of people that succeed or fail (Anderson, 2009), the qualitatively different opportunities students have to participate due to teacher positioning (Yamakawa et al., 2005), the inflexibility of some rights and duties that come with certain student positions and genders (Barnes, 2003, 2004; Evans, 1996; Ritchie, 2002), and teacher and student positions of authority (Herbel-Eisenmann & Wagner, 2010; Wagner & Herbel-Eisenmann, 2009, 2013). This list does not include all educational phenomena researched through positioning theory. Contexts such as online learning were not included because they were not as relevant to this study. However, it does provide a comprehensive account of the types of educational contexts explored through positioning theory. In the following sections the focus of the empirical research is introduced, the participants and research settings are defined, and outcomes of the positioning theory research described.

Students identified as being positioned as powerless and disadvantaged were the focus of Davies and Hunt's (1994) classroom-based research. Davies and Hunt sought to make the binary of teacher and student more visible, challenge and reverse the value of those positions, and where possible remove the binary. Emphasis was given to understanding the positioning of students identified as powerless and marginalised. Data included five videotaped reading lessons and interviews with students from Hunt's classroom practice. Also analysed were the discussions between the two researchers, Hunt as teacher participant and Master of Education candidate and Davies as co-researcher and supervisor.

The binary of power/powerlessness and its relationship to other binaries such as teacher/student, adult/child, and competent/incompetent student was examined. In the binaries focused on, the first (teacher, adult, competent) was perceived to be the privileged and autonomous position and the second (student, child, incompetent) was the disadvantaged and dependent position. Davies and Hunt (1994) identified a trend in their data whereby the first position was regarded as customary and the second, which was defined in terms of its (lesser) relationship to the first, was a deviation from the first that required a departure from "teaching-as-usual" (p. 391).

The discourse of teaching-as-usual is created and sustained by teachers and competent students. Competent students know how to behave, what to expect, and how to skilfully play the game of successful student. Such a classroom context is often taken for granted as the way things are done in classrooms and not visible to other participants (Davies, 1982). Students who disrupt this order are perceived to be problem students, and become marked, marginalised members of the classroom. The problem lies with the students because they are unable to play the game of the classroom and therefore cannot function within the classroom norms. Neither teachers nor competent students have unequivocal or permanent power. Nonetheless, the "marked positions" of bad or incompetent student means it is near impossible for those students to position themselves or to be positioned by others "as having power or being able to act in powerful or agentic ways" (Davies & Hunt, 1994, p. 405). Incompetent students are positioned reflexively and interactively as not having the words or actions that are legitimate within the community.

Teacher knowledge and authority were found to limit positions made available to students (especially students who viewed themselves as powerless in classroom interactions) because of preconceptions regarding what success and competence looked like (Davies & Hunt, 1994). Limiting available positions meant some students chose their lesser status and became locked into "repeated patterns of powerlessness" (Davies & Hunt, 1994, p. 389). Also limiting was how students positioned each other. Negative positioning by students of each other that went against the instructions of the teacher was found to be more powerful and sustaining than positioning by the teacher. Competent students, who saw their positioning being compromised because teachers wanted less competent students included, refused to allow such positioning. They were unwilling to become marked as incompetent through the positioning. This action reiterated and reinforced to the marked incompetent students that they were outsiders and unsuccessful members of the class. Davies and Hunt (1994) posited that most students would choose to be acknowledged as competent. For this to happen they recognised that in some classrooms a disruption to the dominant discourse of teaching-as-usual and the deconstruction of the position of the "teacher-as-one-who-knows" would be needed (p. 406). No actual examples of this are provided but the premise is that when teachers treat difference with high regard and something interesting to be listened to, the students contributing the difference become part of the norm.

Two storylines were evident in the research of Davies and Hunt (1994). First, students who were successful within the teacher accepted ways of doing and learning mathematics were considered behaviourally competent. Conversely, students who approached their mathematics in ways different to the teacher approved methods were considered disruptive. The second storyline related to the power differences available to the two groups of students. Competent students were positioned as powerful, and disruptive students as powerless. As the teacher participant in her own research, Hunt attempted to create a new storyline whereby behaviours different to those normally seen within "teaching-as-usual" were increasingly accepted by all students.

Two other new storylines developed because of Hunt's actions. The first was created by the students who positioned themselves as competent. Students positioned as competent did not accept the novel behaviour of others and continued to position others as disruptive and not belonging to their view of an acceptable classroom (Davies & Hunt, 1994). The other storyline was created by the students positioned as disruptive or incompetent. Students positioned as disruptive chose to accept their negative positioning because the task of changing their positioning was perceived to be impossible. The actions of the powerfully positioned participants and the marginalised students became social acts. The teachers' and competent students' words and actions became taken-as-shared and the less competent students' positions were also accepted within the group.

Students in both groups attempted to sustain their positioning of competent and incompetent, and their words and actions focussed on that sustainment became social acts.

Moving beyond the binary of powerful/powerless, Anderson (2009) found that positioning could historically and culturally locate a student as a "certain kind of person" (p. 293). The kind of person the student was positioned as included success or failure. Fifth grade students (aged 10 & 11) and their mathematics teachers from three at-risk schools in mid-western U.S.A. public school classes participated in Andersons' (2009) research. Over a 14 week timeframe, students were observed and videoed working individually on mathematical problems, discussing the problem in small groups, self-evaluating the quality of the group discussion, and participating in the whole-class plenary to conclude the lesson. This cycle of activities was very different to students' usual mathematics learning routine which more often entailed working independently from text books or working with the teacher at the blackboard. Four students were purposefully selected and analysed because of their "animated discussions and contested evaluations of participation" (Anderson, 2009, p. 295).

The "kinds of person" (Anderson, 2009, p. 292) one is or is able to be depends on the resources for meaning-making (words, actions, tools, interactions with others) available to us, what we see ourselves as being able to do with our resources, and what others allow us to do with our resources. Therefore, certain kinds of people are not always available or accessible to all participants. How quickly any student is recognised as any particular 'kind of person' is dependent on local contextual factors such as the teacher's and students' histories and experiences with each other and biographical data such as gender, ethnicity, socio-economic status, and age. A student defined in one classroom as a quiet kind of person could be positioned as smart with a different teacher or unconfident with another (Anderson, 2009).

The positioning of students as a success or failure by their teacher or peers was influenced by more than the context in which the positioning occurred. As well, positioning as success or failure was influenced by interwoven experiences of "what came before, what happened now, and what happened after" (Anderson, 2009, p. 301). Also influencing the positioning of a student as a success or failure was the acceptance or rejection of the positions by the participant and others. For example, Anderson (2009) found that when the label of failure assigned to one student was shared by his teacher and peers, the "stickiness of that label grew" (p. 306). A ripple effect continued to construct the student as a failure, and limit his resources and opportunities to shake the label of the kind of person that fails. In a similar recommendation to that of Davies and Hunt (1994), Anderson (2009) advocated for classrooms where norms that limited student opportunities and positions were challenged.

Anderson (2009) revealed a dichotomous storyline that developed according to the ways in which students were positioned as certain kinds of student. Students positioned as successful had greater rights than those positioned as failing. A second storyline showed that once positioned as a certain kind of person, the contexts and interactions that led to the initial positioning continued to reinforce it. If a storyline positioned a student as succeeding or failing, the student stayed that way and his or her label grew. Similar to Davies and Hunt's findings (1994), the social acts that became significant in Anderson's (2009) research were the words and actions of the students positioned as successes or failures and the teachers whose words and actions confirmed those positions. In Anderson's research, historical words and actions also became social acts as they continued to reinforce the positioning of students.

In two studies, Barnes (2003, 2004) researched the kinds of people students can be within mathematics lessons in three senior classes (ages 16 to 18) in Melbourne, Australia. Fourteen positions were identified: Manager, Helper, Facilitator, Humourist, Spokesperson, Expert, Outside Expert, Critic, Collaborator, In Need of Help, Outsider, Entertainer, Audience, and Networker. Ten mathematics lessons were observed, videoed, recorded, and transcribed over a period of three weeks. Additional data included interviews, field notes, and examples of students' work. Each position was given an empirically-observed behavioural description that inferred the rights and duties associated with the position: "Manager: initiates work, invites ideas, interprets instructions, gives orders or makes suggestions about who should do what, or how they should tackle the task" (Barnes, 2003, p. 3). Some positions were considered to be more desirable such as Expert and Collaborator. Less desirable positions included Entertainer, Networker, and Outsider. Similarly to Anderson (2009), Barnes (2004) warned that the "exclusive occupancy of any position by one individual may have negative consequences for both group and individual" (p. 14) because with each position came a set of fixed rights and duties. A student who is always positioned as Expert may dominate the group, inhibit the opportunities for others to be similarly positioned, limit the discussion to one point of view, or limit the experiences others can have in articulating their ideas. Students' opportunities may be restricted or they may restrict others because of the rights and duties that are assumed and imposed on them through that positioning (Harré & van Langenhove, 1999).

Barnes (2003, 2004) identified a storyline whereby the longer a student was positioned in a less desirable position such as Entertainer, Outsider, or Networker, the more difficult it became for them to move to a more desirable position. As with Anderson's (2009) findings, the adhesiveness of the label limited students' and their peers' rights and duties and their opportunities to learn. In Barnes' (2003, 2004) research the words and actions that sustained a student's position were the social acts. These words and actions could be advantageous or disadvantageous to the student depending on the desirability of the position. Advantageous and disadvantageous social acts had a comparable social force as neither could be easily challenged or changed.

A further study by Yamakawa and colleagues (2005) determined that it was the teachers' duty to position all students as having the right to contribute to their own and others' learning. Two nine-year-old students from a private elementary school in north-eastern U.S.A. were the focus of this research within an inquiry-based mathematics classroom. Findings showed that because of the teacher's interactive positioning of these two students, they had qualitatively different opportunities to participate in the mathematics learning. The teacher's positioning of the students also created qualitatively different identities for the students as learners of mathematics. One student for instance, provided explanations of mathematical thinking that fitted with the norms of the classroom. This student was positioned by the teacher as a mathematical thinker to whom other students should pay attention. The teacher regularly revoiced this student's thinking and made comments to others in the group such as “this is something some of you might want to write down” (Yamakawa et al., 2005, p. 9). Revoicing occurs when teachers repeat, rephrase, or expand an explanation (O’Connor & Michaels, 1996). When used appropriately, such as in this example, revoicing can be used to fine-tune mathematical thinking, clarify or highlight constructive ideas, increase accessibility to the ideas, or move the discussion in a more productive direction.

Another student in Yamakawa and colleagues (2005) study used mathematical strategies and provided explanations that were considered to be advanced and beyond the sociomathematical norms of the classroom. This student was positioned outside the community and was seen as having advanced strategies that were not useful for others. The teacher’s comments included “That’s the way a lot of your parents would do it. That doesn’t mean it’s the right way. And it’s a very confusing way to a lot of people” (Yamakawa et al., 2005, p. 10). The rights and duties of these two students within their interactive positionings could be considered privileged, but one position allowed for significantly more student participation.

The teacher was responsible for creating two different storylines for two similar students in Yamakawa et al’s (2005) research. The storyline depended on whether the students’ mathematical thinking matched their teacher’s thinking. The storyline for the student whose thinking corresponded with the teacher’s entailed being praiseworthy, having a valuable contribution to make to the group, and being worth paying attention to. This student’s thinking was given a social force because it was acknowledged and appreciated by the teacher. The storyline for the second student, whose mathematical thinking was different to the teacher’s, was that although his idea was good it was not valuable to the group; it was old-fashioned, confusing, and whilst correct, not necessarily the right way to approach the problem. This student’s thinking had the social force of what not to do and as such also became a social act significant to the group. Through these different storylines and associated social acts the two students concerned also had very different opportunities to be heard and to contribute to the group’s learning.

Student gender was identified by Evans (1996) and Ritchie (2002) as a complex influence over the rights and duties that girls and boys give to themselves and others. Participants in Evans' (1996) research were members of one peer-led literature discussion group in a 5th grade class (aged 11 & 12) in a multicultural school in Wisconsin, U.S.A. The group comprised three boys and two girls. Observations, including video and audio recordings, took place over a two-week period. Evans found that the boys in the group marginalised the girls by positioning themselves in the more powerful position of helper and the girls to the position of needing to be helped. The boys achieved this positioning through two strategies. First, they positioned themselves as having the right to tease and the girls as having the duty to accept being teased. Their second strategy was to block the girls' attempts to stop or disrupt the teasing and to thwart the girls' attempts to reposition themselves. One girl repeatedly rejected the boys' marginalisation, so they became more aggressive with their teasing toward her. However, as she had been positioned as powerless by the boys, this girl was never able to disrupt the positioning and reposition herself as having rights within the group. The other girl accepted the marginalisation and was consequently left alone by the boys. There was no reported evidence in Evans' (1996) research of boys marginalising other boys.

The focus of Ritchie's (2002) research also considered the differing rights and duties afforded to girls and boys working in a group. Data for this research, including videoed lessons and post-lesson interviews with groups and pairs of students, were taken and re-examined from previous research (Roth, Tobin, & Ritchie, 2001). Participants included three groups of 6th grade (aged 12 & 13) students from New York, U.S.A. — one mixed gender, one group of boys, and one group of girls — engaged in science tasks and discussions. Ritchie (2002) identified that the mixed gender group was unable to complete the science tasks because of the different rights and duties afforded to the girls and boys. Initially, the two girls who participated in the same and mixed gender groups blamed the boys for the lack of productivity for the mixed gender group. The girls claimed a "good student storyline" (Ritchie, 2002, p. 45) for themselves and accused the boys of thinking they were too hopeless because "we were girls and they were boys and they thought boys were better than girls" (Ritchie, 2002, p. 51). However, analysis of the girls' only group showed that the two girls were responsible for the lack of task completion. In the girls only group these two girls acknowledged that they thought co-operation meant they had a right to have all their ideas accepted without challenge and admitted they could be very bossy. Ritchie (2002) advocated positioning theory as a lens for going beyond one level of analysis. Had Ritchie not sought to further understand the positioning, he may have accepted the discourse of male dominance claimed by the two girls. Through positioning theory, Ritchie (2002) was able to “make visible that which is usually invisible to teachers and researchers” (p. 35).

The storylines and social acts in Evan’s (1996) and Ritchie’s (2002) research differed depending on the gender of the students and the depth to which understanding of the positioning was sought. The storyline in Evans’ (1996) research was that the boys had more rights and the girls had more duties; therefore, the boys’ social acts had more authority. If girls tried to disrupt their less desirable positioning the boys became more aggressive in their teasing and positioning. The second storyline was that submissive girls would be left alone but assertive girls would be badgered until such time that they accepted their positioning. With more in-depth analysis, Ritchie (2002) noted that the “good girl” storyline claimed by the two girls was, by their own admission, not completely true.

Positioning theory provided the framework for Wagner and Herbel-Eisenmann’s (2009, 2013) three-year longitudinal case study research which focussed on teacher authority in mathematics lessons. Teacher authority included the teacher’s content knowledge and their position as teacher. Wagner and Herbel-Eisenmann’s (2013) research traced the changes in one teacher’s authority from his position as the only mathematics teacher in a rural high school to one of many at a large urban high school in Atlantic, Canada. Student participants were Year 9 to 12 (aged 15 to 18) mathematics and science students. Data included interviews with the teacher and students, and video and audio-recorded consecutive lessons.

Authority was categorised in four ways (Wagner & Herbel-Eisenmann, 2013): first, “personal authority” which was usually flagged with the words “I want you to” and implied students should rely on their teacher (p. 483); secondly, disciplinary authority indicated by “we need to” and “we have to” and suggesting students follow the rules of (school) mathematics (pp. 483-484); thirdly “more subtle discursive authority” which implied a “sense predetermination” and included statements such as “we are going to” (p. 484); and The fourth categorisation of authority was uncommon and required “personal latitude” whereby students made decisions and had authority (p. 484).

The researchers agreed that students should develop their own mathematical authority. Their dilemma was what form of authority and how much authority the teacher should cede for this to happen. At the smaller high school, the teacher described his frustration with students’ over-reliance on him or textbooks as the source of mathematical knowledge, their lack of initiative, and their preference for being told what to do. However, when examining the teacher’s practice, Wagner and Herbel-Eisenmann (2013) found that his words did not complement his actions. Many of the directions given by the teacher demonstrated his personal authority because he asked “students to do things without giving reasons for them to do these things” (p. 488). The discipline of mathematics was given authority when the teacher told the students to follow the rules. Mathematics was positioned as being predictable through the teacher’s statements such as “so we’re going to get 4188” (p. 488). Students’ authority or personal latitude was rarely observed because the teacher positioned himself to answer questions and clarify misconceptions.

Teachers’ positioning of themselves and their students at the larger high school differed. The difference occurred because as well as teaching mathematics and science, the teacher also taught students to accept more authority within the mathematics lessons and taught himself to have less. Students positively responded to their positioning of personal latitude and were soon asking questions, seeking alternative strategies, demanding clarifications, and providing answers and explanations (Wagner & Herbel-Eisenmann, 2013).

Wagner and Herbel-Eisenmann (2013) concluded that there was a difference in how teachers were positioned for authority in mathematics lessons. Teachers could be “an authority in mathematics” and they could be “in authority” (italics in original, p. 491). The storyline for teachers who are an authority is that they have the required content knowledge to facilitate learning; teachers in authority have power within the lesson to decide what happens, when, and how. The problematic storyline for students is that unless they have opportunities to be in authority of their learning they may not become an authority. For teachers and students as an authority or in authority the storylines and associated social acts differ.

4.5 Summary

Positioning theory provides a comprehensive framework for examining and understanding the interactions that occur within teacher-led ability-based mathematics groups. The emphasis in positioning theory is on the positions from which teachers and students engage in intentional and authentic interactions that occur naturally between groups of people. The talk, text, and actions of teachers and students is purposeful, co-constructed, and unscripted. Teachers and students have changing positions within interactions that could include explainer, comprehender, challenger, or defender. The extent to which they can, or have to explain, comprehend, challenge, or defend is dependent on the rights and duties they have within the group. Those rights and duties are afforded or constrained by the past, present, and future interactions of the group. The past, present, and future interactions of the group establish its institutional practices, that is, what is expected and accepted within the group.

Each feature of a social episode has a mutually determining relationship with the other and a change in one feature can guide or disrupt interpretation of the other (Moghaddam et al., 2008). Participants' presumed, adopted, or ascribed positions influence the developing storyline and their social acts. Positions may afford or constrain participants' social acts within a storyline as to what is considered possible, proper, or required (Harré & Slocum, 2003). What participants say and do and the illocutionary force of their words and actions is influenced by the positions they are in and influences the path of the storyline (Harré, 1991). The rights and duties assumed or given by those positioned in the storyline can constrain or extend it. Storylines within the social episodes provide the framework for, and are influenced by, positions and social acts by providing clues about the availability and appropriateness of positions and guiding the interpretation of actions as social acts (Slocum & van Langenhove, 2004). A change in the storyline affects both position and social acts. Storylines can be altered through the presence or absence of particular positions, participants challenging the positioning of the first converser, and by means of their own positioning giving the storyline a new twist (Harré, 1991; Slocum & van Langenhove, 2004). An altered storyline can affect the initial social force of a social act and therefore shape the conversation and drive the outcome in a different direction. Social acts can take on different meanings depending on the storyline in which they occur and the rights and duties of those who utter them.

In the next chapter I discuss the methodology of my study. Included in this discussion are the research questions, the qualitative research paradigm, and reasoning behind the methodological choice of case study. Details of research settings and participants and data sources are provided. The processes for data analysis and ethical considerations within this study are also explained.

Chapter Five: Methodology

> The "something" that qualitative research understands is not some set of truisms about communication but the awful difficulties groups face in mapping reality. The qualitative researcher is an explorer, not a tourist. Rather than speeding down the interstate, the qualitative researcher ambles along the circuitous back roads of public discourse and social practice. In reporting on that journey the researcher may conclude that some of those paths were, in fact, wider and more foot-worn than others, that some branched off in myriad directions, some narrowed along the way, some rambled endlessly while others ran straight and long, and some ended at the precipice, in the brambles, or back at their origin. (Pauly, 1991, p. 7)

5.1 Introduction

This research began with an intrinsic interest in why some students are not achieving in mathematics and a concern for those students for whom mathematical understanding and success is unrealised. In particular, I was interested in how teachers position themselves and the students in their lowest and highest mathematics groups in ways that afford or constrain the sharing of mathematical know-how.

The key research question and supporting sub-questions developed throughout this study are presented in Section 5.2. Section 5.3 outlines the qualitative research paradigm underpinning this research. In this section I reviewed the ontological epistemological, and methodological premises I bring to my research. Case study, the methodological premise selected for this research, is discussed in Section 5.4.

The research setting is outlined in section 5.5 and includes the reasons behind selecting Tasman and Pacific Schools (pseudonyms for the participating schools), a description of both schools, and of the participating teachers within each school. Pseudonyms have also been used for participating teachers and students. Section 5.6 describes the sources of data gathered.

The process describing the analysis of data is explained in Section 5.7. Section 5.8 explores the ethical considerations and implications of qualitative classroom-based case study research and explains how I remained ethically true to myself, the participants, and the research throughout this study. Ways to enhance and ensure the trustworthiness of my findings are considered in Section 5.9.

5.2 Research Question

The key research question this study addresses is:

> How do teachers in New Zealand primary schools position themselves and students in their lowest and highest mathematics strategy groups so that mathematical know-how can be shared?

The sub-questions underpinning the key research question are:

1. What acts of teacher positioning with the lowest and highest strategy group afford or constrain the sharing of teacher and student mathematical know-how?
2. What storylines are created by each teacher and group when shared know-how is afforded or constrained?
3. What social acts become significant for each teacher and group when shared know-how is afforded or constrained?
4. What impact could positioning have on student individual and shared learning

This study examined the positionings teachers select for themselves and their students and the effects such positioning have on opportunities for shared mathematical know-how. The positionings students select for themselves, their peers, and teachers are recognised within the findings but were not the focus for this research.

5.3 The Qualitative Research Paradigm

A research paradigm is the “basic belief system or world view that guides the investigation” and provides the basis for interpreting and understanding social reality (Cohen, Manion, & Morrison, 2000, p. 9). The objective of qualitative research is to understand the meanings of an experience in depth, within its context, from the participant’s perspective, and with as little disruption to the natural setting as possible. Meaning in qualitative research is achieved by “studying things in their natural settings, attempting to make sense of, and interpret, phenomena in terms of the meanings people bring to them” (Denzin &

Lincoln, 2005, p. 3). Sense is made of a social phenomenon by the emphasis the researcher places on interpretation and through the researcher "contrasting, comparing, replicating, cataloguing, and classifying the object of study" (Creswell, 2003, p.198).

In this study a qualitative research paradigm was used to examine teachers' acts of positioning, to reason about those positionings, and to interpret relationships and consequences between positioning and shared mathematical know-how (Corbin & Strauss, 2008; Davies & Harré, 1990). Qualitative research methods were chosen because of their capacity to emphasise contexts, meanings, and individuals' interpretations. The context of this research was naturalistic because the major data collection was undertaken during normal teaching situations.

The personal biography of the researcher, his/her beliefs, perspectives, principles, and premises are defined within a research paradigm. Researcher premises shape how they see the world and act in it and guide the researcher's actions from selection of research topic, to research methods, to writing the final report (Denzin & Lincoln, 2011). Guba and Lincoln (1994) proposed three premises linked to the personal biography of the researcher: ontology, epistemology, and methodology.

The characterisations given to these premises in this research are adapted from, and correlated with, their use in social and qualitative research and positioning theory (Crotty, 1998; Harré, 1997; Harré & van Langenhove, 1991; Ponterotto, 2005). The ontological and epistemological beliefs and methodological position I bring to this research are underpinned by, and include, my gender, racial, cultural, ethical, and socioeconomic perspectives (Denzin & Lincoln, 2011). What I believe about reality defines what I understand as legitimate knowledge and how I obtain that knowledge, which in turn influences how I go about the research and what techniques I apply.

5.3.1 Ontology, epistemology, and methodology

Within social research, ontology and epistemology sit alongside each other and inform the theoretical perspective of what is, and what it means to know (Crotty, 1998). Ontology is concerned with the nature and reality of being and addresses such questions as “what is the form and nature of reality and what can be known about that reality” (Ponterotto, 2005, p. 130). Within a positioning theory framework ontology is considered to comprise the talk, text, and actions people use in their everyday lives (Harré & Secord, 1972; Slocum & Van Langenhove, 2004). Epistemology is “a way of understanding and explaining how we know what we know” (Crotty, 1998, p. 3). Questions such as what is knowledge, how is knowledge acquired, what kinds of knowledge are possible or authentic, and what is the relationship between the research participant (knower) and the researcher (would-be-knower) are posed within an epistemology (Crotty, 1998; Guba & Lincoln, 2005; Ponterotto, 2005).

In Chapter Two, the theoretical perspective inherent in this study, social constructivism, was outlined. In terms of ontology social constructivists believe multiple possible realities can be constructed and refute the notion of a single true reality. The researcher is an explorer looking for new understandings rather than a tourist who is there to notice what already exists (Pauly, 1991). The nature of reality in this research was constructed through the talk, text, and actions between the participants, the experiences and responses of participants, the participants and myself, and our interactions (Davies & Harré, 1999; Ponterotto, 2005). The participants and I brought personal and cultural resources to the research including our own and others’ actual and remembered talk, text, and actions. We all drew on personal and cultural resources to construct our realities. As such, different and layered experiences and interactions can result in “multiple meanings of a phenomenon in the minds of people who experience it” (Ponterotto, 2005, p. 130). I did not attempt to unearth a single reality about teachers’ positioning and shared mathematical know-how. Instead the approach was to collect open-ended, emerging data with the intent of developing a theory or pattern of meaning through close observations, careful documentation, and thoughtful analysis of the research questions (Creswell, 2003).

Social constructivism embraces the individual and collective talk, text, and actions of people as an epistemology, a way of knowing (Ernest, 1998). Knowledge is the individually and collectively, constructed and shared, ideas and meanings, of people interacting together and with their environment (Ernest, 1994, 1996). Individual knowledge is fashioned, restructured, and reinforced by the relationships of the group in which it is created. Collective knowledge is the result of individuals contributing and reorganising their mathematical understandings within the group. As such, different people may construct different knowledge in different ways.

A social constructivist epistemology recognises that “the dynamic interaction between researcher and participant is central to capturing and describing the lived experience of the participant” (Ponterotto, 2005, p. 131). A relationship existed between me and the teachers and students in this research as I was the NDP advisor for both schools from 2004 to 2006 when they completed their NDP professional development programme. Both principals agreed teachers felt more comfortable about sharing and discussing their practice because of that relationship. The good relations and sense of rapport between researcher and participants was seen to lead to feelings of trust and confidence and it was considered that there was less chance of teachers teaching in staged or artificial ways (Hitchcock & Hughes, 1989). Teachers discussed how they saw the study as an opportunity to personally and professionally benefit both as a learner and a teacher, and as de Vaus (2001) found, “people are remarkably generous and willing to participate in studies where they believe it will do some good” (p. 84).

The existing position of membership within both schools meant I could not claim to have complete objectivity (Angrosino & Mays de Perez, 2003). However, I chose to take a non-participant observer role in the observations because I wanted events to be influenced by teachers and students. To remove myself from the observed activity I did not interact with either the group being videoed or other people in the classroom.

Methodology refers to the theoretical and philosophical processes and procedures of the research (Denzin & Lincoln, 2011). The methodological premise behind a research paradigm includes the assertions, models, and notions that explain how data may be best interpreted so that new knowledge may be discovered. Assertions, models, and notions emanate from the researcher’s position on ontology and epistemology. The bounded and socially situated nature of this research within the highly subjective social phenomenon of teaching and learning meant a qualitative case study was an appropriate methodological choice. To follow, the reasons behind this choice and the potential misunderstandings or over-simplifications of case study research are discussed.

5.4 Case Study

A case study is embedded in the qualitative research paradigm in that it "makes the world visible in different ways … transforms the world … [and hopes] always to get a better understanding" (Denzin & Lincoln, 2005, pp. 3-4). Understanding is sought from finite data collected and analysed primarily by the researcher within an inductive and iterative investigation (Merriam, 2009). Case study research is exploratory and can resonate with the reader's own experiences and existing understandings, provide insights into how things became the way they are, and generate discoveries of new learning. The end product of a qualitative case study is a "rich, thick description of the phenomenon under study" (Merriam, 2009, p. 43). Merriam (2009) defined case study as "an intensive, holistic description and analysis of a single entity, phenomenon, or social unit" (p. 46). This definition and the work of Merriam (1998, 2009) guided this research.

The principal characteristic of case study is the intrinsically bounded and particularised object of study — the case (Merriam, 2009). The case can be an individual, programme, event, group, institution, or community. It must be bounded and particularised so that what is, and is not, to be studied is clearly defined. By concentrating on an explicit, real-life situation, case study seeks to gain an in-depth understanding of that phenomenon and increased meaning for those involved.

There are two cases in this research. The first is the positioning acts of seven teachers with their lowest and highest mathematics groups that consistently resulted in mathematical know-how being shared or constrained. The second case is the inconsistent positioning acts of five teachers. These two cases are presented as the Findings, Chapters Six and Seven. The cases are bounded and particularised for three reasons: first, because of my intrinsic interest in teacher positioning that affords or constrains the sharing of mathematical know-how; secondly, because of the focus on teachers' mathematics positioning practices;

and thirdly, by the curriculum focus on mathematics. The data were delineated by limiting the observations of each teacher to their lowest and highest strategy groups and three consecutive mathematics lessons with each group. Each case is a rich, thick description of the lived experiences of the participants that illustrates and describes the phenomenon of teachers and positioning in mathematics.

As a qualitative method of research, case study has been said to survive within a "curious methodological limbo" because it is widely used by many but held in low regard by others (Gerring, 2004, p. 341). Flyvbjerg (2006, 2011) identified five misunderstandings about case study that can undermine its use and credibility. The five misunderstandings are as follows:

1. General theoretical knowledge is more valuable than concrete case knowledge.
2. One cannot generalise on the basis of an individual case; therefore, the case study cannot contribute to scientific development.
3. The case study is most useful for generating hypotheses; that is, in the first stage of a total research process, while other methods are more suitable for hypothesis testing and theory building.
4. The case study contains a bias toward verification, that is, a tendency to confirm the researcher's preconceived notions.
5. It is often difficult to summarise and develop general propositions and theories on the basis of specific case studies. (Flyvbjerg, 2011, p. 302)

Whilst these misunderstandings tend to emanate from a quantitative methodological perspective, they are still important to consider. Case study allows for contextualised frameworks that acknowledge the relationship between participants and the related circumstances relevant to the study (Cohen, Manion, & Morrison, 2007). As such, knowledge generated through case can be both theoretical to begin with and become more concrete through analysis. The context dependent knowledge created through case study is as valuable as universal context-free findings (Flyvbjerg, 2006).

Case study generates new thinking that has validity not entirely dependent upon the cases from which it is drawn. New thinking can be applied to, or used, to appraise other studies. It is primarily the responsibility of the researcher to select both a case and a case study method from which generalisations and transformations of knowledge can develop. However, some of the responsibility of generalising from any research lies with the reader because they determine what resonates with their own experiences and existing understandings (Merriam, 2009; Stake, 2012). Some may know of teachers and students who were positioned in similar ways and may be able to contextualise research findings to teaching and/or learning situations of their own (Merriam, 1998). It is within that contextualisation to other situations that summaries and generalisations can occur and that the reader can make the findings more personally meaningful.

Through case study research, hypotheses can be both generated and tested. The reason for this is that knowledge is recognised as having more than statistical significance. Hypothesis can be generated at any stage of the case study analysis and tested against its own or other case study findings. The inclusion of deviant, unusual, or problematic cases and quantitative data within the case study can also provide sources of theory development and hypothesis testing (Flyvbjerg, 2011; Merriam, 2009).

Qualitative research recognises that each researcher brings their own unique perspective to the study (Merriam, 1998). Case study researchers in particular constantly make judgements about the significance of their data, what to observe, include, analyse, and report. Whilst researchers are responsible for minimising any bias they may bring to the research, it should also be noted that the personal qualities of researchers combined with their data may be seen as virtuous (Merriam, 1998). The researcher bias could enhance the quality of the case study research and findings. I sought to reduce possible negative influences of researcher bias by presenting evidence from the data to support my findings and engaging a peer debriefer to review my perceptions, insights, and analyses.

In many ways the suggested limitations of case study research are also its strengths. An emphasis on difference, ambiguity, intuitiveness, subjectivity, perspective, complexity, and integrity could be seen as a limitation or strength, depending on the questions asked and their relationship to the end product (Merriam, 1998). All research, be it quantitative or qualitative, should be driven by the problem, the unit of analysis and not the methodological choice (Flyvbjerg, 2006; Merriam, 2009).

5. 5 The Research Settings

This section describes both the schools and participating teachers. The school roll, ethnic make-up, gender, decile rating1, organisation, and timetabling of mathematics teaching and learning of each school are tabled and discussed. The teaching experience and NDP professional learning and development and teaching experience of each teacher are presented as tables.

5.5.1 The schools

A purposive sampling approach (Merriam, 1998) was used to select the two schools which were from a similar geographic region. The schools were recruited to participate in this research study because of their commonalities and differences. Commonalities included both schools having participated in the NDP professional development for three years with me as their advisor, both schools ability group their students according to the students' numeracy strategy and knowledge assessment results, and both follow the NDP recommended organisation for teaching mathematics (Ministry of Education, 2007c). Differences included one school having a static staff with the other experiencing a total turnover of staff and principal in the three years preceding this study, the decile ratings of each school, the socioeconomic and ethnic backgrounds of students. By purposefully selecting schools with commonalities and differences, the findings from this case study are more likely to be found in other New Zealand schools, thus supporting the transferability of this research (Denscombe, 2010).

The descriptions of Tasman and Pacific School, both contributing schools,2 are presented in the following table.

[1]Every state school in New Zealand is given a decile rating from 1-10 by the Ministry of Education. A school's decile rating indicates the extent to which the school draws its students from low socioeconomic communities. For example, decile 1 schools are the 10% of schools with the highest proportion of students from low socioeconomic communities. These are called low decile schools. Decile 10 schools are the 10% of schools with the lowest proportion of students from low socio-economic communities. These are called high decile schools.

[2] A school which has only primary students aged 5 to 11.

Table 5.1: Pacific and Tasman School Data

	Pacific School		Tasman School	
Decile	10		4	
School Roll	Stable 187		Fluctuating 100 – 130	
Full-time teacher equivalent	8		5	
Gender	Male	43%	Male	59%
	Female	57%	Female	41%
Ethnicity	New Zealand European	78%	New Zealand European	40%
	New Zealand Māori	15%	New Zealand Māori	26%
	Samoan	3%	Samoan	20%
	Fijian	0%	Fijian	4%
	Asian	3%	Asian	4%
	Other	1%	Other	6%
English as an additional language learners	4%		20%	

Pacific School was divided into three syndicates. These were the junior school students (new entrant to Year 2, aged 5 & 6); middle school students (Years 3 & 4, aged 7 to 9), and senior school students (Years 5 & 6, aged 10 & 11). Mathematics was usually taught before lunch (Years 1 to 6) and after lunch in the new entrant class.

Tasman School was organised in two syndicates. These were the junior school students (new entrant to Year 3, aged 5 to 7) and senior school students (Years 4 to 6, aged 8 to 11). Mathematics teaching and learning usually occurred before lunch for the Year 2 to 6 students and after lunch for the new entrant and Year 1 students.

5.5.2 The participants

The following table outlines the designated position of each teacher, their years of teaching experience, year group taught, and NDP experience. Pre-service NDP experience occurred whilst the teacher was completing their teacher qualification and in-service whilst teaching at a school. The demographic data of the 12 teachers is combined to enhance their anonymity.

Table 5.2: Teacher Demographics

Name	Designated Position	Years Teaching	Year Group Taught	NZ NDP Experience
Jenna	Deputy Principal	24	New entrant	In-service
Brooke	Deputy Principal	15	New entrant	Nil
Lisa	Teacher	1	1	Pre-service
Delphi	Teacher	5	1 and 2	Nil
Sheridan	Teacher	3	2	In-service
Naomi	Teacher	20	2 and 3	Nil
Hannah	Teacher	6	2 and 3	In-service
Faith	Teacher	23	4	In-service
Greer	Teacher	2	4 and 5	Pre-service In-service
Chelsea	Teacher	2	4 and 5	Pre-service
Paula	Assistant Principal	11	5 and 6	In-service
Kendra	Teacher	11	5 and 6	In-service

The primary school year in New Zealand is from late-January to mid-December and is divided into four 10-week terms. It was agreed not to gather any data during the first term to allow teachers and students to develop relationships and establish classroom routines. Data collection began in week five of the second term in 2007 at Pacific School and in week seven at Tasman School.

5.6 Data Sources

The in-class observations needed to be a typical and true representation of what normally occurred during number strategy lessons in these classrooms. To retain the naturalness of the setting as much as possible I timetabled the observations to when mathematics usually occurred for each class and established observation protocols with teachers and students. The usual placement of teachers and their strategy groups within the classroom was discussed and the best placement for the video camera determined. Recording guidelines were established within each class such as not walking in front of the camera and not touching the audio recorder placed in front of the teacher. In classrooms where the teacher or students were not familiar with me, we practised recording a lesson.

Each teacher was video and audio recorded for three consecutive lessons teaching their lowest and highest strategy stage group, resulting in 72 lessons observed and transcribed. The three consecutive lessons with each group took place within the same two week timeframe for the teachers in both schools.

Interviews with teachers or students were not conducted as part of data gathering; the danger in asking questions is that participants could have been inadvertently positioned through the questions in ways that influenced their teaching decisions (Partington, 2001). Standardised procedures for video and audio recording the lessons were developed and followed for each of the 72 lessons. One camera was used in each classroom; it was placed on a tripod and maintained a wide angle shot of the teacher and group. Schoenfeld (1988) found that a focus on the teacher is often sufficient as it explains a significant proportion of what takes place in the classroom. I found this to be true for my research. The choice to use both video and audio recordings was made to provide two sources of evidence; if one source was not clear at any point the other could be relied on.

Knowing they are to be videoed, teachers may try to do an especially good job or may do extra preparation for the lesson. Whilst concerns of teachers giving a performance are valid, teachers are also constrained by what students expect and by their own repertoire of teaching practices (Hiebert, Gallimore, Garnier, Givven, et al., 2003). Students may also perform (more positively or negatively) for the camera but as I was known to many of the students my presence in the room again was commonplace. As noted in Section 5.3.1, a relationship of trust and respect existed between me and the teachers and was believed to lessen any pressure the teachers felt to showcase best practice.

I accept that audio recorded lessons are best interpreted as “a slightly idealised version of what the teacher typically does in the classroom” (Hiebert, Gallimore, Garnier, Bogard, et al., 2003, p. 7). I acknowledge that I am offering only an extract from a larger performance, and also that to view the videos, or read the lesson transcripts is not the same as experiencing the actual lessons.

Three data sources were used in this research: observations, video and audio recordings, and archival records. A preliminary process of data analysis began as soon as data began to be collected. The reasons for, and purpose of each data source is discussed in the following section.

5.6.1 Observations

Observations are a core and important source of data in qualitative research. They occur within a natural setting and make it possible to record behaviour as it happens, thus representing a candid and first-hand record of the phenomenon of interest (Merriam, 2009). Increased understanding of the context can occur because through observations, normal and routine interactions and their meanings can become less ordinary (Wiersma & Jurs, 2009).

As each lesson was being recorded I completed observations as a running account. Observations included my emerging insights, hunches, and tentative hypotheses that provided me with the opportunity to refine and reformulate questions and document the work in progress (Merriam, 2009). The purpose of this was twofold. First, the observations provided me with the opportunity to note anything that was not likely to be picked up by the recordings. Secondly, they provided information I could correlate and compare with other data.

Written observations, including field and personal notes were recorded from five minutes before the lesson began to five minutes after the lesson ended. Field notes comprised a running account of anything that was happening that was not likely to be picked up by the video or audio recording and unsolicited comments from teachers that occurred before the recording began and after it was completed. Personal notes encompassed notes to myself regarding my feelings, impressions, and reactions. Theoretical notes were added after the observations and included my hunches, possible emergent categories, hypothesis, and trends. Field, personal, and theoretical notes were added to throughout the duration of the research, allowing me to "plot the progression of my thinking" (Gillham, 2000, p. 24).

The content of different notes often overlapped so I digitised my observations at the end of each data gathering day, separating them into the three categories of field, personal, and theoretical notes The emphasis for all notes was on describing what occurred in plain terms without evaluating and avoiding inferences, generalisations, vague terms, or catch phrases. When completed, notes were included in a lesson transcription table overview for each teacher.

5.6.2 Video and audio recordings

Video can provide the most inclusive and least intrusive way of accurately capturing what happens in classrooms. The capability of videotaping to record moment-by-moment unfolding events has made it a “powerful and widespread tool in the mathematics education research community” (Powell, Francisco, & Maher, 2003, p. 406). There are both advantages and disadvantages in the use of video to collect and analyse data (Herbert & Pierce, 2007; Merriam, 2009; Otrel-Cass, Cowie, & Maguire, 2010, Robson, 2011; Wiersma & Jurs, 2009). These are discussed next.

Moment-by-moment unfolding rich behaviour, complex interactions, subtle nuances, non-verbal communications, insights into emotions, and depth of understanding of concepts can be captured through video. Video data are considered dense, and as such can be viewed in different technological ways such as real time, slow motion, frame by frame, forward, and backward. The permanency of video data means the data can be viewed and re-examined multiple times, from multiple points of view, through varying perspectives (Wiersma & Jurs, 2009). The process of analysing data can be cyclic: observed, coded, evaluated, and then observed again.

However, video cannot capture everything; the positioning and focus of the camera immediately makes a sampling choice by including and excluding certain events. Selections are made (and potentially limited) on the basis of the technology being used and the theoretical interests and assumptions of the data gatherer, thus constraining and shaping the possibilities and limitations of data records and later analyses and presentation of results. There is a danger that the researcher may focus only on what interests them.

To overcome such potential problems I selected appropriate video equipment, developed competent videography techniques, and planned and documented systematic recording strategies consistent with clearly defined research purposes. Experience and expertise were gained in collecting video data through my previous mathematics research (Higgins & Tait-McCutcheon, 2006; Tait-McCutcheon, 2008; Tait-McCutcheon, Drake, & Sherley, 2011; Tait-McCutcheon & Sherley, 2008).

5.6.3 Documentation and archival records

Documents and archival records were also part of my data collection and they provided a secondary source of data in this research. Neither contain the unmitigated truth and both were written for a specific reason and/or audience beyond this study. I needed to remain aware of the reasons these documents were produced, the contexts in which they were written, and the age, authenticity, and accuracy of the documents (Merriam, 2009).

Documents gathered included Education Review Office (ERO) reports, mathematics policy documents, mathematics long and short-term plans, and artefacts such as the group and individual modelling books. ERO reports were included because they provide an external view of the mathematics teaching and learning programmes in Pacific and Tasman Schools. They also provided an objective source of information as they have not been altered due to the presence of the researcher (Merriam, 2009). The mathematics policy documents and long and short-term plans of each school provided me with a big picture of what the schools intended to happen in terms of teaching and learning mathematics. The group and individual modelling books provided me with insight into what actually happened.

Archival records can be used in conjunction with other sources of information to build knowledge from the case study research (Yin, 2003). Cumulative student achievement data from the NDP database website were used within this study to corroborate teacher placement of students in their strategy groups.

The use of multiple primary sources of evidence, which included video and audio recordings, transcriptions, and participant observations, and secondary sources of data such as documents and archival records increased my opportunities to learn and helped confirm the findings.

5.7 Data Analysis

Data analysis is the process of making meaning by interpreting what participants have done and said and what the researcher has seen and read (Merriam, 1998).

Qualitative data analysis requires a fluid, evolving, dynamic approach that includes contrasting, comparing, replicating, cataloguing, and classifying from concrete data toward more conceptual levels (Denzin & Lincoln, 2011). The process of analysing qualitative data should be "relaxed, flexible, and driven by insight gained through interaction with data rather than being overly structured and based only on procedures" (Corbin & Strauss, 2008, p. 12).

Consistent with the use of case study, data collection and analysis have informed one another in an iterative manner. The intent of my collaborative and concurrent approach to data collection and analysis is to present research findings that are "parsimonious and illuminating" (Merriam, 1998, p. 162). A constant comparative method (Corbin & Strauss, 2008) was chosen as the most appropriate method for data analysis.

Constant comparison provides a method of data analysis for examining when, why, and under what conditions themes occur in observations. Theory is built from data within and between levels of conceptualisation (Corbin & Strauss, 2008; Merriam, 2009). Tesch (1990) proposed "forming categories, establishing the boundaries of the categories, assigning the segments to categories, summarising the content of each category, and finding negative evidence" (p. 96) as some of the tasks that could be compared and contrasted.

Units of data within one data source were compared and then used again to compare units of data across multiple sources in order to reduce data to salient categories and themes. According to Guba and Lincoln (1994), to be considered a unit of data two criteria must be met; first, the data should have meaning and not require additional information to be interpretable, and secondly, the data should reveal information about the study and motivate the reader to think beyond the data. My role was to identify meaningful (and potentially meaningful) units of data that caused immediate thinking and further metacognitive thinking.

5.7.1 Transcriptions

Transcriptions of video data are important because they allow for further identification of themes that are "above, beyond, and beside" (Powell et al., 2003, p. 422) those already suggested or determined through viewing video data. The printed, sequential rendering of speech may also reveal more about the data and a transcription allows for longer consideration of the dialogue. Transcripts provide a means for reporting evidence of findings in participants' own words. Structured classroom excerpts from each teacher's individual video observation created contextualised descriptions of their positioning decisions and students' learning opportunities.

Transcripts of episodes of video recordings have been used to present evidence for interpretations. As the video segments themselves are not available to the reader I have made the relevant features of the visual materials accessible through transcription and description. Transcription for each of the 72 lessons was completed by me using the same following process to ensure consistency:

1. Watch and listen to the video recording and transcribe the dialogue
2. Print the first draft transcription, listen to the audio recordings and add/amend any dialogue missed/incorrectly noted from the video recording
3. Listen to the audio recording again making any required edits
4. Watch and listen to the video recording adding non-verbal actions (body movements and participants' inscriptions) to the transcript
5. Format the transcript as a rich text document and import into Transana (Woods & Fassnacht, 2007).
6. Watch and listen to the video recording and follow the transcription in Transana. Complete a conversation coding analysis scheme
7. Watch and listen to the video recording and follow the transcription in Transana. Time code the transcript to synchronise with the video
8. Run a macro tool to convert the Transana time codes to minutes, seconds, and hundredths of seconds
9. Format the transcriptions back into Microsoft Word
10. Continue to edit transcriptions throughout analysis phase.

Even though it is near impossible to render an exact, genuine transcript of the positioning events captured on video, I have what Powell et al. (2003) describe as "close approximations to being exact and genuine for my research purposes" (p. 411).

5.7.2 Coding of data

At the completion of the data collection and transcription process I began the task of analysing the video data within its entirety. The analytic approach taken was “sit, look, think, look again” (Pirie, 1996, p. 556). Video of the lessons was viewed and reviewed, transcripts of the lessons were read and reread, and both were interpreted and categorised for evidence of teacher positionings. Categories of teacher positionings had not been proposed or predetermined prior to observing the video as I did not want to “blind” myself or “make it difficult to notice unanticipated” positionings (Powell et al., 2003, p. 423). Merriam (1998) suggests I was “having a conversation with the data, asking questions of it and making comments to it” (p. 181). References to the field, personal, and theoretical notes I made added to the comments and questions I was able to make.

Through a process of open-coding the units of data, I identified potential themes from real examples within the transcripts. Open-coding is the process of breaking down, examining, comparing, conceptualising, and categorising participants' words to identify and develop themes and interpret data. Researchers are guided in the codes that they develop by their theoretical framework, their research questions, and the nexus of what they observe (Powell et al., 2003). Coding was completed in this research by attributing meaning to events within the video, identifying key concepts in the video, and labelling those that were most significant.

To increase the manageability of the volume of data available to me (72 lesson transcripts), a process of data reduction was required. “Data reduction is a form of analysis that sharpens, sorts, focuses, discards, and organises data” (Miles & Huberman, 1994, p. 11) and is therefore a necessary part of the analysis process. Before I began the analysis of teacher positionings I realised I needed to frame periods of positioning rather than each individual act of positioning as it occurred. A different act of positioning could occur with each new dialogue, so I drew on the construct of a Social Episode (Harré, 1998b), described in Chapter Four, to frame the positionings as an event. Each social episode began with an act of teacher positioning that resulted in mathematical know-how being shared and was concluded when the discussion reached a natural conclusion. This process provided me with indicators of which chunks of dialogue to code and why, and for identifying data not relevant to this study which could be disregarded.

Data were analysed in the same order they were gathered. Pacific School first followed by Tasman School and teachers of the oldest students first, through to the youngest. In each phase of analysis I started with the transcripts of Paula, Pacific School, Years 5 and 6, lowest then highest group. The examples from Paula's transcripts formed the basis for the initial set of codes. This set of codes was altered and added to as I worked through all the data. I then completed each phase by analysing the videos and transcripts of the six teachers at Pacific School and all Tasman School teachers. The four phases of data analysis are described below. Throughout these phases I referred to the participant observations, documents and archival records for corroborating and contradicting evidence that would reiterate, enhance, or challenge my initial findings.

For the first phase of analysis I read the lesson transcripts and looked for examples of teachers and students sharing their mathematical know-how. Examples included one student explaining to another why they were incorrect, a teacher modelling multiplicative thinking with an array showing 3 groups of 5, and a student using their written recording to defend their strategy. I identified the teacher positioning that preceded the mathematical sharing and coded the positioning as teacher talk, text, or actions. Teacher talk, text, and actions were selected as codes because discourse in positioning theory includes talk, text, and actions of any kind (Davies & Harré, 1990). Table 5.3 shows the six codes used for describing the teacher positioning as reflexive (self) or interactive (other) and whether the positioning occurred through teacher talk (T), text (Tx), or actions (A).

Examples of the phase one reflexive positioning codes include the teacher revoicing a student's explanation (RPT), recording a student's explanation (RPTx), and providing a model for students to discuss (RPA). Phase one interactive positioning examples include the teacher asking students to discuss their mathematical disagreements (IPT), record their own or a peer's thinking (IPTx), and provide a model of their own or a peer's thinking (IPA).

Table 5.3: Phase one codes

Teacher	Talk	Text	Actions
Reflexive Positioning	RPT	RPTx	RPA
Interactive Positioning	IPT	IPTx	IPA

I reviewed the transcripts to determine if any episodes of shared know-how had been missed and as I did this I coded all episodes according to the teacher positioning and their words, texts, or actions. The teacher positionings were coded and constantly compared in order to define and refine their properties (Willis, 2006). Relationships between the codes were noted, trialled, and grouped as themes. As the themes were further considered I looked for conflicts, such as cases that negated initial concepts, negative instances, or erroneous conjectures and considered these cases as theory was created (Powell et al., 2003). Relationships amongst the codes were closely examined and codes that appeared to go together were grouped; from these groupings tentative concepts were developed. The concepts then started to build categories through which theory was being created.

In the second phase of analysis I identified the mathematical contexts in which the teachers' positionings occurred. The individual contexts identified were grouped as: strategy errors, misconceptions, and self-corrections; difference, efficiency, and sophistication of mathematical explanations; connections and relationships; and patterns. These contexts provided me with insight into the types of triggers that prompted teachers' positionings that led to mathematical know-how being shared.

For the third phase I placed the positioning codes and mathematical contexts onto a matrix. From here, I plotted the teachers' positioning acts according to the codes and contexts. This process enabled me to determine each teacher's pattern of positioning with their lowest and highest strategy group. P indicates Pacific School and T Tasman. The initial letter of the teacher's name is the second letter of the code. The third letter L or H refers to the lowest or highest strategy stage group.

Table 5.4: Examples of teacher placement on the positioning codes and contexts matrix

	RPT	RPTx	RPA	IPT	IPTx	IPA
Errors						
Misconceptions						
Self-corrections				PLL		PLL
Difference	PLL					PLL
Efficiency	THL PNH	PNH		PGL THL THH PNH		THL
Sophistication	THL			THL PLL		THL
Connections	PGL PGH	PGL PGH	PGL	PGL	PGL	
Relationships						
Patterns				THH	THH	

At this point in the analysis it became apparent that not all teachers' positionings resulted in mathematical know-how being shared because not all teachers and their groups were represented on the matrix. For example, there was little or no evidence of Naomi at Pacific School positioning her lowest group or Lisa from Pacific School positioning students in her highest group to share their know-how. Some teacher talk, text, and actions appeared to constrain the sharing of know-how, so I repeated phases one, two, and three of the analysis, this time looking for examples of teacher positioning that constrained mathematical know-how. I did not uncover any different positions or mathematical contexts but student behaviour was added as a non-mathematical context because for one teacher group behavioural expectations strongly influenced when know-how could be shared. Throughout phases one, two, and three field, personal, methodological, and theoretical notes were compared and contrasted, and confirmed or refuted with my on-going observations and analysis.

At phase four of the analysis I established themes to describe the positioning pattern of each teacher with their lowest and highest strategy group. These categories were derived from the codes and contexts determined in phases one and two. Category examples are provided in Table 5.5.

Table 5.5: Examples of themes of affording and constraining teacher positioning

Affording Teacher Positions	Constraining Teacher Positions
Teacher provides a model/written recording for students to discuss	Teacher asks students to watch/listen – no discussion
Teacher requires students to solve mathematical disagreement	Teacher explains why an answer is correct/incorrect
Teacher seeks student explanation and justification	Teacher explains for students and instead of students
Teacher appropriates student difference/efficiency/sophistication explanation for discussion	Teacher limits opportunities for creative solution methods
Teacher highlights mathematical connections/relationships	Teacher protects student from mathematical/cognitive conflict
Teacher expects students to help each other correct/self-correct errors/misconceptions	Teacher praises correct answers
Teacher allows students space to experience mathematical conflict	Teacher seeks correct answers only

In the final phase of the analysis I revisited each transcript and noted evidence of each affording and constraining positioning theme. Evidence of teacher talk, text, and actions within each theme was closely examined and tested for robustness. At this point I was able to determine the patterns of positioning of each teacher with their lowest and highest strategy stage groups. The result of this grouping was that I identified seven teachers who positioned both groups in the same way and five teachers who positioned students in the lowest and highest groups differently.

5.8 Ethical Considerations

Ethical principles were paramount in planning and conducting this research, as evidenced by approval granted by the Victoria University of Wellington Faculty of Education Ethics Committee (See Appendix A). Babbie (2007) contended that "the fundamental ethical rule of social research is that it must bring no harm to research subjects" (p. 27). Harm, defined as physical and psychological, requires diligence on behalf of the researcher to ensure participants' voluntary participation, informed consent, anonymity and confidentiality. These principles and their relationship with this research are described in the following sections.

I had a previous professional relationship with both schools so it was important that the potential participants in this research did not feel compelled or coerced into taking part. Fundamental to the ethical integrity of voluntary participation is informed consent and the participants' right to freedom and self-determination. Habibis (2006) noted that participants must be "fully informed about what the research is about and what participation will involve, and that they make the decision to participate without any formal or informal coercion" (p. 62). Freedom upholds their right to refuse to participate or to withdraw at any time and self-determination "places some of the responsibility on the participant should anything go wrong" (Cohen et al., 2007, p. 52).

Voluntary participation and informed consent were sought and ensured through the following processes. Principals were given an information sheet (see Appendix B) which outlined the research purpose and questions, my identity, what participation would entail, proposed use of data, participants' rights, and how they would be protected. They were asked to gain consent from their Boards of Trustees for their school to participate and for their teachers to be approached to participate. Teachers were given the same information sheet (see Appendix B) and I responded to questions regarding expectations, time commitments, and potential findings. I was aware of the dangers of considering all teachers within a school a captive audience (de Vaus, 2001) and of the principal being perceived as the gate-keeper (Miller & Bell, 2002) who controls access to the teachers. I contacted each teacher to ensure that their participation was completely voluntary.

Principals and teachers were given a consent form (see Appendix C and D) which reiterated their rights and responsibilities within the research. It was agreed that I would come back to the school in a fortnight to answer any further questions and obtain consent. Twelve teachers consented to their participation in this research and one teacher from Pacific School chose not to participate as she was in a relieving position and unsure she would remain at the school for the duration of this study.

Each participating class was spoken to about the purpose of the research and their rights and responsibilities. Informed consent for students was obtained from them and their parents/caregivers so as to be sensitive to the participating students' welfare (see Appendix E and F). An information sheet written specifically for the students was included in the information and consent package sent to parents. Students and their parents were given the right to withdrawal at any time. This allowed for students to change their minds as it could not be ensured that all students fully understood the implications of their consent at the onset of the study (Habibis, 2006). All students and their parents gave permission for them to participate in the study and no students were withdrawn from the study.

Video recordings of data require more in-depth considerations of informed consent. This is because participants were asked to give their consent before the recording occurred and material that may be unbecoming or damaging to a participant could be used. Hall (2000) warns against the ethical issue of “repurposing” where video data can be obtained by a different researcher and used for a different purpose. In this situation, even when participants have consented to the use of their recorded voice and body images, they have not consented to their images being repurposed (Powell et al., 2003). I met the ethical requirements of video recorded data by giving participants the option of interrupting or discontinuing a recording session, obtaining progressive levels of consent whereby participants were able to sight and consent to actual video recordings if they so wished, and destroying all recordings within five years of the study’s completion (Roschelle, 2000).

Participants can be harmed by the failure to honour promises of anonymity or confidentiality. Whilst each school, and the teachers and students within each school community, were given a pseudonym, anonymity could not be claimed as teachers at each school knew who was participating in the research and therefore information had been collected from identifiable respondents (Babbie, 2007). Confidentiality has been protected by restricting access to the data to me and my colleague peer-debriefer who signed a confidentiality agreement. The guarantee of the researcher not to identify any participants or their responses publicly has contributed to maintaining confidentiality for the participants and their schools.

5.9 Trustworthiness of Findings

The trustworthiness of my research can be tested and affirmed by considering the reliability, credibility, transferability, dependability, and confirmability of the qualitative research methods (Lincoln & Guba, 1985). Each criterion and the processes employed are discussed in the next section.

5.9.1 Reliability

Triangulation within qualitative research is recommended as a way to confirm emerging findings and the reliability of conclusions by looking at the same phenomena using different methods or looking at the same phenomena from different points of view (Merriam, 2009). Triangulation occurred in this research in three forms: participant sources, data sources, and data analysis.

Participant sources in this research were triangulated by including different schools, teachers, and students. Differences within these three groups included decile rating and staff changes, teachers' NDP experience, and students' achievement results. Data sources included video and audio recordings, transcriptions, participant observations, and documents and archival records. Video data in particular offer specific enhancements to triangulation in data analysis in terms of testing and refining data, interpretations, and findings (Powell et al., 2003). Data analysis was triangulated by analysing the positions, storylines, and social acts evident within each social episode. Positions, storylines, and social acts and the relationships between them are important when attempting to understand the meaning given to social episodes (Moghaddam et al., 2008; van Langenhove & Harré, 1999).

5.9.2 Credibility

The credibility in qualitative research is derived from the researcher's presence and depends on the ability and effort of the researcher as the instrument (Merriam, 2009). Credibility asks, does this research ring true and do the research findings represent a plausible conceptual interpretation of the data drawn from the participants' original data? I have increased the credibility of this research by recognising and clarifying my experiences with mathematics and existing relationship with participants. In doing this, I have provided the reader the opportunity to evaluate how these experiences and relationships might have influenced my observations and interpretations.

Credibility of this research has been enhanced through the processes of member checking and peer debriefing (Cohen et al., 2007). Participating teachers were given the opportunity to view their video and read the transcripts. One teacher could not be contacted. The remaining 11 teachers viewed their videos and/or read the transcripts of their lessons. The teachers who read their lessons confirmed that the transcriptions represented a true record of them (Angrosino & Mays de Perez, 2003).

Peer debriefing occurs when researchers' perceptions, insights, and analyses are reviewed with a colleague who, whilst outside the context of the study, has a general understanding of the nature of the study. A doctoral colleague with shared research experiences was asked to make more explicit my implicit beliefs, to test my working hypothesis, and to serve as a catharsis (Guba & Lincoln, 2005; Merriam, 1998). My colleague was asked to annotate six lesson transcriptions of four teachers and to identify acts of positioning that afforded or constrained the sharing of know-how. We then compared and discussed our initial findings. The few inconsistencies in findings that arose were discussed until consensus was achieved. A plausible level of objectivity has been attained through an observer agreeing with what is occurring in given situations (Angrosino & Mays de Perez, 2003). The iterative and inductive-deductive nature of this research provides assurances of the fit between the raw data and what emerges as the research findings.

5.9.3 Transferability

The degree to which the results of each case study can apply to other contexts, settings, or participants beyond the bounds of this inquiry comprises its transferability (Cohen et al., 2007). From a qualitative perspective, transferability is first my responsibility as I am the one doing the generalising (Mertens, 2005). However, the contention within naturalistic research is that no true generalisation is really possible as all observations are defined by the specific contexts in which they occur, and knowledge gained from one context may not have relevance for other contexts or for the same context in another time frame (Lincoln & Guba, 1985). In a traditional study it is the obligation of the researcher to ensure that findings can be generalised to the population; in a naturalistic study the obligation for demonstrating transferability belongs to those who would apply it to the receiving context (the reader of the study). The thick descriptions used to tell the story of teacher positioning will, to some extent, provide transferability for the reader and “accurate explanations and interpretation of the events” to a different setting (Cohen et al., 2007, p. 405).

5.9.4 Dependability

Qualitative research must be dependable and consistent rather than replicable because it is “holistic, multi-dimensional, and ever changing and is not a single, fixed, objective phenomenon waiting to be discovered” (Merriam, 1998, p. 202). This research is dependable because I have maintained the rigour of the data collection, data analysis, and theory generation through an audit trail which allowed me to “walk people through my work, from beginning to end, so they can understand the path I took and judge the trustworthiness of my outcomes” (Maykut & Morehouse, 1994, p. 146). The audit trail contains the data collection process, how categories were derived, and how decisions were made throughout the study. Field notes, hunches, and ideas contribute to the audit trail.

Data have been gathered from the natural setting and I have been both reflective and intuitive by including my own experiences, thoughts, and feelings in annotations. I have considered and reported on how my observations may have affected the participants and how I have been affected by what I have observed (Patton, 2002). Intuitiveness is concerned with my responses to the various stages of the research process. I have self-monitored, analysed, and provided further evidence of credibility by showing that my interpretations of the data are reasonable.

I have attempted to remain open to the nuances of increasing complexity throughout the analysis phase by watching and re-watching, listening and re-listening, and reading and re-reading the video, audio, and transcripts to see what materialises. Convergent lines of inquiry have been discovered through multiple sources and multiple pieces of evidence. The use of multiple sources was a methodological choice to pre-empt possible criticism regarding the issues of dependability and reliability and to validate the construction of findings. The lines of inquiry have addressed issues of dependability through the use of pattern-matching, which tested existing theories and explanation- building, and developed an outline of what was happening. Both pattern-matching and explanation-building were used to advance and contradict existing and hypothetical theories.

5.9.5 Confirmability

Confirmability refers to how well the inquiry's findings are supported by the data collected (Lincoln & Guba, 1985), the degree to which the results could be confirmed or corroborated by others, and the degree to which the findings are the product of the focus of the inquiry and not of the biases of the researcher.

Confirmability is achieved when constructions, assertions, and facts can be tracked to their original sources and when the logic behind their construction leads to an explicitly and implicitly coherent and corroborating whole (Mertens, 2005). I have achieved confirmability through documenting the procedures for checking and rechecking the data, including the negative instances that contradict initial findings, and conducted a data audit trail to examine the data gathering and analysis processes for potential bias.

5.10 Summary

This chapter has outlined the methodology underpinning this study. The ontological epistemological, and methodological premises I brought to this research through my personal biography were explored. The research settings and participants were introduced and the data sources and processes of analysis explained. I also attended to both the ethical considerations and the trustworthiness of this study.

Throughout this chapter I have illustrated how this study was an exploration in mapping the social reality of teaching and learning mathematics. I positioned myself as an explorer as I wanted to discover and understand this reality in ways that I could then enlighten others. The theoretical and philosophical structures applied to this research within its methodology have been iterative and circulative. As Pauly (1991) suggested, I have pursued the possible directions acts of teacher positioning can take within mathematics lessons and have analysed the varying pathways and destinations of those acts. The analysis of teachers' acts of affording and constraining teacher positioning are presented as excerpts of the transcripts and discussions in Chapters Six and Seven.

Chapter Six: Consistent Teacher Positioning

6.1 Introduction

The key research question this thesis was founded on is as follows: How do teachers in New Zealand primary schools position themselves and students in their lowest and highest mathematics strategy groups so that mathematical know-how can be shared?

The literature chapters drew attention to independence, judgement, and creativity as constructs of students' mathematical know-how and the need for teachers to position themselves and students so that mathematical know-how could be shared. In this chapter and the next, excerpts are presented from the observations, field notes, and unsolicited teacher comments regarding teachers' positioning of themselves and their students in the lowest and highest mathematics strategy groups. This chapter presents the case study of the seven teachers in this study who had a consistent approach to positioning themselves and students in both groups. Consistency between both groups occurred when students were similarly afforded or constrained in their opportunities to share their mathematical know-how and participate in the know-how of others.

Teacher positioning decisions that afforded the sharing of mathematical know-how included teachers modelling and assisting students to build connections between their existing knowledge and new strategies, teachers providing and promoting different, efficient, and advanced mathematical explanations, and teachers expecting students to identify and correct their own and others' errors. Teacher positioning that consistently constrained the sharing of know-how emphasised correct answers and limited flexibility with which students could approach their problem solving.

Section 6.2 presents the positioning practices of Greer (Pacific School, Years 4 & 5), and illustrates how she consistently positioned students in both groups to share their mathematical know-how. Greer positioned herself to provide students with models and representations and positioned them to notice and apply connections between their existing knowledge and strategies and their next learning steps. Hannah's positioning practices (Tasman School, Years 2 & 3) are the focus of section 6.3. Hannah positioned herself and students in both groups to share their mathematical know-how by highlighting the value of students applying different, efficient, or advanced strategies. Section 6.4 examines the positioning practices of Delphi (Tasman School, Years 1 & 2). Delphi emphasised the importance of students applying their existing knowledge to their new strategies, knowing how to determine mathematical difference, then applying the most appropriate strategy to their problem solving. The positioning practices of Jenna (Pacific School, New Entrants) are explored in section 6.5. The excerpts in this section illustrate how Jenna stressed the importance of students monitoring their own and others' mathematical thinking and the value of students helping each other in her lowest strategy group. Jenna emphasised mathematical explanations, justification, and argumentation with her highest strategy group. Kendra (Tasman School, Years 5 and 6) is the focal point of section 6.6. Students in Kendra's groups were positioned to resolve their mathematical arguments and answer their mathematical enquiries. Section 6.7 explores the teaching and positioning practices of Sheridan (Pacific School, Year 2). Sheridan promoted students' errors as valuable teaching and learning tools and positioned students to correct errors for themselves and each other.

Chelsea's positioning decisions, described in Section 6.8, were consistent but did not appear to afford students opportunities to share their mathematical know-how. Students in the lowest group were positioned more to provide answers than to share their mathematical know-how, and students in the highest group were positioned to apply specific strategies rather than explore the appropriateness of different strategies. Section 6.9 summarises the positioning decisions of the seven teachers included in this chapter.

The positioning of teachers and students reported here is in the form of excerpts taken from the three sequential mathematics lessons. The excerpts include the talk, text, and actions of the teachers and students. A table is provided before each analysis and includes: the number of students in the group, year level, age, and gender of the students, strategy stage of the group, and the achievement expectation of each group according to the MoE end-of-year curriculum expectations (MoE, 2009). According to the MoE (2009) students are identified as achieving one year above, at or as expected, one year below, or well below (more than one year) the expectation for their year level.

Each excerpt includes an introduction to, and explanation of, the observed event, and is followed by an analysis that highlights the positioning of teachers and students, the evolving storylines and social acts and a commentary that supports the analysis. The excerpts are coded according to Table 6.1.

Table 6.1: Excerpt script codes

Code	
Bold script	Teacher's name
Italicised script	Dialogue
Standard script	Written mathematics
[square brackets]	Actions of participants

The excerpts were analysed in terms of three features of positioning theory: positions, storylines, and social acts (as explained in Chapter Four). These excerpts were selected because they illustrated the teacher's consistent positioning patterns for themselves and students in both groups. Greer, Hannah, Delphi, Jenna, Kendra, and Sheridan made positioning decisions that enabled students in both groups to share their mathematical know-how. Chelsea made consistent decisions that constrained opportunities for know-how to be shared by students in both groups.

6.2 Greer, Pacific School, Years 4 and 5

Table 6.2: Greer's strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	10	7
Year Level	4	5
Age	8 and 9	9 and 10
Gender	1 boy and 9 girls	4 boys and 3 girls
Strategy Stage	Stage 5: Early additive part-whole thinking.	Stage 7: Advanced multiplicative to early proportional part-whole thinking
Achievement	As expected	As expected

Students in Greer's lowest and highest strategy groups were expected to share their own mathematical know-how and listen to, and apply, the mathematical know-how of others. Greer positioned students to notice connections within their existing mathematics knowledge and their next learning steps, and between their own and others' mathematical know-how. The following excerpts from Greer's teaching showed how she modelled and highlighted mathematical connections then supported students to make connections and apply them to their new learning.

6.2.1 "Have a think about these two strategies – what do you notice?"

In the first lesson with Greer's lowest strategy group, students were learning how "to solve multiplication problems using arrays" (MoE, 2007f, p. 15). Greer presented an array of 5 three-bunny strips and asked the students to record what the materials were showing and the total number of bunnies. Students shared their different recorded strategies: 3 + 3 + 3 + 3 + 3, 5 + 5 + 5, and 5 x 3. Greer highlighted two strategies for students and asked them to discuss what they noticed about the two strategies:

> *Look a bit closer at two of these strategies. Have a look at Toni's strategy where she has added up in groups of 3 and Poppy's strategy where she said 5 times 3 equals 15. Have a think about these two strategies then talk to your partner – what do you notice about them?*

The following excerpt demonstrates how Greer appropriated students' observations about the two highlighted strategies, how she assisted them to notice the relationship between repeated addition and multiplication, and critique the efficiency of multiplicative thinking.

Participant	Dialogue
Greer	*Right Priya what did you and Erica think about Toni's strategy?*
Priya	*We thought she was plussing by 3 and going up by 3 each time and she went up to 15.*
Greer	*Erica why did she stop at 15?*
Erica	*Because she had plussed 3 — 5 times.*
Greer	*Okay — who talked about Poppy's strategy?*
Rawiri	*Us.*
Greer	*And what did you notice?*

Rawiri	*That 5 times 3 equals 15 cos there is 5 groups of 3 bunnies and like Toni did - the 5 times 3 means 3 plus 3 plus 3 plus 3 plus 3.*
Greer	*Good now have a chat with your partner — is using the addition or the multiplication strategy more efficient — which strategy is quicker?*

Greer asked students to notice a mathematical connection between the 5 by 3 arrays and the different strategies students had recorded. Closer attention was drawn to Toni's additive and Poppy's multiplicative strategies by Greer directing students to look a bit closer at the two strategies and discuss what they noticed. Erica observed that Toni plussed 3 – 5 times and Rawiri agreed that 5 times 3 means 3 plus 3 plus 3 plus 3 plus 3. Students were directed by Greer to discuss the efficiency and speed of the multiplicative strategy. The lesson continued with students working in small groups to explore multiplicative thinking using array-based equipment.

6.2.2 "Okay, tell me what I did?"

The second lesson with Greer's lowest group focussed on students learning "to solve problems about sharing into equal sets" (MoE, 2007f, p. 17). Greer told students she was a pirate captain sharing her treasure of 10 gold coins between two pirates – Marama and Poppy. Students were asked to record their strategy for sharing 10 gold coins between two pirates and decide if their strategy matched Greer's modelling. Greer shared the gold one-by-one alternately between Marama and Poppy and asked students to record what she had modelled. The next excerpt illustrates how Greer appropriated students' prior knowledge of repeated addition and sharing to introduce the concept of division.

Participant	Dialogue
Greer	*Okay, tell me what I did — how many coins did I start with?*
Students	*10.* [Greer records 10 in the modelling book]
Greer	*And what did I do?*
Priya	*Shared them into 2.*
Greer	*And what's the sign that means shared?*
Rawiri	*Divided by.*
Greer	*And what does the divided by sign look like?*
Rawiri	*Line and a dot on top and another dot underneath.*

Greer	[laughing] *You had better write that* [turns the modelling book toward Rawiri].
Rawiri	[records] ÷ 2 =
Greer	*Let's have a look at your ideas — do any of them show 10 divided by 2 — Poppy?*
Poppy	*Sort of I started with 10 but I subtracted not divided.*
Greer	*Tell us more?*
Poppy	*Well I went 10 takeaway 5 takeaway 5.*
Greer	*Okay so repeated subtraction. Priya?*
Priya	*No I timesed* [indicates to her recording] *2 times 5 equals 10.*
Greer	*Allie?*
Allie	*Yes because I drew 2 pirates and I shared out the gold and they got 5 each.*
Greer	*Okay so what does the divided by sign mean?*
Rawiri	*Halved* [Greer writes *halved*]
Greer	*Halved do we agree?*
Erica	*No not always halved.*
Greer	*What then?*
Erica	*Shared.* [Greer records *shared* in the modelling book]
Greer	*Anyone else?*
Jen	*Splitting.* [Greer records *split* in the modelling book]
Greer	*And how do they need to be shared or split?*
Erica	*Equally.*
Greer	*Yes or?*
Rawiri	*The same.*
Greer	*Good yes everybody has to get the same so it has to be equal or the same. Now have a chat about Poppy's repeated subtraction and Allie's picture showing 10 coins divided by 2 pirates.*

Greer asked students to record her model of 10 gold coins shared between two pirates. Students shared their strategies and critiqued how well their recordings represented Greer's model. Greer and students clarified that the model denoted division and Poppy and Priya observed their strategies did not correctly represent Greer's model because they used subtraction and multiplication. Allie felt she had represented the model correctly because she drew 2 pirates and I shared out the gold and they got 5 each. Greer emphasised the concept of division by discussing the division sign, highlighting the connection between repeated subtraction and division, and asking students to unpack and describe the action of dividing. Students discussed the relationship between Poppy's and Allie's strategies and the mathematical connections between repeated subtraction and division. The lesson continued with students playing the part of pirate captain and creating division problems for their peers to solve.

6.2.3 Discussion: Positioning to build mathematical connections

Greer positioned herself as having the right to provide models and representations that enhanced the accessibility of the mathematical ideas for students in her lowest group. In Excerpt 6.2.1, she provided materials for students to compare their existing additive knowledge and strategies with new multiplicative strategies. Greer modelled dividing 10 coins between two pirates in Excerpt 6.2.2, and students used the model as the basis for their recording, then critiqued the accuracy with which their recording described division. In both excerpts Greer could have positioned herself to tell students about the connection. Instead, she chose to highlight the connections through her modelling and questioning and guide students to making the connections themselves. Greer had a duty to scaffold the students toward new learning, but not a right to do the work for them.

Students in Greer's lowest group were positioned to share their mathematical know-how by explaining, considering, and comparing their own and peers' thinking, noticing connections, critiquing strategies for efficiency, and recording their thinking. Students were asked to discuss what they noticed about Erica's additive and Toni's multiplicative strategies and to critique the efficiency of the two strategies. Students were also asked to discuss what they noticed about Poppy's repeated subtraction strategy and Allie's picture showing 10 coins divided by 2 pirates. Greer's positioning decisions ensured students in her lowest group had access to their peers' mathematical know-how which extended beyond only needing to listen. Students were expected to interact with each other in mathematically meaningful ways. The reason for mathematical meaningfulness in the excerpts above came from Greer's expectation that students would engage in their own and peers' strategies. Greer's positioning was accepted by students when they volunteered their different strategies, discussed the mathematical connections between the different strategies, critiqued the efficiency of multiplication and division, and applied multiplicative strategies to their problem solving.

Greer's positioning created two storylines evidenced in the excerpts above. In the first storyline, students were expected to share their mathematical know-how, and explain what they knew by elaborating on and critiquing their know-how for efficiency. Students were required to go beyond simply sharing ideas to assessing the ideas for their effectiveness. The second storyline was that existing knowledge was valuable to new learning. Students were expected to reflect on their existing knowledge and strategies and consider how their know-how could be applied to progress their mathematics.

Students' and teachers' actions become social acts and take on a significant force when they become meaningful to the group (Davies & Harré, 1990; Harré & Secord, 1972). Efficient explanations and strategies and mathematical connections were significant to this group because Greer directly positioned students to be more efficient in their strategising and to build connections between their strategies. The spoken and recorded strategies of students became social acts when they were appropriated by Greer for discussion and use. Students' strategies continued to have significance to the lesson as Greer referred to them specifically when asking students to review their understanding of division. The act of using materials and written recordings in both lessons became socially significant when students appropriated and used the materials and recordings to set multiplication and division problems for their peers.

In summary, Greer positioned students in her lowest strategy group to make connections between their existing mathematical know-how and their next learning steps. Such positioning assisted students to progress to understanding more advanced mathematical know-how. The positioning choices Greer made with her highest strategy group and how those positionings similarly assisted students to build connections between their existing and developing mathematical know-how are outlined in section 6.2.4.

6.2.4 "Keep talking about the two strategies and see what you notice"

In the second lesson students in Greer's highest strategy group were asked to work in pairs to solve 7 x 38 using two different strategies and to name the strategies they used. When students indicated they had solved the problem, Greer sought volunteers to explain and name their strategies. Sam named his strategy as place-value partitioning and explained: I went 7 times 30 is 210 and 7 times 8 is 56 and then I added 210 and 56 and it's 266. Greer recorded Sam's strategy in the group modelling book and asked, did anyone do it differently? Chanelle, who was new to Pacific School, stated I did an algorithm. The video showed Greer looking surprised and some students looking puzzled. Greer informed me after the lesson that she had not expected algorithms to be suggested because she had not introduced them as a pen and paper strategy for solving multiplication problems. The next excerpt shows how Greer supported students to unpack and understand Chanelle's different working-form strategy by asking them to follow and explain each strategic step.

Participant	Dialogue
Greer	*Okay so from the beginning what was the first step?*
Chanelle	*7 times 8 equals 56.*
Greer	*Kane — where did the 7 times 8 come from?*
Kane	[points to the equation] *The 7 and the 8 ones in 38.*
Greer	*Chanelle?*
Chanelle	*Yip.*
Greer	*Okay what's next Chanelle?*
Chanelle	*You put the 6 under the 7 and you carry the 5 and write it above the 3.*
Greer	*Why? Any ideas guys? No! Chanelle can you help?*
Chanelle	[points to the equation] *The 6 is 6 ones so it goes in the ones column and the 5 is 5 tens so it goes in the tens column.*
Greer	*Okay so what's next?*
Chanelle	*7 times 3 is 21 plus the 5 tens is 26 and then the 6 goes in the tens and the 2 in the hundreds and the answer is 266.*
Greer	*Okay so what I'd like you to do now is to keep thinking about Chanelle's strategy. What I want you to do is to look at Sam's place-value strategy*

	[indicates Sam's recording 7 x 30 + 7 x 8] *and Chanelle's algorithm and I want you to keep talking about the two strategies and see what you notice.*

Greer engaged students in Chanelle's strategy by asking them to explain the steps she took. To assist students to develop understanding of the algorithm strategy Greer asked them to discuss similarities between Sam's place-value strategy and Chanelle's algorithm. Students commented that the strategies were very similar but the way the problems were set out was different. Meg noted that Sam did 7 times 30 and 7 times 8 and added the answers in his head and Chanelle did 7 times 8 and carried, and then 7 times 30 was 210. Nigel commented that Chanelle's strategy was a lot more confusing than Sam's – Sam's you can do in your head easy but Chanelle's – it's hard to keep track of. At the end of this lesson Greer informed me she was going to change the plan for the third lesson. Instead of moving on to learning about division strategies, Greer decided to give students more time to explore Chanelle's different algorithm strategy for solving multiplication problems.

6.2.5 "What did you notice about the strategy I used?"

The third lesson with the highest group began with Greer explaining that algorithms could also be called "standard written form" (MoE, 2007c, p. 5) or "working-form" (MoE, 2007f, p. 42). Students were asked to discuss how they would solve 12 x 86; would they use a place-value strategy or an algorithm? In pairs, students attempted to solve 12 x 86 using place-value partitioning but soon found: we can't keep track of each step in our head! Students looked to Chanelle to explain the algorithm for solving 12 x 86 but Chanelle commented: we didn't get to those big numbers at my last school.

Greer told students she was going to model solving 12 x 86 using a long working-form algorithm, which she described as being very similar to the place-value strategy Sam had used. Greer introduced a long working-form algorithm as the scaffold between Sam's place-value strategy and Chanelle's short working-form algorithm. Students were asked to observe and notice the connection between what Greer said and what she recorded.

Participant	Dialogue
Greer	*Okay I'm going to solve 12 times 86 using a different algorithm, watch what I'm doing and give me a thumbs up when you think you can explain it.* *2 times 6 equals 12* *2 times 80 equals 160* *10 times 6 equals 60* *10 times 80 equals 800* [Greer said and recorded each step simultaneously] 86 x12 12 160 60 800
Greer	*Okay so — have a talk with the people beside you — what did you notice about the strategy I used — what did you notice between what I said and what I recorded?* Students discussed what they noticed about Greer's explanation and her written recording.
Greer	*So what did we notice?*
Nigel	*Well first you did 2 times 6 is 12 and you wrote down 12.*
Greer	*Then?*
Meg	*Then 2 times 80 is 160.*
Greer	*So where did the 80 come from?*
Meg	*The 2 ones and the 8 tens?*
Greer	*And how do we know they are 8 tens and not 8 ones?*
Sam	*Cos they are in the tens column.*
Greer	*Good - next?*
Sam	*10 times 6 is 60.*
Chanelle	*10 times 80 is 800.*
Greer	*And what do we need to do now?*
Kane	*Add them up?*
Greer	*Yip can you do that for us Tess?*

Students explained the steps Greer took to complete the long working-form algorithm and Greer drew students' attention to the place-value connections within the strategy. At the conclusion of the third lesson with her highest strategy group Greer informed students: Now your challenge before we meet again is to use Chanelle's short form algorithm to solve 12 times 86. Students were observed on the video looking sceptically at each other, so Greer asked: What do you have to support you with this challenge? Students discussed having Chanelle's and Greer's recording of a short and long-form algorithm and the answer. Greer reminded students they also had their place-value knowledge.

6.2.6 Discussion: Positioning to build mathematical connections

Greer's positioning of herself with her highest strategy group was similar to the positioning with her lowest strategy group. Rather than demonstrating what to do, she positioned herself to provide models to scaffold students' understandings of different strategies. She also positioned herself as having the right to share her mathematical know-how and for students to unpack and notice her strategies and apply what they had learned to solve 12 x 86 using Chanelle's short working-form algorithm.

Students were positioned to share, observe, and explain their own and others' familiar and different strategies and to notice connections between their existing and new mathematical know-how. In Excerpt 6.2.4, Sam explained a familiar place-value partitioning strategy to solve 7 x 38, and Chanelle used a different algorithm strategy. Greer positioned Chanelle as having the right to explain her different strategy and students as having a duty to unpack and notice her long working-form algorithm strategy and then to apply what they had learned to solve 12 x 86 using short working-form. Greer decided to use Chanelle's mathematical know-how to advance the understandings of the whole group.

The way in which Greer positioned herself and students in her highest strategy group created similar storylines and social acts to those created with her lowest group. Connections between existing and new know-how and efficiency with strategies were again stressed by Greer as being significant to students' learning. Students were expected to use their existing knowledge, to explain and justify their mathematical reasoning, understand the mathematical reasoning of others, and make connections between existing and new knowledge and strategies. The explanations of Sam's place-value strategy and Chanelle's algorithm became social acts when they were appropriated by members of the group. The strategies became significant to the developing storyline because they provided the basis for discussion and assisted in moving the mathematical know-how of students forward.

To conclude, Greer positioned students in her highest strategy group to notice and build on connections between their existing knowledge and strategies and their next learning steps. Greer appropriated students' different strategies by positioning them to share their different mathematical know-how and expanding students' access to, and experience of, different strategies. Such positioning may have assisted students to build connections between their existing mathematical know-how and the different mathematical know-how of their peers.

6.3 Hannah, Tasman School, Years 2 and 3

Table 6.3: Hannah's strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	6	5
Year Level	2	3
Age	6	7
Gender	1 boy and 5 girls	4 boys and 1 girl
Strategy Stage	Stage 4: Advanced counting	Stage 5: Early additive part-whole thinking
Achievement	As expected	As expected

The following excerpts and discussions illustrate how Hannah consistently positioned students in her lowest and highest strategy groups to share their own and engage with others' mathematical know-how. Evidenced in the following excerpts is the emphasis Hannah gave to students in both groups sharing and unpacking their different, efficient, and advanced mathematical know-how. The impact the focus on students' mathematical know-how had on progressing students' learning is described below, first with Hannah and the lowest group, then the highest group.

6.3.1 "Oh wow, that's different"

In the second lesson with Hannah and her lowest strategy group students were learning to count on or back to solve addition and subtraction problems. Students had been creating word and number problems for each other to solve using their counting-on strategies. Hannah confirmed with students that they were using a counting-on strategy and not a counting-from-one strategy. As the following excerpt outlines, Hannah expressed her surprise when Portia explained her stage 5 early additive part-whole strategy and Imogen suggested a different part-whole strategy.

Participant	Dialogue
Hannah	*How did you work out 8 plus 5 Solomon?*
Solomon	*I went 8 — 9, 10, 11, 12, 13* [holds up 5 fingers].
Hannah	*Okay Portia how about you?*
Portia	*It's 13 because 8 plus 2 is 10 and 10 plus 3 is 13.*
Hannah	*Oh wow, that's different, that's, well that's part-whole thinking and okay, can you tell us all about that again? Listening guys this is very interesting.*
Portia	*I added on 2 to the 8 to make it 10 and then I added on the 3 to the 10 to get 13.*
Hannah	*Can anyone else explain Portia's strategy?*
Imogen	*No but I went 5 and 5 and 3 is 13.*
Hannah	*Oh wow another different way, okay, well these are very advanced strategies – very efficient strategies – I think we need to stop counting on and explore Portia's and Imogen's strategies – Portia, Imogen, explain your strategies again and I am just going to grab some equipment so that you can model your thinking for us too.*

Hannah responded to Portia's and Imogen's explanations with oh wow and described the explanations as being different, interesting, advanced, and efficient. The girls' explanations were appropriated by Hannah when she asked them to explain and model their know-how for their peers. To support the girls with their explanations Hannah provided equipment that included tens frames, number lines, and Slavonic abacuses. The importance of the part-whole strategies was stressed by Hannah when she commented well these are very advanced strategies – very efficient strategies.

6.3.2 Discussion: Positioning for efficient thinking

Hannah positioned herself as having the right to highlight examples of students' different, interesting, advanced, and efficient mathematical know-how. Portia's and Imogen's strategies were mathematically different because they required a more advanced part-whole way of thinking about the numbers. This group was transitioning to stage 4: advanced counting but the girls' strategies would be considered stage 5 part-whole thinking (MoE, 2007b). Therefore, their strategies would be considered advanced or sophisticated. The girls' strategies were noted by Hannah as being more efficient, that is, they required fewer steps than other strategies and were easier to keep track of. In assuming the right to highlight explanations that were different, Hannah simultaneously positioned students as having a duty to suggest, understand, and apply different strategies. Students' acceptance of this positioning is evidenced by Portia and Imogen sharing their stage 5 part-whole strategies even though they had been asked by Hannah to model stage 4 counting-on strategies.

The positioning of Hannah and her lowest group created three storylines. First, students were positioned by Hannah to do more of the enquiring, explaining, and modelling. Hannah positioned herself to introduce the learning intention, pose the first two or three problems, ask questions, and provide equipment. Students were positioned to create number and word problems to be solved, model, record, and explain their mathematical know-how, and listen to, and use the mathematical know-how of others. In a parallel second storyline, Hannah expected students to become more efficient with their mathematical know-how so that they could problem solve in quicker, more efficient ways. The third storyline is that students can influence the planned direction of the lesson and progress the lesson through their more advanced mathematical know-how. The actions of the teacher and students became social acts when they were given significance or importance by the group. Efficient and sophisticated explanations became social acts with this group when Hannah highlighted their significance for students and their mathematical advancement. Portia's and Imogen's part-whole strategies became important when Hannah questioned students about them, reiterated that the strategies made their problem solving more efficient, and when she changed the lesson plan to explore Portia's and Imogen's advanced strategies.

In review, Hannah positioned students in her lowest group to share their mathematical know-how by creating, modelling, and solving problems. Alongside from the expectation that mathematical know-how would be shared was an agreement to understand that the difference, efficiency, and sophistication of the know-how were important. Hannah's positioning of herself and students in her highest group are explored in the following sections.

6.3.4 "Which strategy do you think is more efficient?"

Within each lesson with her highest group Hannah asked students to "pretend I don't know" and suggested "if I didn't understand this how would you help me to understand?" In doing so she positioned students to ensure they were being understood by her and their peers and she promoted them as co-teachers within the lesson.

Students in Hannah's highest strategy group were learning how to use tidy numbers to add to and subtract from 100 in their third lesson. Tidy numbers are numbers that end in at least one zero. Students were discussing how to solve the problem of Garry Grasshopper visiting at number 56 and needing to get home to number 100 quickly, recorded as 56 + ☐ = 100. Hannah facilitated the opportunity for students to strategise more efficiently by bringing a less efficient approach to their attention. She began the lesson by claiming she only knew how to count in ones and asked students for some help.

Participant	Dialogue
Hannah	*Should he* [referring to Garry Grasshopper] *go like this?* [counts slowly] *56, 57, 58, 59, 60…*
Coral	*No.*
Hannah	*Or is there a faster, easier way?*
Coral	*He could hop 40.*
Hannah	*He could hop 40? — How would that sound?*
Students	*56 - 66, 76, 86, 96.*
Hannah	*Then?*
Lee	*96 — 100.*
Hannah	*Could Garry have jumped another way to 100?*
Jin	*56 to 60, then 60 to 100 — 44.*
Hannah	*Oh is that faster or easier?*
Students	*Yes.* *No.*

Hannah	*Okay pair up and write the number sentence for the two ways Garry could jump from 56 to 100. Talk about which strategy you think is more efficient, which strategy would get Garry home quickest.*

Hannah provided a less efficient model of counting on in ones for students to review and critique. The purpose of the counting model was to illustrate that jumping to and with tidy numbers was a more efficient strategy than counting on in ones. Strategies that make problems easier to solve are both easy to understand and easy to manipulate. When discussing the efficiency of the two strategies 56 + 4 + 40 = 100 and 56 + 40 + 4 = 100, students decided that both ways were efficient, but according to Lee it depended if you wanted to add the big (56 + 40 + 4) or the small (56 + 4 + 40) numbers first. Students agreed that being efficient meant attending to the numbers in the task and applying the most efficient strategies to the task.

6.3.5 Discussion: Positioning for efficient thinking

Hannah's positioning of herself and students in the highest group was similar to her positioning with the lowest group. With both groups Hannah positioned herself to draw students' attention to quicker, faster, and easier ways of problem solving. Hannah deliberately modelled an inefficient counting strategy to encourage students to look for a faster, easier way to solve the addition problem in Excerpt 6.3.4. Students in Hannah's highest group were also positioned to do most of the work. Using a similar sequence to that of her lowest group, Hannah introduced the learning intention, asked the first two or three problems, and provided the equipment. Students had a duty to share answers, strategies, models, and written recordings to create number and word problems, and to listen to and use the mathematical know-how of others. As with the lowest strategy group, these duties were readily accepted by students in the highest group.

The storylines and social acts were similar in Hannah's two groups. In related storylines students in the highest group were expected to model and explain their mathematical know-how and listen to the mathematical know-how of others. Similarly, they were expected to become more efficient with their mathematical know-how so that they could problem solve in quicker ways. Using different, efficient, and advanced explanations became important social acts as they featured strongly in students' discussions about their mathematics. Students discussed the efficiency of applying their number knowledge of patterns and of counting on in tidy numbers. Jin's pattern became socially significant for the highest group when Hannah questioned students about the pattern and assisted students to connect their knowledge of patterns with quicker problem solving.

In conclusion, Hannah positioned students in her highest group to share their mathematical know-how by creating, modelling, and solving problems. She ensured students had opportunities to do more of the mathematics work than she did. As with her lowest group, there was also an expectation that students would enhance their mathematical know-how by applying strategies that were different, efficient, or sophisticated.

6.4 Delphi, Tasman School, Years 1 and 2

Table 6.4: Delphi's strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	8	7
Year Level	1 and 2	2
Age	6 and 7	7
Gender	5 boys and 3 girls	4 boys and 3 girls
Strategy Stage	Stage 3: Counting from one by imaging	Stage 4: Advanced counting
Achievement	Year 1: As expected Year 2: Below	As expected

The following excerpts and discussions illustrate how Delphi positioned students in her lowest and highest strategy groups to share their mathematical know-how and benefit from hearing peers' mathematical know-how. In particular, Delphi emphasised with both groups the importance of knowing how to determine the mathematical difference between strategies and apply the most appropriate strategy, and how to utilise connections between existing and new knowledge.

6.4.1 Telling a different number story

In the first lesson with Delphi's lowest strategy group, students were learning how to add and subtract to and from 5. Delphi placed blue and brown pieces of material in front of students and told them the blue material was a pond and the brown a rock. She placed plastic frogs in front of students and asked: Can I get some help counting these frogs please? Students counted and agreed there were 5 frogs. Delphi asked Lelei to place 3 frogs in the pond and 2 frogs on the rock and directed students to record the number story in your modelling books. Ethan shared his number story 3 plus 2 equals 5 and Delphi asked if anyone had a different way to record the problem. Wiki proposed 3 and 2 is 5 as being different and Delphi asked the group if the two equations were different. The following excerpt illustrates how Delphi positioned students to debate and determine what it meant to provide an explanation that was mathematically different.

Participant	Dialogue
Delphi	*Oh who could tell us why they think those two equations are different or the same?*
Sefina	*They are the same numbers, they aren't different numbers.*
Wiki	*But I didn't say mine like Ethan did. I said 3 and 2 is 5.*
Delphi	*Okay — so Sefina says they are not different because the numbers are the same and Wiki says they are different because she read her equation differently. Anyone else? Can anyone else think of a different way to record our equation?*
Ihaka	*Could you do 2 plus 3 equals 5?*
Wiki	*No cos they are the same numbers!*

Sefina did not think 3 plus 2 equals 5 and 3 and 2 is 5 were different because the numbers were the same. Wiki thought the two number stories were different because the words she used were different in her number story. Delphi put the responsibility for deciding if the strategies were different back to the students and asked for reasons as to why the equations were similar or different, and if anyone else could think of a different way to record our equation? Ihaka's question – could you do 2 plus 3 equals 5 – caused further confusion because, according to Wiki, the equation 2 + 3 = 5 was not different to 3 + 2 = 5 because it used the same numbers.

Delphi facilitated a discussion where students were supported to explore the notion of mathematical difference. Eventually, as the following excerpt illustrated, students agreed that how the numbers "told the story" made the explanation or number story different.

Participant	Dialogue
Graham	*Well I think 3 plus 2 equals 5 tells a number story one way and 2 plus 3 equals 5 tells a number story another way and that's what makes it different.*
Delphi	*So it's the story the numbers tell that makes the explanations different or the same. Okay so Wiki let's go back to when we shared 3 plus 2 equals 5 and 3 and 2 is 5 — do those numbers tell a different story or do they tell the same story?*
Wiki	*They tell the same story.*
Delphi	*Why?*
Wiki	*Because they are both 3 frogs in the pond and 2 frogs on the rocks.*
Delphi	*Okay everybody what about 3 plus 2 equals 5 and 2 plus 3 equals 5 do they tell the same story or a different story?*
Students	*Different story.*
Delphi	*Why?*
Graham	*Well 3 plus 2 equals 5 says the frogs in the pond are first and 2 plus 3 equals 5 says the frogs on the rock go first. It tells it differently.*
Delphi	*So when we think about sharing a different explanation we need to remember Graham's excellent idea about telling a different story.*

Students became confused regarding what constituted mathematical difference when comparing their number stories. Delphi provided them with the opportunity to sort out the confusion for themselves. She drew attention to Graham's suggestion by asking him to repeat it. Graham suggested that how the number story was told determined if the explanation was different. Delphi asked Wiki to reflect on her previous claim that using different words such as 'and' for 'plus', and 'is' for 'equals', meant she was providing a mathematically different explanation. Wiki recognised that 3 plus 2 equals 5 and 3 and 2 is 5 told the same story because they both say 3 frogs in the pond and 2 frogs on the rocks. Delphi sought further explanation as to why the number story was different and Graham explained that by changing who went first in the number story (frogs in the pond or frogs on the rock) made the number stories different. Students were reminded that when we think about sharing a different explanation we need to remember Graham's excellent idea about telling a different story. Students explored addition to 10 in the second lesson and Delphi reminded students to consider if their explanation was telling a different story before they shared strategies they believed were different.

6.4.2 Discussion: Positioning for mathematical difference

Delphi positioned herself in the three lessons with her lowest strategy group as having the right to assist students to determine and apply mathematically different strategies and to build connections between their existing and new learning. In Excerpt 6.4.1, Delphi helped students to come to an understanding that for a strategy to be different it needed to tell a different story. To be able to identify mathematical difference, students need to understand the explanations that have already been discussed to be able to judge the extent of the similarities and differences. Delphi positioned students to share their different number stories and strategies and reflect on the differences and similarities between them. This positioning provided additional learning opportunities for students and extended their cognitive activity beyond simply solving the task to comparing the different ways the task could be solved.

Delphi's positioning of herself and students in these three lessons created two storylines. In the first storyline students in Delphi's lowest group were expected to share, explain, critique, compare, defend, model, and record their own and their peers' mathematical know-how. The second storyline stressed the importance of being able to identify and describe mathematical difference in explanations. In Excerpt 6.4.1, Delphi provided the opportunity for students to discuss what it meant to be different and develop their own definition that highlighted the numbers telling a different story. Students' explanations became significant social acts when they were promoted by Delphi. Graham's idea of the numbers telling a different story became important when Delphi referred to his strategy, when she directed Wiki to reflect on her example of mathematical difference in consideration of Grahams' idea, and when she reminded students to remember and apply Graham's excellent idea to their future problem solving.

To summarise, Delphi had a right to expect students to understand mathematical difference and suggest mathematically different strategies, and a right to position students to reflect on and use existing know-how to advance their mathematics. Students had a duty to share their number stories, defend the mathematical difference of their number story, determine what constituted mathematical difference, reflect on their definitions of difference, and apply their new understandings. These positions were readily accepted by students and there were no examples in the lessons of students refusing the positioning or trying to position someone else to do the work for them. Delphi's teaching and positioning decisions with her highest group are described in the following sections.

6.4.3 "Now that is very interesting"

In the second lesson with Delphi and her highest strategy group, students were learning how to "add by counting on when the larger number is given first" (MoE, 2007e, p. 18). In this lesson Delphi drew students' attention to interesting strategies they used to solve their addition problems. The first problem Delphi asked students to solve was 3 + 63.

Participant	Dialogue
Delphi	*Okay who has an answer and a strategy for working that out?*
Pio	*Easy 3 and 3 is 6 so it's 66.*
Delphi	*Oh interesting Pio — could you record that for us please? Did anyone else solve that differently?*
Joseph	*Yes* [holds up 3 fingers] *64, 65, 66.*
Delphi	*Very clever Joseph. Can you write that down for us please? 64, 65, 66. Now can you have a think for a minute — are Pio's and Joseph's strategies different — and why are they different?*

Pio and Joseph shared their strategies and Delphi asked students to consider whether they thought Pio's and Joseph's strategies were different and why. Connor thought the two strategies were different and he used the names of the strategies to explain the difference – Pio doubled and Joseph counted on. Delphi confirmed Connor was correct and presented the next problem for students to think about and then discuss: 5 plus 91 jellybeans. In the following excerpt Delphi highlights Pio's really interesting strategy of swapping the numbers.

Participant	Dialogue
Delphi	*Okay who wants to go first?*
Pio	*Oh me Miss I went 95 plus 1 equals 96*
Delphi	*Wow that's a really interesting strategy to use Pio — you need to tell us how you knew to work it out that way.*
Pio	*I swapped the numbers; I swapped the 5 and the 1 around so I didn't need to go 91 plus 5, I just go 95 plus 1 and its 96.*

Delphi	*That really is very clever — have a talk to someone about Pio's strategy — how did his strategy make his adding easier?* Students discuss Pio's strategy in pairs.
Delphi	*Great discussing guys. Okay I have a question for you. If I solved 91 plus 5 jellybeans by counting on* [holds up fingers] *92, 93, 94, 95, 96 — would that be different to how Pio swapped the numbers and then counted on? Have a think and then we will have a talk.* Students discuss Delphi's question in pairs.
Delphi	*Okay what do we think — would those two strategies be different?*
Tyson	*No because Pio and you counted on.*
Pio	*Yes because I swapped before I counted on — I made the 1 the 5 and then it was 96 — I done it easier.*
Delphi	*Oh now that's interesting — Pio says his way made the problem easier to solve. So Pio is saying that 95 plus 1 is easier to work out than 91 plus 5. Do we agree?*
Students	*Yes.*
Delphi	*So maybe Pio's strategy is a bit different because he swapped the numbers but what is really interesting is that he made the sum easier to solve. I wonder if there is another way that would make our adding easier.*
Rohan	[giggles] *Eat some of the jellybeans before we start adding!*
Delphi	[laughs] *Yes well that would make it easier — but how about a way where we didn't have to eat the equipment! Okay I am going to give you five different sums to talk about and solve. What strategies have we discussed today and you can use?*
Aroha	*Counting on.*
Tyson	*Doubles.*
Pio	*Swapping numbers.*

Delphi drew students' attention to Pio's 'swap-the-numbers' strategy first by asking him to repeat his strategy and then by asking students to decide if Pio's strategy was different to the counting-on strategy she modelled. Tyson did not think the strategies were different because both Pio and Delphi counted on. Pio defended his strategy and claimed it was different because he swapped the numbers before he counted on and had therefore done it easier. Delphi highlighted for students that using an easier strategy was also interesting. Students were asked to review the strategies they could bring to their addition problems – and students shared counting-on, doubles, and swapping numbers.

6.4.4 Discussion: Positioning for mathematical difference and ease

Delphi positioned herself with her highest strategy group in comparable ways to how she positioned herself with her lowest group. She again had the right to assist students to recognise, distinguish, and apply mathematically different strategies. In Excerpt 6.4.4, Delphi facilitated discussions to compare different strategies where students used their doubles knowledge, counting-on strategies, and Pio's 'swap-the-numbers' strategy to solve the problems. Delphi provided a counting-on strategy to solve 91 + 5 and positioned students to determine if it differed from Pio's 95 + 1 strategy. This led to Pio introducing the notion of an easy explanation to the discussion with his claim that I made the 1 the 5 and then it was 96 – I done it easier. The positioning of students in the highest group was also comparable with students in the lowest group. Students had a duty to share, record, justify, compare, and defend their own and others' mathematical know-how. Each of these positions was willingly accepted by students in the highest group and at no time did they attempt to position Delphi to do any of the work for them by refusing or deferring their mathematical duties.

The first storyline for students in Delphi's highest group was common to her lowest group. Students had a duty to share, explain, critique, compare, defend, model, and record their own and peers' mathematical know-how. The second storyline was that students were able to distinguish strategies and suggest ones that were different. Therefore, identifying different and easy strategies as social acts was important to students. Pio's 'swap-the-numbers' strategy was also given social significance because Delphi highlighted the strategy as being interesting to students, asked Pio to repeat his explanation, and asked students to engage with the strategy by comparing it with another she provided.

In conclusion, Delphi's positioning with her highest group was consistent with that of her lowest group. The positioning afforded opportunities for students to share different strategies, compare the attributes that constituted mathematical difference, and explore what made a particular strategy easier. Students willingly accepted these positions across the three lessons.

6.5 Jenna, Pacific School, New Entrants

Table 6.5: Jenna's strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	4	6
Year Level	New Entrant	New Entrant
Age	5	5
Gender	3 boys and 1 girl	1 boy and 5 girls
Strategy Stage	Stage 1: One to one counting	Stage 4: Advanced counting
Achievement	Below	As expected

Students in Jenna's lowest and highest strategy groups were expected to share their mathematical know-how and listen to, and apply, the mathematical know-how of others. Jenna positioned students to monitor, explain, and substantiate their own and others' mathematical know-how. The following excerpts from Jenna's teaching show how she advanced the understandings of students in both groups by positioning them to observe and comment on each other's explanations and recordings.

6.5.1 "Great checking guys"

The focus for the first lesson with Jenna and her lowest strategy group was for students to learn how to make and record sets and numbers to 10. Materials included fingers, blocks, and counters. The following excerpt shows how Jenna positioned students to help each other by providing models, assisting with counting sequences, and checking each other's counting.

Participant	Dialogue
Jenna	*Okay can everybody please look at Xiang — how many fingers is he holding up — and are you holding up the same amount?*
Duncan	*He got* [counts Xiang's fingers] *1, 2, 3 and me got* [counts own fingers] *1, 2, 3.*
Esmee	*Xiang go* [counts Xiang's fingers] *1, 2, 3 Esmee got* [counts own fingers] *1, 2, 3.*
Jenna	*Great checking guys, what about you Ned?*
Ned	*1, 3, 4.*
Jenna	*Oh hang on – can we all help Ned to check Xiang's fingers and his own?*
Students	[counting Xiang's fingers] *1, 2, 3 and* [counting Ned's fingers] *1, 2, 3.*
Jenna	*Okay Ned have another go.*

Ned	*1, 2, 3.*
	Students continued to make sets to 10 with materials and to record in the modelling book how many in each set.
Jenna	*Duncan — how many counters have you got?*
Duncan	*1, 2, 3, 4.*
Jenna	*Is he right?*
Duncan	*Yes.*
Xiang	*Yes.*
Jenna	*How do you know he is right Esmee?*
Esmee	*Duncan count, he go* [holds up 4 fingers] *1, 2, 3, 4.*
Jenna	*Great checking Esmee — well done!*

Jenna asked students to look at Xiang's model, notice how many fingers he was holding up, and use his model to monitor if they were holding up the same number. Duncan and Esmee checked they were correct by counting Xiang's fingers then their own. Ned had difficulty counting three fingers and Jenna asked students can we all help Ned to check Xiang's fingers and his own? Duncan and Esmee modelled counting Xiang's and Ned's three fingers. Jenna directed Ned to have another go and Ned correctly counted his three fingers. Students were required to check incorrect and correct answers. Duncan correctly counted his four counters and Jenna asked students to check the accuracy of his counting, decide if he was correct, and justify why he was correct. Jenna reiterated the importance of students monitoring their work when she congratulated Esmee for her great checking.

6.5.2 "Oh! We'd better check it again"

The learning intention for the third lesson with Jenna and her lowest group was learning how to "add and subtract small numbers on materials" (MoE, 2007e, p. 7). A picnic was the context for this lesson and students were placing pictures of fruit on to cardboard picnic baskets. Esmee was attempting to put 4 oranges onto her basket, and as illustrated in the next excerpt when she looked for confirmation she was correct, Jenna put the responsibility for checking and deciding if she was right back on Esmee, then the other students.

Participant	Dialogue
Esmee	[points to the 3 oranges] *Is that 4?*
Jenna	*Is it 4 Esmee — you check and see?*
Xiang	[glances at Esmee's oranges] *No 3.*
Esmee	*1, 2, 3, 4.*
Jenna	*Touch the oranges while you count them.*
Esmee	*1, 2, 3, 4* [touches one orange twice].
Xiang	*No 1, 2, 3* [touches each orange].
Jenna	*Oh we'd better check it again* [Duncan and Ned focussed on Esmee's oranges].
Esmee	[Esmee points to each orange] *1, 2, 3.*
Students	Students returned to organising their own picnic baskets.
Jenna	[Esmee added a fourth orange to her basket] *Do you need that one to make 4? Put that one in and count them again. See if you have got 4, see if you were right?*
Esmee	[smiling] *4 I got 4!*
Jenna	*Is that right, have you get 4 oranges? Good girl, good counting.*

Esmee positioned Jenna to confirm she had four oranges by asking Is that 4? Jenna did not accept the positioning and gave the responsibility for answering her own question back to Esmee and suggesting she check and see. Esmee had difficulty counting her oranges and Jenna enlisted the others' help. As soon as Jenna said oh we'd better check it, other students stopped what they were doing, focussed on Esmee's picnic basket, and counted 1, 2, 3 as she pointed to each orange. When Esmee added one more orange to her basket, Jenna questioned her about needing it and suggested she count them again to see if she was right. Esmee grinned when she counted the fourth orange and happily announced 4 I got 4! Jenna facilitated the opportunity for Esmee to self-correct with the help of her peers and ensured Esmee had the final responsibility and reward for knowing she was right. Later in the same lesson Duncan put five, instead of the required four oranges, on to his picnic basket. The next excerpt demonstrates how Jenna appropriated Duncan's self-correction and asked him to explain his mathematical know-how and related actions.

Participant	Dialogue
Jenna	*What have you done Duncan? Why did you take one off?*
Duncan	*Um.*
Jenna	*Why did you have to take 1 off?*
Duncan	[shakes his head side to side] *Because it not right, it's not 4.*
Jenna	*Oh wasn't it 4? What did you have to do then to make it right?*
Duncan	*Take 1 off and it is 4.*
Jenna	*Right — you had to take 1 off — great checking and fixing up Duncan!*

Jenna noticed that Duncan had taken corrective action by changing his set from 5 to 4 and asked him to explain why he took one orange off. Duncan had the self-awareness to notice that his set of 5 was not right because it's not 4 and the strategic knowledge to know that he needed to take one off to correct his set to four. Jenna positioned Duncan to reflect on, and repeat, his self-correction and praised him for his great checking and fixing up.

6.5.3 Discussion: Positioning for self-regulating

Jenna positioned herself with her lowest strategy group as having the right to ensure students were explaining, monitoring, and proving their own and peers' mathematical know-how. Students were positioned by Jenna to use Xiang's correct model and to critique and justify Duncan's correct counting strategy. Esmee was positioned to self-correct her counting strategy and when she was unable to, Jenna called on others for help. Jenna noticed Duncan's self-correction and asked him to explain his action. Explaining, monitoring, and proving their own and their peers' mathematical know-how positioned students as having a duty to provide counting models, represent their thinking on materials, pay attention to others' explanations, help each other, and justify their correct answers and self-corrections. Jenna expected students to share their mathematical know-how and attend to peers' mathematical know-how. Students in the lowest group readily accepted their duties and actively participated in their group's know-how.

A prevalent storyline within Jenna's teaching with her lowest group is the importance of students becoming self-aware about their own and others' mathematical know-how. Jenna ensured students had opportunities to notice their own and others' mathematics through the questions she posed and the expectations she held. This was done by asking students to check they were correct, explain how they knew they were correct, and positioning students to help each other. The awareness students were expected to have, regarding their own and peers' mathematical know-how, was a prevalent storyline with this group. Jenna set the expectation that students would participate in each other's know-how by positioning them to provide models, explanations, and assistance as required. Students accepted this positioning and on many occasions were observed volunteering their examples and help. The correcting of models such as Xiang's model of three fingers, errors such as Esmee's miscounting, and Duncan's self-correction became significant social acts for this group when they were appropriated by Jenna and other students.

In summary, Jenna's positioning decisions with her lowest strategy group supported students to share their own and experience others' mathematical know-how. Know-how was shared and experienced through students providing counting models and representations, and checking their own and others' correct and incorrect models and representations. In positioning students to monitor and reflect on their own and others' learning, Jenna assisted students to develop self-regulated learner skills. The similar positioning choices Jenna made with her highest group are illustrated through the following excerpts.

6.5.4 "How do we know who is right?"

The learning intention for all three lessons with Jenna's highest strategy group was learning how to "add tens to a number by counting on in tens or adding the tens together" (MoE, 2007e, p. 22). Jenna began the first lesson with her highest group by scattering three jellybeans and asking students to decide how many jellybeans. After answers and explanations had been shared, Jenna scattered seven, then 11 jellybeans, and again answers and strategies were shared. The fourth scattering contained 87 jellybeans and Jenna asked students to predict the number of jellybeans. Suggestions of 20, 40, and 100 were offered but students were unable to give a mathematical reason for their answers. Ainsley suggested counting the jellybeans in groups of 10 and Jenna directed students to work together to count the jellybeans into sets of 10, and put the sets of ten into a canister. The following excerpt illustrates how Jenna positioned students to determine who was right and why.

Participant	Dialogue
Jenna	*Okay so how many have we got altogether?*
Students	*87.*
Students	*No 78!*
Jenna	*Is it 87 or 78, how do we know who is right?*
Shane	*Count them.*
Jenna	*Good idea — let's use Ainsley's idea to count in tens.*
Students	[pointing to the canisters] *10, 20, 30, 40, 50, 60, 70 80.*
Jenna	*Okay so what would we keep counting in now – what would be next?*
Students	[pointing to the single jellybeans] *81, 82, 83, 84, 85, 86, 87*
Jenna	*So how many jellybeans?*
Students	*87*
Jenna	*Who thinks Ainsley's idea of counting in tens is easier than trying to work out that great big muddly pile of jellybeans? Was it easier that way?*

Jenna gave the responsibility for determining the correct number of jellybeans to the students by asking: Is it 87 or 78, how do we know who is right? Shane suggested counting the jellybeans and Jenna agreed counting was a good idea and specified the type of counting by stating let's use Ainsley's idea to count in tens. By counting in tens and ones students determined there were 87 jellybeans. Jenna further engaged students with the counting in tens strategy by asking students Who thinks Ainsley's idea of counting in tens is easier than trying to work out that great big muddly pile of jellybeans? This lesson concluded with Jenna putting students into pairs, giving each pair a pile of fewer than 100 jellybeans and asking them to count in tens to work out how many jellybeans they had.

6.5.5 "Who can tell us or show us how they know that they are right?"

The second lesson began with students sharing the total number of jellybeans they were given at the end of the first lesson and explaining their strategy for knowing how many they had. Jenna asked students to explain how they knew three pots would contain 30 jellybeans and in doing so elicited different examples of mathematical justification. Students explained how they knew they were correct and they provided explanations that included repeated addition, skip counting, and multiplication. Shane, Lilly, and Ainsley referred to their written recordings and modelled their thinking on materials to ensure their explanation was clear to others. By asking students to further explain their strategies, Jenna provided the opportunity for all students to hear examples different to their own. In the same lesson, Ainsley claimed there were 100 jellybeans because they had 10 pots of 10 jellybeans, and 10 times 10 equals 100. In the following excerpt Jenna positioned Ainsley and her peers to explain her multiplicative thinking.

Participant	Dialogue
Ainsley	*And there's 100 altogether.*
Jenna	*Why do you think there are 100 altogether?*
Ainsley	*Because 10 times 10 equals 100.*
Jenna	*But why do we have 100 altogether — are you sure?*
Ainsley	*Yes — we have 10 pots and 10 jellybeans and 10 times 10 is 100 so 10,* [holds up 1 finger] *20,* [holds up 2 fingers]
Students	[chant and hold up fingers] *30, 40, 50, 60, 70, 80, 90, 100.*
Jenna	*Do you think Ainsley is right?*
Lilly	*Yes.*
Jenna	*Why do you think she is right Lilly?*
Lilly	*Because she counted up in tens.*
Jenna	*What do you think Candace? Do you think she might be right?*
Candace	*Yip.*
Jenna	*Why do you think she might be right?*
Candace	*Because if there is 10 packets and they all have ten there is 100* [counts each pot] *10, 20, 30, 40, 50, 60, 70, 80, 90, 100 and 10 times 10 is 100.*
Jenna	[places 10 packets of 10 in front of Candace] *So there's your ten packets of jellybeans so are you saying that's 100?*
Candace	*Yes because 10 pots of 10 is 100.*
Jenna	*Bianca — is she right?*
Bianca	*Yes it’s the same as plussing them* [points to each pot] *10 plus 10 plus 10 plus 10 plus 10 plus 10 plus 10 plus 10 plus 10 plus 10.*

Jenna positioned Ainsley to share her multiplicative know-how and positioned students to make sense of, and explain, Ainsley's strategy. Ainsley discussed the relationship between skip counting and multiplicative thinking, Candace reiterated this, and Bianca described an additive relationship. By expecting students to explain each other's correct answers and strategies, Jenna positioned students to engage with other ideas at a higher cognitive level. As well as unpacking and explaining their own ideas, students had to understand others' strategies so they could also unpack and explain them. Students had access to their group's mathematical know-how, explanations, and understandings.

6.5.6 Discussion: Positioning for mathematical justification

Jenna positioned herself with her highest strategy group as having a duty to support students to individually and collectively know when and why they were correct. This positioning was similar to the way she positioned herself with her lowest group whereby students had a duty to check and justify why they were correct. In Excerpt 6.4.4, students determined if the correct number was 87 or 78. Shane, Lily, and Ainsley defended their strategies for knowing there were 30 jellybeans and Candace and Bianca justified Ainsley's multiplicative strategy in Excerpt 6.4.5. In each of these excerpts students accepted their duty to reason and justify they and others were correct. A consequence of Jenna's positioning of students was that they had access to the explanations and justifications of each other's mathematical know-how. In Ainsley's case this was mathematical know-how that advanced the learning of the whole group because she introduced and used a more advanced multiplicative strategy. Students having access to different or advanced know-how is reliant on teachers incorporating different and advanced know-how into the lesson. By positioning herself as having the right to expect students to explain and justify their answers, Jenna simultaneously positioned herself as having the right not to be the one providing the explanations and justifications. Students have a duty to check their own and others' mathematical know-how and a duty to know why they and others were correct or incorrect. As with the lowest group, Jenna's positioning decisions assisted students to develop self-regulatory skills.

There were three storylines apparent in Jenna's teaching with her highest strategy group. The first storyline was that students were expected to share, explain, discuss, model, justify, and record their own and others' mathematical know-how. A second storyline was that students were expected to be able to explain how they knew they were correct. This expectation resulted in students hearing the answers of others but, more importantly, hearing the reasoning and understanding behind their answers. The third storyline, evident in the first excerpt, was students had a responsibility to correct any misconceptions or errors. The shared expectation that students were able to explain their own and others' mathematical know-how was a significant social act for this group. Students' skip counting, additive, and multiplicative strategies explanations became important when Jenna positioned them to explain and justify their strategies.

In conclusion, Jenna's positioning decisions with her highest strategy group supported students to share their own, and experience others' mathematical know-how. Know-how was shared and experienced through students proposing counting methods, discussing strategies and determining correct answers, and sharing and proving their own and peers' strategies.

6.6 Kendra, Tasman School, Years 5 and 6

Table 6.6: Kendra's strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	8	4
Year Level	5 and 6	6
Age	10 and 11	11
Gender	4 boys and 4 girls	3 boys and 1 girl
Strategy Stage	Stage 6: Advanced additive to early multiplicative part-whole thinking	Stage 7: Advanced multiplicative to early proportional part-whole thinking
Achievement	As expected	As expected

Kendra appeared not to want to have the position of sole authority with either her lowest or highest group. This was most evident when, at the beginning of all six lessons, she asked students to remind her if she was talking too much or if she started doing all the work such as explaining, modelling, or recording. The following excerpts illustrate how Kendra positioned students in the lowest group to reconcile their mathematical disagreements and how questions from students in the highest group influenced their learning opportunities.

6.6.1 "Right you two - Prove it"

In the first and second lessons with Kendra and her lowest strategy group students were learning how to "solve multiplication problems by taking some off and putting some on" (MoE, 2007f, p. 32), for example, solving 9 x 3 as (10 x 3) – 3. The first lesson finished with students disagreeing about how to determine the correct amount to compensate. Kendra asked students to have a think about a strategy we could use that would help us to know how much to compensate and we will talk more in our next lesson. The second lesson began with Kendra recapping the disagreement and asking if anyone had a strategy for how they could know how much to compensate. Ruby stated that the number that got compensated was the number at the front. Wiremu disagreed with the number that stayed the same got compensated. Ruby's argument is that the amount compensated is the first number in the equation. Wiremu's argument is the number that stays the same is compensated, and depending on how the problem is recorded that number could be the multiplier (number of times the set is repeated) or the multiplicand (size of the set). The following excerpt illustrates how Kendra encouraged Ruby and Wiremu to share their mathematical know-how by explaining, arguing, and proving their compensation strategies and assisting others to understand their strategies. The excerpt began with Kendra giving Ruby and Wiremu a felt-pen, indicating either side of the modelling book to them, and saying Right you two – prove it!

Participant	Dialogue
Ruby	[records in the modelling book and says] 3 x 10 = 30 3 x 9 = 27 *3 times 10 equals 30 and 3 times 9 equals 27* [indicates to her recording] *See you take away the first number. You take away 3. 30 takeaway 3 is 27.*
Wiremu	[records in the modelling book and says] 5 x 8 = 40 6 x 8 = 48 *No 5 times 8 is 40, and 6 times 8 is 48, and you add on 8 because the 8 stays the same.*
Kendra	*Any questions for Ruby or Wiremu?*
Wiremu	*Yes put it the other way — put it like 10 times 3 and 9 times 3.*
Ruby	*Why?*
Wiremu	*Because then it's the second number — the 3 stays the same and that's the one you take away!*

Kendra	*Okay can we just have thumbs up if you understand what Wiremu is saying or a thumb sideways if you are still not too sure?*

Kendra asked the group to consider how they could solve their dilemma of how much to compensate and Ruby and Wiremu volunteered their verbal and recorded compensation strategies. Kendra did not say who was right or wrong; instead, she asked the group if they had any questions for Ruby or Wiremu. Wiremu challenged Ruby regarding the way she set out the problems and used his own mathematical reasoning to evaluate the sensibleness of his and others' ideas. Students were asked to self-monitor how well they understood Wiremu's justification of the correctness of his strategy. Through upright and sideways thumb actions students assessed themselves as having full or partial understanding. At this point, Wiremu was repositioned by some students (those with sideways thumb actions) and Kendra to explain his strategy further and ensure he was being understood by others.

Participant	Dialogue
Kendra	*Okay Wiremu some of us need a bit more of an explanation — can you explain your reason again and maybe make sure we are with you as you explain?*
Wiremu	*Okay,* [speaks slowly in a higher pitch] *now Ruby said you take away the first number and take away 3 so on this one* [indicates Ruby's recording] *3 times 10 equals 30 and 3 times 9 equals 27 and she took away 3 the first number. Are you with me?*
Students	[laughing] *Yes.*
Wiremu	*Good. But if we turned the numbers round and did* [records] *10 times 3 is 30 and 9 times 3 is 27 then* [indicates the 3] *it's the second number that's the same but we still take away 3. Yes?*
Students	[laughing] *Yes.*
Wiremu	*Good. But I reckon that you take away the number that stays the same because* [indicates the equations] *in 3 times 10 and 3 times 9 the 3 is the same and in 10 times 3 and 9 times 3 the 3 is the same and in both you take away the 3 to work out the 3 times 9 or the 9 times 3. Okay?*
Students	[laughing] *Okay!*
Kendra	*Do I really sound like that?*
Students	[laughing] *Yes!*

Kendra	*Oh dear! Right I would like you to have a chat with your partner about Wiremu's same number strategy and if you have any questions – ask him!*

Wiremu adopted a falsetto and honey-tone to his voice and explained his 'same number' strategy in greater detail to the group. Students responded positively to Wiremu's new persona by smiling and laughing with him. Kendra sought reassurance of students' understanding by having them apply and test Wiremu's 'same number' strategy throughout the remainder of the lesson. Wiremu's position of authority within the lesson was sustained and the group was positioned to ensure they had appropriated Wiremu's strategy in a meaningful fashion.

6.6.2 Discussion: Positioning for intellectual autonomy

Kendra positioned herself with her lowest group to facilitate the sharing of mathematical know-how between students, to review the explanations and understandings of the know-how, and to ensure students took responsibility by arguing for, and defending their know-how. Through positioning herself as a facilitator, Kendra simultaneously positioned students to be active. Students were positioned to represent, compare, explain, argue, challenge, and justify their mathematical know-how. Kendra positioned them to solve their own mathematical dilemmas and this positioning was accepted by Ruby and Wiremu. They were positioned by Kendra as authorities within the lesson because she directed them to explain, model, and justify their mathematical know-how to others. Both students had the right to explain themselves and the duty to ensure they were understood. Students, through their self-assessment and in agreement with Wiremu, had a duty to ensure they understood his explanation. They accepted Kendra's positioning decisions and they did not look to her to sanction Ruby's or Wiremu's explanations. Kendra's positioning of herself and students facilitated opportunities for students to develop intellectual autonomy because they were expected to reflect on their learning, negotiate their differences, and work toward a shared understanding.

Four different storylines were evident in the excerpts above. In one storyline there was an accepted expectation that students solved their own mathematical dilemmas. Kendra did not provide any answers or strategies; instead, she positioned students to solve any mathematical confusion themselves. This storyline was shared by Kendra and students because there were no examples of students positioning Kendra to provide answers in any of the lessons. The story Kendra was telling was that she was not the authority in the lesson and that students needed to share and listen to each other. A second storyline was students had the right to share their mathematical know-how and a duty to ensure their know-how was understood by others. This storyline was evident in Ruby's and Wiremu's willingness to share their strategies and their attempts to defend their strategies to each other and their group. In a third storyline it was acceptable to contest the mathematical reasoning of others. This can be seen in Wiremu's challenge of Ruby's first-number strategy and Kendra's appropriation of Wiremu's challenge. The renegotiated nature of authority between the teacher and students which Kendra initiated when she positioned Wiremu as teacher was the fourth storyline. By positioning Wiremu as teacher, Kendra reduced the gap in status between herself and the students.

There were four social acts arising from the actions of Kendra and students in the lowest group. The first social act was the expectation students would work through their disagreements and misconceptions and come to a shared agreement before they moved forward with their mathematics. The recordings that Ruby and Wiremu made to represent their mathematical thinking became social acts when they were referred to by both students and used to illustrate their verbal descriptions. Ruby referred to her recording when she justified her explanation and Wiremu referred to both recordings when he explained his same number strategy to the group. The strategies of Ruby and Wiremu were given significance by Kendra when she asked others in the group to direct any questions they had to them. Finally, Wiremu's mathematical reasoning became a social act when Kendra gave it force and significance by referring to it as Wiremu's same number strategy. The importance of Wiremu's strategy was sustained when students applied and tested his same number strategy by solving problems such as 6 x 499 and 6 x 501.

To review, Kendra positioned students in her lowest group to solve their own mathematical dilemmas, to argue for their answers and strategies, and to understand and apply each other's mathematics. Kendra's positioning decisions promoted a sense of intellectual autonomy amongst students in the lowest group. From the positions given, students were expected to become more aware of, and draw on, their own and others' capabilities when making judgements, exploring, reasoning, and conjecturing about their mathematics.

The next section examines the positioning decisions of Kendra for herself and her highest strategy group. Questions from students about their mathematics learning were a common feature of the lessons taken by Kendra with her highest strategy group. The ways Kendra appropriated students' questions and used the questions as a means to extend students' mathematics learning beyond the initial expectations of the lesson are illustrated in the following excerpts.

6.6.3 "Can we use what we have worked out so far?"

The first and second lessons with Kendra and her highest strategy group focused on students learning how to "solve multiplication problems with powers" (MoE, 2007f, p. 73). The first lesson concluded with Tama asking can you have a power to the one and the zero. Kendra suggested students explore powers to 1 and 0 in the next lesson and they joked with her that she did not know the answer and would have to find out. Kendra laughed and commented how fabulous it was to work with this group because she learned so much! At the conclusion of this lesson Kendra admitted she was not sure how to model or explain powers to 1 and 0 and that she did need to double check.

The second lesson began with Kendra repeating Tama's question and asking if anyone had an answer or suggestion. There were no suggestions from the group, so Kendra presented a table, Figure 6.1, she had drawn in the modelling book.

10^0	**10^1**	**10^2**	**10^3**	**10^4**	**10^5**	**10^6**

Figure 6.1. **Table for exploring 10^1 and 10^0**

Kendra directed students to talk with a partner and between you can you think of anything we could fill in, in our table? If the first row was a question what would the answer in the second row be? The following excerpt illustrates how Kendra facilitated the opportunity for students to review their existing knowledge of exponents and use that knowledge to consider examples and emerging patterns on the table. Kendra asked students to complete a table they could use as a problem solving tool to answer Tama's question.

Participant	Dialogue
Kendra	*Shardae? Any ideas?*
Shardae	*We thought that 10 to the power of 2 would be 100 because 10 times 10 is 100.*
Kendra	*Do we agree?*
Shardae	[records 100 below 10^2]
Kendra	*Any other ideas?*
	A discussion follows where the students suggest and record 1 000, 10 000, 100 000, and 1 000 000 as the numeral for the exponential forms 10^3 to 10^6.
Kendra	*Well done guys. Now can we use what we have worked out so far to answer Tama's question about 10 to the power of 0? Looking at our chart what patterns can you see in the numbers?*
Joel	*10 to the power of 2 has 2 zeroes and 10 to the power of 3 has 3 zeroes and …*
Kendra	*Okay so can we use what Joel has noticed to find 10 to the power of 1 and 10 to the power of 0?*
Tama	*Oh, oh, oh it's gonna be 10 and 1.*
Kendra	*Why?*
Tama	*Cos 10 to the power of 6 is 6 zeroes and 10 to the power of 5 is 5 zeroes and…*
Students	[chant with Tama] *… 10 to the power of 4 is 4 zeroes, and 10 to the power of 3 is 3 zeroes, and 10 to the power of 2 is 2 zeroes and 10 to the power of 1 is one zeroes and 10 to the power of 0 is none zeroes*
Kendra	[laughing] *Okay so what does 10 to the power of one equal?*
Students	*10.*
Kendra	*And what does 10 to the power of 0 equal?*
Students	*1.*
Kendra	*So can there be powers to 1 and 0?*
Students	*Yes.*

Kendra asked students to share their existing knowledge and Shardae volunteered we thought that 102 would be 100 because 10 times 10 is 100. Kendra sought agreement with Shardae from the group then asked her to record 100. Students continued discussing their ideas and completed the table for 102 to 106. Following Kendra's direction to look for patterns within the table, Joel noticed that 10 to the power of 2 has 2 zeroes and 10 to the power of 3 has 3 zeroes. Tama appropriated Joel's pattern and the group collectively determined that 10 to the power of 1 is one zeroes and 10 to the power of 0 is none zeroes.

6.6.4 "How could we use our table to work out what 10^{-1} would be?"

Later in the same lesson, Shardae asked can you have powers that are negatives? Kendra put Shardae's question to the group. Shardae, referring to the table used earlier, suggested could we put some more columns in before 0? Kendra gave Shardae a felt-pen and told her to insert what she thought they needed in the table used in the first lesson. Shardae added three columns to the left of the table and labelled each 10-3, 10-2, and 10-1. The group was directed by Kendra to have a korero with a partner and then we will talk as a group – how could we use our table to work out what 10-1 would be?

Participant	Dialogue
Kendra	*Okay does anyone have any ideas to share?*
Harry	*We noticed that the pattern is — goes down by ten — each time.*
Kendra	*Tell us more.*
Shardae	*Well 1000 divided by 10 is 100 and 100 divided by 10 is 10 and 10 divided by 10 is 1.*
Tama	*Oh it's one-tenth!*
Kendra	[laughing] *Okay Tama tell us more.*
Tama	*1 divided by 10 is one-tenth, oh and then one-tenth divided by 10 is one-hundredth, and one-hundredth divided by 10 is one-thousandth, and then oh it would go on for negative infinities!*
Kendra	*Is that what you and Harry meant Shardae?*
Shardae	*Yes down by 10 or divided by 10.*

Harry and Shardae noticed the pattern goes down by 10 each time and expanded on this by describing 1000 divided by 10 is 100 and 100 divided by 10 is 10 and 10 divided by 10 is 1. From Harry and Shardae's observation, Tama determined 10-1 was one-tenth and Kendra asked him to tell us more. Tama elaborated on his finding and Kendra confirmed with Harry and Shardae that Tama had interpreted their observation correctly.

6.6.5 Discussion: Positioning for mathematical inquiry

Kendra positioned herself with her highest strategy group similarly to the positioning she gave herself with the lowest group. Her position was again facilitative because she supported students to answer their own mathematical inquiries into their learning, but she did not assume the right to position herself to provide answers. Kendra positioned herself to scaffold students by providing a table, asking for ideas, drawing attention to the patterns in the table and students' observations, and expecting students to make sure their explanations were understood by their peers. Students accepted this positioning and they worked collaboratively by sharing their mathematical know-how and building on, and appropriating, each other's ideas to reach agreement and answer their mathematical inquiries.

The most prominent storyline with Kendra and her highest group was that questions were welcome and were expected to be solved by the students. In the excerpts above, as with other lessons with Kendra, students posed questions about their mathematics. Each question was answered by the students. A second storyline was about the importance of students using their existing mathematical knowledge. In Excerpt 6.6.3, students used what they knew about 102 to 106 to determine if 101 and 100 were possible exponential forms and what the numeral would be. At Shardae's suggestion the table was used to determine if negative exponents were possible.

The initial table presented by Kendra and expanded by Shardae and the students suggested strategies were significant social acts within the two lessons. The use and completion of the table became a social act and had significance for the group when it was used by students for problem solving. Students applied the pattern they had previously created to determine the numeral for 10-1, 10-2, and

10-3. Shardae and Harry observed that the numeral goes down by ten. From Shardae's and Harry's observation and by applying their mathematical rule, Tama was able to strategise that 10-1 would be 1/10th because 1 divided by 10 is 1/10th. Words and actions took on a social significance when they were given meaning by other participants in the interaction. Students' suggested and recorded explanations became social acts when they were accepted and appropriated by others in the group. For instance, Joel identified the pattern between the exponents and the numeral and Tama used the pattern to identify 101, Shardae suggested extending the table into negative exponents, and Harry observed the numeral went down by 10 each time. Shardae's observation of dividing by ten was appropriated by Tama to determine 10-1, 10-2, and 10-3.

In summary, Kendra's positioning decisions with her highest group were similar to those she made with her lowest group. Kendra appropriated students' questions and made room in the lessons for the questions to be explored and answered by them. Students in this group appeared motivated to bring challenging questions to their learning and advance their own and others' mathematical know-how.

6.7 Sheridan, Pacific School, Year 2

Table 6.7: Sheridan strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	4	10
Year Level	2	2
Age	6	6 and 7
Gender	4 boys	5 boys and 5 girls
Strategy Stage	Stage 3: Counting from one by imaging	Stage 4: Advanced counting
Achievement	Below	As expected

The positioning of Sheridan and students in her lowest and highest strategy groups is discussed in the following excerpts. Sheridan positioned students in both groups to work collaboratively to correct their own and peers' errors and misconceptions, to explore advanced explanations and written recordings, and to critique their own and apply others' mathematical know-how.

6.7.1 “That was a very good discovery”

The focus for the first lesson with Sheridan and her lowest strategy group was to use counting materials to create groups of ten. In a previous lesson students created individual “10 Posters” by gluing leaves, photographs of their fingers and toes, and pictures from magazines in groups of 10 on to paper. In this lesson Sheridan asked students to individually check each poster showed 10. The objects were counted and each poster was placed in the “is 10” or “is not 10” pile. The “is 10” pile was double checked by Sheridan and students; any that were not 10 were placed on the “is not 10” pile. The following excerpt outlines the discussion of the “is not 10” posters and illustrates how Sheridan positioned students to monitor and correct their mathematical know-how.

Participant	Dialogue
Sheridan	*Right have a look at this poster for me — is it 10?*
Students	*No.*
Sheridan	*Oh what is it then?*
Students	*1, 2, 3, 4, 5, 6, 7, — 7.*
Sheridan	*Oh what could we do to make it 10? Have a think about that, we have 7 on our poster but we want to have 10.*
Mason	[Oliver draws 3 circles] *He made it 10.*
Sheridan	[to Oliver] *Have you made it 10 — so what did you do?*
Oliver	[holds up 3 fingers] *3.*
Sheridan	*3 what love?*
Oliver	[holds up 3 fingers] *3 more fingers.*
Sheridan	*Can anyone tell us how Oliver has fixed up our 10?*
Regan	*7 and 3 makes 10.*
Sheridan	*Oliver can you write that on our poster please? I think what Oliver did is he knew we had 7, everybody hold up 7 fingers, and then he counted on how many more fingers would he need to make 10? Can you guys count on from your 7?*

Sheridan asked students to check if the poster showed 10 and to suggest ways to correct the poster. Students agreed there were 7, not 10 pictures and Sheridan enquired as to what we could do to make it 10. Mason noted Oliver made it 10 and Sheridan asked Oliver to explain what he had done. She questioned students: Can anyone tell us how Oliver has fixed up our 10 and Regan described

Oliver's actions as 7 and 3 makes 10. Sheridan commented on Regan's number story as a very good discovery. By revoicing 7 and 3 makes 10 as 7 plus 3 equals 10, asking Oliver to record Regan's number story, and revoicing and modelling Oliver's strategy, Sheridan provided the opportunity for all students to access and experience more advanced mathematical explanations and recordings.

Later in the same lesson students identified one poster had 11 objects, not the required 10. The following excerpt shows how Sheridan positioned the boys to collaboratively solve the problem of one poster having 11 objects and learn about recording subtraction number stories.

Participant	Dialogue
Sheridan	*Excellent — right let's have a look at this poster — someone put this poster in the 'not 10' pile – I wonder why?*
Students	*1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.*
Mason	*That's not 10 that's 11.*
Sheridan	*How are we going to fix it so we have enough — we have too many with 11 — we need 10.*
Regan	*Cross one out.*
Sheridan	*Do the rest of you agree — would you cross one out — or would you do something differently?*
Regan	*Cross one out.*
Sheridan	*What do you think Curtis?*
Curtis	*Cross one off.*
Sheridan	*And what would be our number story — our subtraction number story — wow you guys are doing so well!*
Regan	11 takeaway 1 is 10.
Sheridan	*Excellent Regan — could you record that for us please*?

Sheridan brought the poster to the students' attention and asked them to determine why it was in the "not 10" pile. Students counted and concluded there were 11 objects. Sheridan asked for suggestions of how to fix their problem of having too many with 11. Regan suggested crossing one out and Sheridan sought agreement or suggestions for different strategies from his peers: Do the rest of you agree – would you cross one out – or would you do something differently? Regan agreed with crossing one out. Sheridan enquired as to what the subtraction number story would be and Regan suggested the number story 11 take away 1 is 10.

6.7.2 "We have two different answers – what should we do?"

In the second lesson with Sheridan and her lowest group, students were learning to image numbers up to 5, then 10, to solve addition and subtraction problems. Imaging requires students to "image visual patterns of the objects in their mind and count them" (MoE, 2007a, p. 3). Using cardboard replicas of oranges and baskets, students were placing the oranges on to two baskets and determining how many oranges there were altogether. Sheridan introduced the concept of adding with or from zero. Students were asked to model, then solve, the following problem: we have 1 orange in this basket and no oranges in this basket – how many oranges do we have altogether?

Participant	Dialogue
Regan	*Equals 1.*
Oliver	*Equals 10.*
Sheridan	*Oh we have two different answers – what should we do?*
Mason	*Check them Miss, check who is right.*
Sheridan	*Great idea – right Regan why are you right?*
Regan	*Look 1, it's just 1, there ain't no more oranges so it's just 1.*
Sheridan	*Okay so what about you Oliver, what happens when you check?*
Oliver	[points to his recording 1 + 0 = 10] *1 plus 0 equals 10 — see a one and a zero makes a 10.*
Sheridan	*Can anyone see the problem here? Would anyone like to share what they think or what they noticed?*
Regan	*I counted and it is 1, cos 1 and none is 1* [lifts up the orange] *there is only 1, there isn't 10 Oliver.*
Sheridan	*Oliver — can you count the fruit too please?*
Oliver	*1.*
Sheridan	*So what do you think happened with your number story? Anyone? Does anyone think they know what happened with Oliver's number story?*
Students	[shrugs and mumbles]
Curtis	*He done it wrong?*

Sheridan	*That's okay that's a very good teaching point for me and a learning point for you – we will look at this in our next lesson* — [writes in the modelling book] *how to record number stories with a 0.*

Sheridan noted two different answers were given and asked the group what they should do. Mason's suggestion to check the answers was appropriated by Sheridan when she called on Regan and Oliver to explain why they thought they were correct. Regan was correct but Oliver had made a recording error. Oliver recorded 1 + 0 =, then combined the digits 1 and 0 together to make what he identified as 10. Regan reiterated his strategy was to count the fruit and, when prompted to count by Sheridan, Oliver stated there was one piece of fruit. Sheridan sought suggestions from the group to explain what happened with Oliver's number story but this was met by shrugs and mumbling. Sheridan made the decision at that time to continue with the students solving addition and subtraction problems to 5 and then 10, and to make how to record number stories with a zero the teaching point for the next lesson.

After this lesson Sheridan indicated she had been unable think of a way to help Oliver through his recording confusion that did not involve telling him what to do. Instead, Sheridan made the decision to explore number stories with zeroes in the next lesson and gave herself some time to think of ways to support students to assist Oliver to self-correct his recording error.

6.7.3 Discussion: Positioning for self-regulating strategies

Sheridan positioned herself with her lowest strategy group to provide opportunities for students to notice, explain, monitor, review, justify, and record their own and peers' mathematical know-how. Sheridan expected students to review their own and others' work and make any corrections. Students were expected to check their mathematics, offer suggestions on how to correct others' mathematics, agree on which strategy to use, and model and record their corrective strategies. In excerpt 6.7.1, students reviewed their previous work, identified if they were correct or incorrect, determined strategies to self-correct, explained and revoiced their own and others' strategies, and suggested ways to record their mathematical know-how. Through reviewing and correcting their 10s posters, students were required to engage at a more critical level with the task.

There were two storylines prevalent in Sheridan's teaching with her lowest group. The first storyline was that completing the mathematics work was not enough; work should also be reviewed, checked, and corrected where necessary. When two different answers were provided for 1 + 0 in Excerpt 6.7.2, students knew to suggest checking as the means to determine which answer was correct. The second storyline was that reviewing, checking, and correcting were the responsibility of the students. In the same excerpt discussed above, students were unable to assist Oliver with his recording error 1 + 0 = 10 and, as noted by Sheridan, she was unable to think of a way to assist Oliver. Instead of telling Oliver why he was incorrect, Sheridan appropriated the error as a very good teaching point for me and a learning point for you. Sheridan gave herself the responsibility of bringing the error back to the group's attention but the responsibility for correcting the error would be with the group. Reviewing, checking, and correcting their work are important social acts for this group and they gained significance because of Sheridan's emphasis and expectations. Errors were also significant social acts for this group as the errors they made became their next learning steps.

In conclusion, Sheridan expected students in her lowest group to monitor their own and others' mathematical know-how, and to correct and learn from any errors students made. Students had a duty to pay attention to the mathematics they and their peers were explaining and, where required, correct each other's thinking. These duties were accepted by students as they noticed errors, provided the strategies to correct the errors, and progressed the mathematical know-how of the group with advanced addition and subtraction number story recordings. Sheridan's teaching and positioning decisions with her highest group are described in the following sections.

6.7.4 "Can anyone help me help Eve?"

The learning intention for all three lessons with Sheridan's highest strategy group was learning how to "add tens to a number by counting on in tens or adding the tens together" (MoE, 2007e, p. 22). In the first lesson, Sheridan modelled 20 using two pots of 10 beans and, indicating to the equipment of pots of 10 beans and single beans, asked students how can we change our 20 to 28? Eve suggested add some more of them and Sheridan asked her to show us, do it for us, use the equipment to show 28, and explain what you are doing as you go? Eve grouped 2 pots of 10 and stated they were 20; she added another pot of 10 and claimed the total was 23. When questioned by Sheridan, Eve added another pot of 10 maintaining and now it's 24. Sheridan instructed the group to listen to what Eve is telling us and Eve added 3 more pots of 10 and continued with her explanation: Then it goes 25, 26, 27. At this point, Eve was asserting 9 pots of 10 represented 27. Eve's confusion between the place value of tens and ones and how Sheridan facilitated the opportunity for students to assist Eve with her misconception is outlined in the excerpt below.

Participant	Dialogue
Sheridan	*Okay does anyone understand what Eve is saying there? Can anyone help me help Eve?*
Cooper	[picks up one pot] *How much is this?*
Students	*10.*
Cooper	[picks up one bean] *How much is this?*
Students	*1.*
Cooper	[to Eve] *So how can 9 of these* [indicates 9 pots] *be 27?*
Eve	*What?*
Sheridan	*Thanks Cooper I see where you were headed with that — can anyone else help Eve?*
Lyn	*Can I show her what I think?*
Sheridan	*Sure!*
Lyn	*Okay Eve — this is 10 [gives Eve a pot] and this is 1* [gives Eve a bean]*. So if we count the pots — you count with me.*
Lyn & Eve	[count two pots of 10] *10, 20.*
Lyn	[moves 7 pots to the side and replaces them with 7 beans] *And then we count the beans…*
Lyn & Eve	*21, 22, 23, 24, 25, 26, 27*
Sheridan	*So Eve, if you added the 1 bean that Lyn gave you, how many would we have?*
Eve	*28.*
Sheridan	*Ah so what happened before?*
Ilana	*She meant ones but used tens.*

Sheridan	*Is that what you did?*
Eve	*Yes.*
Sheridan	*So when you had 9 pots of 10 what did you really have? Shall we count them together?*
Students	*10, 20, 30, 40, 50, 60, 70, 80, 90.*
Sheridan	*Okay and Eve thank you for sharing your ideas because you really made us all think and making us think is a good thing.*
Eve	[grins]
Sheridan	*Okay so if we go back to our 27 and I added another pot of 10, talk to your neighbour — how many do we have altogether?*

Eve claimed 9 pots of 10 represented 27. Sheridan positioned students to participate in Eve's mathematics by asking: Perhaps someone doesn't agree that Eve has got it quite right. Cooper highlighted the difference in value between a pot (10) and a single bean (1) but Eve did not find this helpful. Lyn volunteered to show Eve what she thought, gave Eve one pot and one bean to hold, and instructed Eve to count with me. Together Lyn and Eve counted the two pots of 10 and the seven single beans. It appeared Eve had corrected her place value misconception when she correctly answered Sheridan's question – if you added the 1 bean that Lyn gave you, how many would we have? Sheridan thanked Eve for sharing her ideas and making everybody think and Eve grinned.

6.7.5 "Who agrees with Reuben - that it's 60 plus 20 is 62?"

In the second lesson Sheridan overheard Reuben claiming that 60 plus 20 equalled 62: Because it's 6 and 2 so you take away the zero. Sheridan recorded 60 + 20 = 62 in the modelling book and questioned Reuben so you go like this do you? Reuben agreed the recording was correct so, as the next excerpt shows, Sheridan drew on the group to critique Reuben's idea and help him to correct his place-value misconception.

Participant	Dialogue
Sheridan	*Who agrees with Reuben that 60 plus 20 is 62? Who disagrees? Alright Cooper would you like to tell him why you disagree with him.*
Cooper	*Because they are tens and so they can't be ones.*
Sheridan	*Would you like to explain that a bit more — somebody else? Because I don't know if Reuben would understand that. Wade, can you explain that to him a bit more?*

Wade	*Well if you have 60 and if you have 2 in the ones that would make 62 but it's 20 so if you have 6 and 2 more its 80 because it's tens.*
Sheridan	*Do you get that Reuben?*
Reuben	*No.*
Sheridan	*Not really? Okay Wade can we try your ideas together?*
Wade	*Yip*
Sheridan	*Reuben how many do we have here?* [points to the 6 pots of 10] *Can you count them – how many is there? You can count them in tens if you want to.*
Reuben	*10, 20, 30, 40, 50, 60 — 60.*
Sheridan	*60 so there is 60 there. How many is here Reuben?* [points to the 2 pots of 10]
Reuben	*10, 20 – 20.*
Sheridan	*So would you agree that this is 60 plus 20?*
Reuben	*Yes.*
Sheridan	*Yes? Everyone agrees with that?*
Students	*Yes.*
Sheridan	*So Wade, what do you think would help Reuben next?*
Wade	*Put them all together and count them up, put the 60 and 20 together and count them up, all of them.*
Sheridan	*Alright, so Reuben can you move the group of 60 and the group of 20 together and then can you count them for us?*
Reuben	*10, 20, 30, 40, 50, 60, 70, 80, oh 80*
Sheridan	*So what do you think happened before?*
Reuben	*I thought the 2 were ones not tens.*
Sheridan	*I see, and if they were ones and the question was 60 plus 2 would Reuben's 62 have been correct?*
Students	*Yes.*
Sheridan	*It would wouldn't it?*

Reuben suggested 62 was the answer to 60 plus 20. Sheridan asked Reuben to explain his answer and drew the group's attention by asking who agreed or disagreed with him. Cooper and Wade explained why they disagreed with Reuben but Reuben did not find their explanations helpful. At this point, Sheridan asked Wade if they could work together to help Reuben and he agreed. Reuben was directed to count the 6 pots of 10 and the 2 pots of 10 and Sheridan sought Wade's advice on what Reuben should do next. Wade's suggestion was for

Reuben to put the 60 and 20 together and count them up. Reuben skip-counted to 80 to get the correct answer and explained that his error was he thought the 2 were ones not tens. The group was questioned regarding the correctness of Reuben's initial interpretation of the problem as 60 + 2 = 62 and they confirmed this would have been correct.

6.7.6 Discussion: Positioning for self-regulating

Sheridan positioned herself with her highest group in similar ways to her lowest group. Students in the highest group were provided opportunities to notice, explain, monitor, review, and justify their own and their peers' mathematical know-how. Sheridan also expected students to engage with others' mathematical know-how. In Excerpt 6.7.4, Eve was positioned to defend her claim that 9 pots of 10 equalled 27, correct her place value error, and apply her improved place value understandings. Eve's peers were positioned to challenge her strategy and provide suggestions for her to correct her strategy. Reuben also demonstrated an incorrect place value strategy in Excerpt 6.7.5, and his peers were positioned to question his strategy and help him to correct his error. Sheridan had the right to include the group in individuals' errors and students had the duty to help their peers correct their errors.

The storylines and social acts occurring in Sheridan's teaching with her highest strategy group were similar to those present with her lowest group. As with the lowest group, errors were seen as shared teaching and learning opportunities. Students took on a teaching role when they helped each other correct errors. In the excerpts above, Cooper, Lyn, Wade and the group as a whole assisted Eve and Reuben to correct their place value errors. Students appeared to accept this positioning from Sheridan and willingly assisted peers. Students assisted their peers by asking questions and providing models that promoted understanding. Reviewing, checking, and correcting their work were key social acts for this group and they became important because of Sheridan's emphasis and expectations.

To summarise, Sheridan's positioning decisions with her highest group supported students to share their own and experience others' mathematical know-how. Know-how was shared and experienced through students explaining their strategies, identifying correct and incorrect strategies, and suggesting and modelling corrective strategies. As with the lowest group, errors were treated as valuable teaching and learning tools by Sheridan.

6.8 Chelsea, Tasman School, Years 4 and 5

Table 6.8: Chelsea strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	7	5
Year Level	4	4 and 5
Age	9	9 and 10
Gender	1 boy and 6 girls	4 boys and 1 girl
Strategy Stage	Stage 5: Early additive part-whole thinking.	Stage 6: Advanced additive to early multiplicative part-whole thinking
Achievement	As expected	As expected

Chelsea was consistent in the positioning decisions she made regarding herself and students in her lowest and highest strategy groups. However, the way Chelsea positioned herself and students seemed to constrain, rather than afford, opportunities for students to share and listen to others' mathematical know-how. The following excerpts illustrate the emphasis Chelsea placed on correct answers with her lowest group and on students being limited to specific strategies with her highest group.

6.8.1 "Are there 4 in each group? Or can I see 4 groups of …?"

The learning intention for every lesson with Chelsea and her lowest strategy group focussed on students "learning to solve multiplication problems using arrays" (MoE, 2007f, p. 15). Chelsea began the first lesson by sharing the learning intention. Teulia asked what are arrays and Chelsea indicated the equipment stating: these are. An array of four sets of two goats was shown to students, Chelsea asked them how many have we got there altogether and students chanted 2 plus 2 plus 2 plus 2 equals 8. The next excerpt illustrates how Chelsea questioned and prompted students to find the correct answer.

Participant	Dialogue
Chelsea	*Okay, we have got 8 goats. How can we use it, how can we use multiplication, how can we use sets of* [records x in the modelling book], *how can we use this operation here? How can we turn that into this – okay? Daphne?*

Daphne	*There are 2 sets of …*
Chelsea	*Remember sets means groups of. Ashleigh?*
Ashleigh	*2 sets of 2 equals 8?*
Chelsea	*2 sets of 2?* [indicates 2 sets of 2 goats] *okay this would be 2 sets of 2 just here,* [holds up one set of two goats] *look there's one set* [holds up two sets of two goats], *there's another set, and there is 2 goats in each set just there isn't there* [indicates 2 arrays*], in each group, so what would this be?*
Ashleigh	*2 sets of 4 are 8?*
Chelsea	*Are there 4 in each group? Can I see 2 sets of 4 goats, or 2 groups of 4 goats, or 2 times 4? Or can I see 4 groups of …?*
Students	*2.*
Chelsea	*4 sets of 2 very good. Let's double check that* [points to each array] *1, 2, 3, 4, yes, there are 4 sets there isn't there? There are 4 groups and how many goats in each group?*
Students	*2, 4, 6, 8.*
Chelsea	*2 good so 4 sets of 2 equals 8. 2, 4, 6, 8 and this is where knowing how to count in 2s becomes very handy and you guys can do that can't you? We don't need to go 2 plus 2 plus 2 plus 2 do we? We can write it as* [records 4 x 2 = 8] *4 sets of 2 equals 8. Okay can you guys count in fives?*
Students	*5, 10, 15, 20, 25, 30, 35, 40, 45, 50.*
Chelsea	*That's brilliant, well done, I knew you could do that!*

Daphne began to suggest there are 2 sets of … but was interrupted by Chelsea saying remember sets means groups of and passing the question to Ashleigh to answer. Ashleigh answered 2 sets of 2 equals 8 and Chelsea used the arrays to show why Ashleigh's answer was incorrect and illustrated how many 2 sets of 2 would be. Ashleigh's second suggestion was 2 sets of 4 are 8 and Chelsea questioned her about how many goats were in each array and led students to the correct answer by asking Or can I see 4 groups of …? On hearing the correct answer Chelsea double checked for students by counting each array and confirming there were 4 sets of 2. Daphne skip-counted the 4 sets of 2 and Chelsea highlighted the connection between skip-counting and multiplication. Students' knowledge of skip counting was checked and Chelsea presented a 5 by 5 array for students to record as a multiplication equation.

6.8.2 "No – you have 4 sets of 3 look"

During the third lesson, confusion arose between students regarding how to correctly use the array equipment to represent a multiplication equation. Chelsea asked students to work in pairs using the arrays to represent different equations. Teulia and Ashleigh were asked to model 3 sets of 4 but rather than work together, Teulia created an array showing 3 sets of 4 and Ashleigh created a different array showing 4 sets of 3. In the next excerpt Chelsea explains why Ashleigh was incorrectly representing 3 sets of 4.

Participant	Dialogue
Chelsea	*Can we please have a look at what Teulia and Ashleigh have done? Teulia what are you showing us?*
Teulia	*3 sets of 4.*
Chelsea	*Good. And Ashleigh what are you showing us?*
Ashleigh	*3 sets of 4.*
Chelsea	*No – you have 4 sets of 3 look* [holds up one row of 3 rabbits], *1 set of 3,* [holds up second row of 3 rabbits], *2 sets of 3,* [holds up third row of 3 rabbits], *3 sets of 3,* [holds up fourth row of 3 rabbits], *4 sets of 3. You have made 4 sets of 3 not 3 sets of 4. Now you need to put those away and share with Teulia. Now I want you all to write the answer down. What is the answer to what Teulia is showing us? What is the answer to 3 sets of 4? How many is 3 sets of 4 altogether and everybody write their answer down.*
Students	Worked out 3 sets of 4.
Chelsea	*Okay — who has an answer for us? Kateraina?*
Kateraina	*12.*
Chelsea	*Mei-Lien?*
Mei-Lien	*12.*
Chelsea	*Ashleigh?*
Ashleigh	*12.*
Chelsea	*Peata?*
Peata	*12.*
Chelsea	*Excellent looks like we are all back on track.*

Teulia and Ashleigh were asked to explain their representations for the group. Chelsea showed her agreement with Teulia's representation by stating good. Ashleigh claimed she was showing 3 sets of 4 and Chelsea explained why her representation was not correct. Students were asked to record the answer to 3 sets of 4 and on hearing the correct answer of 12 from four students, Chelsea announced: Excellent looks like we are all back on track. The lesson continued with Chelsea checking students' representations were correct and ensuring they were able to give the correct answer for the multiplication equation.

6.8.3 Discussion: Positioning for correct answers

Chelsea positioned herself to remind students of what they already knew, to explain and model why students' answers were correct or incorrect, and to illustrate the connections between students' observations and the learning intention. In Excerpt 6.8.1, Chelsea reminded Daphne that sets means groups of, she modelled for Ashleigh why her answers 2 sets of 2 equals 8 and 2 sets of 4 equals 8 were incorrect, led students to the correct answer by stating I see 4 groups of, then illustrated the connection between Daphne's skip-counting and recording multiplicative equations. Chelsea explained why Ashleigh was not modelling 3 multiplied by 4 with her 4 by 3 array in Excerpt 6.8.2. Students were positioned by Chelsea to provide correct answers and these were interpreted by Chelsea as students being back on track as evidenced in Excerpt 6.8.2. Students were not positioned to explain their own or others' correct answers and they were not given opportunities to self-correct their incorrect answers or assist others. Chelsea's positioning of herself and students in her lowest group appeared to be more about students sharing their mathematical 'know-what' than their know-how, which meant the focus was on answers rather than mathematical understandings.

The most prevalent storyline between Chelsea and her lowest strategy group appeared to be the importance of correct answers. This storyline may have been established through Chelsea's questioning for correct answers, students not being asked to provide explanations of their answers, and Chelsea describing correct answers as evidence that students were back on track. Correct answers appeared to assure Chelsea that students understood the concepts and that she could progress the learning. Correct answers were the most significant social act developing between Chelsea and her lowest group. This emphasis appeared to constrain the opportunities for students to share their know-how.

To summarise, Chelsea emphasised the need for students to provide correct answers rather than explanations and positioned herself to explain and justify the know-how behind students' correct and incorrect answers. The positioning decisions Chelsea made regarding herself and her highest group are outlined in the following excerpts.

6.8.4 "I'll bet you all went 20 plus 4…"

Students in Chelsea's highest strategy group were learning to "solve multiplication problems by taking some off and putting some on" (MoE, 2007f, p. 32) in their first and second lessons. Chelsea began the first lesson by laying out 4 rows of 5 cubes and telling students there were 4 sets of 5. Students were asked so 4 sets of 5 is how many and they collectively responded 20. The following excerpt illustrates how Chelsea explained the difference between two arrays and provided the explanation for solving 4 multiplied by 6 using the compensation strategy.

Participant	Dialogue
Chelsea	*Yes it is, good, 4 sets of 5 is 20. And it's not 5 sets of 4 is it?*
Students	*No.*
Chelsea	*No because* [points to each row of 5] *I have 1, 2, 3, 4 sets of 5. So what would 4 sets of 6 look like? Okay so let's just actually put them on,* [adds 1 cube to each row] *one more cube on here, and here, and here, and here. So now what am I showing?*
Iosefa	*6 sets of 4.*
Viliami	*4 sets of 6.*
Chelsea	*No it's not 6 sets of 4 because I have 4 sets and there are 6 in each of them. Okay so what is the answer?*
Students	*24.*
Chelsea	*Yes it is and I'll bet you all went 20 plus 4 because we added 4 more didn't we, we added one more to our 4 rows of 5 to give us 4 rows of 6 and so 20 plus 4 is 24. Is that what you all did?*
Students	*Yes.*
Chelsea	*Excellent I thought you might.*
Luke	*I did 2 times 6 is 12 and 12 and 12 is 24.*
Chelsea	*Okay but that's not what I want you to do, what I want you to do is use our compensation strategy.*

Viliami provided the answer 20; Chelsea agreed with him, pointing out there were 4 sets of 5 not 5 sets of 4, and explained why. Chelsea added one cube to each row and asked students to explain what she was showing. Iosefa proposed 6 sets of 4, Viliami suggested 4 sets of 6, and Chelsea explained why Iosefa's answer was incorrect. She then described how she thought students had strategised to work out 4 times 6 and confirmed this with them. Luke recommended a different strategy 2 times 6 is 12 and 12 and 12 is 24. His strategy was acknowledged by Chelsea when she responded okay, but he was not encouraged to apply a different strategy to the problem. Instead, Chelsea directed Luke to use our compensation strategy.

6.8.5 "Being right isn't enough; you have to use the right strategy too"

In the second lesson students were applying the compensation strategy to solve problems like 2 x 21 and 2 x 19. Chelsea asked students what the answer was to 2 times 20 and they responded correctly with 40. She then asked students to discuss how they could use 2 times 20 equals 40 to work out 2 x 21 and 2 x 19.

Participant	Dialogue
Eliza	*Well that's easy double 21 is 42.*
Luke	*Yeah and 2 times 10 is 20 and 2 times 9 is 18 and 20 and 18 is 38.*
Hamiora	*No you have to go 2 times 20 is 40 and 2 more is 42 and 2 times 20 is 40 and 40 take away 2 is 38.*
Luke	*Says who?*
Eliza	*Yeah Hamiora says who?*
Hamiora	*Says Miss, aye Miss we have to use the compensation strategy aye?*
Chelsea	*Yes you do, I want you to use our compensation strategy to solve 2 times 21 and 2 times 19.*
Luke	*But that's dumb — double 21 is 42. I don't need to times it and add just double it man it's two times so you just double it.*
Chelsea	*It's not about what you need to do. It's about knowing all the different strategies you can use. This compensation strategy could be very useful when it comes to solving harder problems.*
Luke	*Okay well give us a harder one then.*
Chelsea	*Alright then guys Luke has asked for a harder problem so he can use our compensation strategy. So how about this. What is 9 times 10?*
Students	*90.*
Chelsea	*90 good. So tell me how you would use 9 times 10 to solve 9 times 11 and 9 times 9?*
Eliza	*9 times 11 is 99.*
Luke	*No that's not using compensation.*

Eliza	*But it's still right.*
Luke	*Yeah but being right isn't enough you have to use the right strategy too.*

Eliza and Luke solved 2 x 21 and 2 x 19 using a doubling and place value strategy. Hamiora stated they were wrong because they had to use the compensation strategy which he explained as 2 times 20 is 40 and 2 more is 42 and 2 times 20 is 40 and 40 take away 2 is 38. Eliza and Luke challenged Hamiora's right to tell them they were wrong and Hamiora brought Chelsea into the disagreement as his back-up. Chelsea reiterated Hamiora's claim that the compensation strategy had to be used. Luke declared that's dumb and questioned why he needed to compensate when he could just double. Chelsea stressed the importance of knowing all the different strategies and the usefulness of the compensation strategy when solving harder problems. Luke's request for a harder problem was accepted by Chelsea and she asked students to solve 9 times 10, then 9 times 11, and 9 times 9. Eliza shared 9 times 11 is 99 and Luke told her the strategy she used was wrong because it was not compensation. Eliza asserted she was still right but Luke claimed that being right isn't enough you have to use the right strategy too.

6.8.6 Discussion: Positioning for specific strategy use

Chelsea positioned herself with her highest strategy group to explain and model students' correct and incorrect strategies for them and to expect students to apply the strategy that was the focus of the lesson. Applying one strategy per lesson was not a requirement of any other teachers in my study. In Excerpt 6.8.4, Chelsea explained the difference between a 4 by 5 array and a 5 by 4 array, clarified why she had modelled 4 sets of 6 and not 6 sets of 4, and explained how she thought students solved 4 times 6. Chelsea did not acknowledge or appropriate the different doubling and place value strategies students suggested in both excerpts or the use of basic fact knowledge in Excerpt 6.8.5. Instead, she emphasised the importance of students applying the compensation strategy. Students had a duty to follow Chelsea's model and explanations and apply the strategy selected for the lesson. In these excerpts there was no evidence of students being provided opportunities to flexibly use their mathematical know-how.

There were two storylines occurring in Chelsea's teaching with her highest strategy group. In the first storyline, applying the strategy that is the focus for the lesson appeared to be more important than applying the strategy that could be more efficient. Students were not given opportunities to trial different strategies and test them for efficiency; instead, they were directed in each lesson to use a particular strategy. In a conflicting storyline some students continued to challenge the need to apply only one strategy in each lesson. Eliza and Luke struggled to understand why they could not apply the strategy of their choice when their answer was correct. The strategy focussed on in each lesson would have held significance as a social act for the group.

To conclude, Chelsea's positioning decisions in regard to herself and students in her highest strategy group limited opportunities for students to share their mathematical know-how. Restrictions to students sharing mathematical know-how occurred through Chelsea's positioning of herself as the one to model and explain the mathematical thinking behind students and incorrect answers. The emphasis Chelsea placed on correct answers as evidence of understanding and her procedural approach to learning about different strategies may have constrained students' opportunities to share their own and experience each other's mathematical know-how.

6.9 Summary

This chapter illustrated the consistent teaching and positioning decisions of seven of the 12 teachers in my study. Six teachers – Greer, Hannah, Delphi, Jenna, Kendra, and Sheridan – positioned themselves and students in their lowest and highest strategy groups to ensure students in both groups had opportunities to share their mathematical know-how. Chelsea was also consistent in her positioning decisions regarding her lowest and highest groups but this positioning did not appear to provide opportunities for students in either group to share their mathematical know-how.

The six teachers whose positioning decisions afforded the sharing of mathematical know-how positioned themselves in five key ways: by providing models and representations; highlighting mathematical connections; emphasising the importance of different, efficient, and sophisticated explanations; stressing the need for students to check their own and others' answers and strategies; and incorporating students' questions and advanced strategies into the learning. Teachers positioned students to share their mathematical know-how in six important ways: by providing opportunities for students to explain, model, and record their thinking; consider peers' thinking; notice mathematical connections; provide and evaluate explanations for difference, efficiency, and sophistication; review and critique their own and others' work for accuracy; and inquire about their mathematics learning. These decisions show how teachers gave themselves facilitative positions and gave their students dynamic positions, which appeared to enable students to realise their teachers' expectations for them to engage in their own and peers' mathematical know-how. Using a facilitative position, it appeared teachers gave the responsibility for undertaking and completing the mathematics tasks to the students. Being positioned to do most of the mathematics work could ensure the mathematics learning of these students progressed more confidently and competently than had the teachers chosen to do the work for them.

Chelsea's decisions appeared to constrain opportunities for students to share their mathematical know-how as she positioned herself to undertake most of the mathematical thinking and modelling. Explanations and models of mathematical know-how were shared more by Chelsea than by students. The expectation of students in both groups was to provide correct answers and apply specific strategies. Not being positioned to do most of the work may have inhibited the depth of mathematics understanding that could enable students to move forward. In meeting Chelsea's expectations of them, students' opportunities to share their know-how may have been constrained.

Chapter Seven presents the case studies of the five teachers in my study who were inconsistent with their positioning decisions. Inconsistent positioning decisions were applied to individuals within a group and to the whole group.

Chapter 7: Inconsistent Teacher Positioning

7.1 Introduction

Chapter Six presented the first case study comprising the seven teachers who positioned themselves and students in their lowest and highest strategy groups to share their mathematical know-how in similar ways. Six of the teachers used positioning to influence opportunities for mathematical know-how to be shared consistently with both strategy groups. One teacher's positioning decisions appeared to constrain opportunities for mathematical know-how to be shared in both groups. This chapter illustrates the teaching and positioning decisions of the five teachers in my study who were not consistent in providing opportunities for students in their lowest and highest strategy groups to share their know-how.

Sections 7.2 to 7.7 present the case of the five teachers whose positioning decisions were inconsistent for both groups. Section 7.2 focusses on Paula (Pacific School, years 5 and 6) and demonstrates how her positioning decisions consistently provided opportunities for students to share their own and experience peers' mathematical know-how with the exception of one male student in both groups. The positioning decisions of Faith (Pacific School, year 4), Naomi (Pacific School, years 2 and 3), and Brooke (Tasman School, New Entrant) are the focus of Sections 7.3, 7.4, and 7.5. These teachers were inconsistent in the positioning decisions they made regarding the students in their lowest and highest groups; students in the highest group had more opportunities to share their mathematical know-how. Section 7.6 explores the positioning practices of Lisa (Pacific School, year 1). She appeared to position students in the lowest group to share and explain their mathematical know-how whilst good behaviour more than mathematical know-how was emphasised with her highest group. Section 7.7 summarises the inconsistent positioning decisions of the five teachers whose positioning practices comprise this chapter. Each case study in this chapter is formatted and presented in the same way as those in Chapter Six: Consistent Positioning.

7.2 Paula, Pacific School, Years 5 and 6

Table 7.1: Paula's strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	7	10
Year Level	6	5 and 6
Age	10 and 11	10 and 11
Gender	2 boys and 5 girls	7 boys and 3 girls
Strategy Stage	Stage 6: Advanced additive to early multiplicative part-whole thinking	Stage 7: Advanced multiplicative to early proportional part-whole
Achievement	As expected	As expected

The decisions Paula made regarding the positionings of herself and the students in her lowest and highest strategy group were consistent with the exception of one male student in each group. The majority of students in both groups were expected to work together to solve disagreements and correct misconceptions. Most student errors were effectively used by Paula as tools for teaching and learning. The following sections outline how two students, Wyatt in the lowest group and Nathan in the highest, appear to be excused from Paula's expectations and positioning decisions.

7.2.1 "Give each other some advice on what you think happened"

The first and second lessons with Paula's lowest strategy group focussed on students learning how to solve 6, 7, and 8 times tables using known 5 times tables. For example 7 x 7 = (5 x 7) + (2 x 7). During the second lesson, students individually solved 8 x 6 as either (8 x 5) + (8 x 1) or (5 x 6) + (3 x 6) then discussed their answer and strategy with a partner. Paula knelt behind each pair and listened to their discussion. On hearing Marama and Apirera suggest different answers, Paula directed them to show each other, and explain what you did. The next excerpt shows how Apirera was able to self-correct her procedural error by participating in Marama's dialogue and written recording.

Participant	Dialogue
Apirera	*I did 5 times 6 equals 30 and then 3 plus 6 equals 9 and that's 39.*
Marama	*I thought it equalled 48! I went 5 times 6 is 30 and 3 times 6 is 18 and that makes 48?*
Paula	*So you've both got different answers? Can you give each other some advice on what you think happened? Everybody listening please – help out where you can.*
Marama	*I broke up the 8 into 5 and 3.*
Apirera	*Me too.*
Marama	*But you plussed the 3 and the 6 not timesed them.*
Apirera	*Aye?*
Marama	*Look* (points to Apirera's equation 5 x 6 + 3 + 6 = 39) *the 3 from the 8 and the 6, you plussed the 3 and the 6 and you got 9 and 39.*
Apirera	*No but I…*
Marama	*… Look* (turns her modelling book toward Apirera and points to her recording (5 x 6) + (3 x 6) = 30 + 18 = 48) *it's 5 times 6 plus the 3 times 6, not 5 times 6 and then plussing!*
Apirera	*No but wait* (records 8 x 6 in her modelling book). *5 times 6 plus 3 times 6 is, 5 times 6 is 30 and 3 times 6 is 18 and 30 and 18 is 48 – oh I get it!*

Paula positioned Apirera and Marama to work together to determine whose answer was correct and why. Apirera correctly split the 8 of 8 times 6 into 5 and 3 but when recording her strategy she added rather than multiplied the 3 and 6. Marama used Apirera's recording (5 x 6) + 3 + 6 = 39 as a tool to show Apirera her error and referred to her own recording (5 x 6) + (3 x 6) = 30 + 18 = 48 to model the correct strategy. Apirera responded to Marama by re-writing the equation 8 x 6 and re-solving the problem. She articulated and recorded her strategy 5 times 6 plus 3 times 6, is 30 plus 18 is 48. As this occurred, Marama nodded her head up and down and smiled at Apirera. The girls continued working together setting themselves three multiplication equations to solve. Apirera was able to apply the 'Fun with Fives' strategy accurately and successfully solve the next three equations. Paula's positioning enabled Apirera and Marama to share their mathematical know-how and work collaboratively to correct Apirera's procedural misconception.

7.2.2 "Just move over here with me"

Excerpt 7.2.1 illustrated how Paula positioned students to collaboratively solve their mathematical disagreements and correct their misconceptions. Through discussions, students had opportunities to share their mathematical know-how and learn from the mathematical know-how of others. A contradiction to this positioning was apparent with one student in the lowest strategy group. When Wyatt (Year 6) disagreed with another student or made an error, Paula drew him aside from the group and worked quietly with him by herself. The following excerpt occurred after Paula overheard Wyatt and Kieran disagreeing about the answer to 6 x 8 in the first lesson. Kieran claimed the answer was 48 and Wyatt disagreed that the answer was 64.

Participant	Dialogue
Paula	*Okay so Wyatt, what was the problem you were asked to solve?*
Wyatt	*6 times 8.*
Paula	*So 6 times 8 is the same as?*
Wyatt	*5 times 8.*
Paula	*And?*
Wyatt	*3 times 8.*
Paula	*Where did you get the 3 from?*
Wyatt	*The 8 because 5 plus 3 is 8.*

Wyatt applied the strategy of using his known 5 times tables but instead of solving the required problem 6 x 8 he solved 8 x 8 as (5 x 8) + (3 x 8). At this point in the dialogue Paula indicated to Wyatt to move to the side of the group with her and Paula spoke quietly to him. In the next excerpt Paula positioned herself as the authority in the lesson by telling Wyatt where he had gone wrong and how to self-correct, and providing him with examples of the same equation to follow.

Participant	Dialogue
Paula	*Just move over here with me. Look you've broken the 8 into 5 and 3 but you've used your 8 again. Can you see that? So you've got 8 times 8. You've timesed 8, 8 times. How could you do something like that but use your 6 instead of your 8? Instead of doing 8 times 8? So you need to break it down as 5 and something else. So you have 5 times something and 5 times something so you have 5 times and break up the 8. If we look back here* [turns back the pages in the modelling book] *when we were doing this. We were working out our - using our 5s – so what was the number that we were breaking up? Let's have a look here, what have we done here? Oh here's that same sum look! So what did we do? What did we do that was different to what you did there? Have a look here. Have a look at this one, have a look at this problem. Can you see what we did?*
Wyatt	*Um?*
Paula	*We've got 6 times 8 so we are breaking our 6 into? 5 and a 1 so we can say that 6 times 8 is the same as 5 times 8 plus?*
Wyatt	*1 times 8?*
Paula	*Can you see that?*
Wyatt	*Yip.*
Paula	*Okay, so what did you do that was different?*
Wyatt	*I did the 8.*
Paula	*Yes you ended up doing 8 times 8 so do you want to try that again using the 6? Okay try that again now.*

Wyatt was informed by Paula of the error he had made you've timesed 8, 8 times, told what he needed to do to correct his procedural error so you need to break it down as 5 and something else, and shown an example of the same problem the group had solved previously. Paula did not make the same positioning decisions with Wyatt that she made with other students. When Apirera and Marama (Excerpt 7.2.1) disagreed about the answer to 8 x 6, they were positioned by Paula to give each other some advice on what you think happened. Wyatt's answer to 8 x 8 was correct with 64 and could suggest he had effectively used his known 5 times tables to solve 8 x 8. Had he been asked to discuss his thinking with another student Wyatt may have been able to recognise that whilst his strategy was correct, he was solving the wrong equation. Paula appeared not to have confidence in Wyatt's ability to self-correct or other students' ability to help Wyatt self-correct. Evidence of this is Paula's positioning of herself as the only person to support Wyatt.

7.2.3 Discussion: Inconsistent individual student positioning

Paula positioned herself and Wyatt differently from others in her lowest group. With the exception of Wyatt, Paula positioned herself to provide opportunities for students to work collaboratively with each other to correct procedural errors and misconceptions. Paula had the right to expect students to share, explain, defend, and record their own mathematical know-how and to listen to, challenge, and appropriate the mathematical know-how of others. Belinda and Kieran were positioned, with assistance from Paula and the group, to share then resolve their strategic differences. Apirera and Marama were positioned to talk through and determine the reasons behind their different answers by explaining their strategies and giving each other advice. There were no exceptions to the positionings of the majority of the group across the three lessons; at no time did Paula position herself as the first person to assist other students. These students readily accepted their positioning and on some occasions sought to assist each other without needing any direction from Paula.

In every lesson Paula positioned herself as the first and only person to assist Wyatt. In Excerpt 7.2.2, Wyatt was given the opportunity to share his mathematical know-how but as soon as Paula became aware of an error or misconception she became the only person with whom Wyatt had the opportunity to unpack his mathematical know-how. There were no examples in the three lessons where Paula positioned Wyatt to resolve a disagreement or error with a peer. Each time Paula positioned herself to work privately with him, he was observed placing his chin on his chest, hanging his head, and looking at the floor.

Paula’s positioning pedagogies differed depending on the student and because of this the storylines and social acts created with her lowest group were divergent, again depending on the students involved. In one storyline most students had a responsibility to work collectively to correct any misconceptions or errors. In a second storyline most students were expected to help each other, work through their disagreements and misconceptions, and come to a shared agreement. Paula’s expectations became a social act as for most students the importance of working collaboratively with peers became significant to the group. The disagreements and collaborations of most students took on a social force when they were publicly used by Paula as part of the lesson.

In a disparate storyline Wyatt did not have the same duties as his peers as he was not expected to work with them to either provide or receive help. He was positioned by Paula to work more privately with her because she spoke in a hushed tone with him and physically positioned him away from other students. At the conclusion of the third lesson with the lowest group, Paula commented that the expectation that Wyatt work only with her was a common occurrence and due to Wyatt's on-going confusion with the mathematical ideas explored at stage 6. Wyatt's opportunities to share his mathematical know-how and listen to the know-how of others were limited by the positioning decisions Paula made for herself and him. The mathematical contributions he made could not become social acts because his peers were not positioned to attend to his know-how and he did not have opportunities to participate in theirs. Wyatt received help from Paula and heard her explanations; he did not experience a position of authority where he provided support for others. Such positioning could limit Wyatt's opportunities for shared and collaborative mathematical progress.

In summary, with the exception of Wyatt, students in Paula's lowest group had opportunities to share their mathematical know-how and benefit from participating in the know-how of peers. Opportunities occurred through Paula's positioning of students to justify and defend explanations, come to an agreement, correct misconceptions, and record their thinking. These opportunities were not available to the entire group as Wyatt did not have the same positioning expectations placed on him by Paula. The positioning decisions of Paula with her highest strategy group are explored in the following two excerpts, and as with the lowest group expectations were different for one male student.

7.2.4 "Talk about what might be different between your strategies"

The third lesson Paula taught her highest strategy group required students to "use working form to solve multiplication problems" (MoE, 2007f, p. 63). Students were instructed to work in pairs discussing and solving 8 x 58 using both long and short working-form algorithms. Paula noticed that Theo and Liam had recorded different strategies and answers.

Participant	Dialogue
Paula	*You look like you have done it two different ways – can you talk about what might be different between what you have done, between your strategies?*
Liam	*I went 8 – 8s are 64 I put the 4 in the ones and carried the 6 to the tens. Then I did 8 times 5 is 40 plus the 6 is 46 and I put that by the 4.*
Theo	*I went wrong.*
Paula	[laughing] *That was quick!*

Liam's strategy was correct and on hearing it Theo immediately acknowledged I went wrong. Paula showed her confidence in Liam helping Theo to correct his place value misconception as she smiled at the boys and moved to listen to the next pair of students. Paula came back to stand behind Theo and Liam and asked Theo so did you work out where you went wrong? Theo's explanation of where he went wrong is illustrated in the following excerpt.

Participant	Dialogue
Theo	*Yip I did 8 times 5 and it should of* [sic] *been 8 times 50.*
Paula	*Oh! So that's - that's really important.*
Theo	*I do that a lot.*
Liam	*Yeah you do do that a lot!*
Paula	*So how could he know that that's 8 times 50 and not 8 times 5 when you look at …*
Theo	[interrupting] *Because it's in the tens column!*

Liam and Theo worked together to rectify Theo's place value error without requiring prompting or reminding from Paula. By participating with Liam, Theo was able to identify and self-correct his error. The dialogue that occurred between Theo and Liam whist Paula spoke to other students was not clear, but the boys were observed talking through their recorded equations. When questioned by Paula, Theo was able to explain he did 8 times 5 and it should of [sic] been 8 times 50 and he could remember this because it's in the tens column! Theo was able to recognise and reconcile his place value error and establish a means by which he ensured he was correct next time.

The two excerpts above illustrate how Paula did not position herself as the only person able to support students with errors or misconceptions. Instead, Paula positioned students who made errors to work with others to correct their strategy and place value understandings. Students were positioned by Paula to show a responsibility toward their own and others' mathematical know-how.

7.2.5 "Interesting"

This excerpt comes from the first of the three lessons from Paula's highest strategy group. Students were solving 7 x 38 and Paula overheard Nathan saying he had gone wrong by working out 7 times 10 but forgetting to include the other two rows of 7 lots of 10. Paula asked Nathan to explain what he did before he realised he had gone wrong. What follows is the sequence of Paula questioning Nathan, him answering her questions, and Paula evaluating his responses.

Participant	Dialogue
Nathan	*I did 7 times 10.*
Paula	*Where did you get the 10 from?*
Nathan	[points to the place value equipment] *From these bundles.*
Paula	*Oh okay did you look at this first row here?*
Nathan	*Yes but then I forgot to do the other 2 rows here.*
Paula	*Okay.*
Nathan	*And so I had 70 and then I did 3 times 16.*
Paula	*So why did you do 3 times 16? Where did you get your 3 times 16 from?*
Nathan	*There.* [points to the equation]
Paula	*Okay so how much is that?*
Nathan	*Well 8 and 8 is 16 and there were um 3 other groups like that.*
Paula	*Okay.*
Nathan	*And I, and I, and then I timesed it and I got 38 and then I went 70 plus 30 equals 100 and then plus 8 equals 108.*
Paula	*Interesting.*
Nathan	*But now I've sort of fixed it up I went 210 plus 38 because it's 7 times 10 is 70 plus 3 equals oh, no times 3, and 70 times 3 is 210 plus 38 so it would be 248.*
Paula	*Excellent. That's a really excellent way of doing that Nathan.*
Frank	[confused] *No but that doesn't …*
Paula	[glaring at Frank] *Ah thank you Frank.*

To solve the problem 7 x 38 Nathan explained that his strategy was 7 times 10 and 3 times 16. Paula attempted to understand Nathan's explanation but perhaps because both his strategy and accuracy were incorrect, she was unable to follow his reasoning and remarked interesting. Paula did not enlist the help of Jasdeep with whom Nathan was working. Nathan then announced that he had fixed it up by going 210 plus 38 because it's 7 times 10 is 70 plus 3 equals oh, no times 3, and 70 times 3 is 210 plus 38 so it would be 248. Nathan corrected his initial error of multiplying 7 x 10 to multiplying 70 x 3 but did not recognise that 7 times 8 does not equal 38. Following Nathan's second attempt Paula commented Excellent. That's a really excellent way of doing that Nathan. Frank challenged the accuracy of Nathan's strategy and answer but was silenced by Paula with a stern look and Ah thank you Frank. By the end of this lesson, three students had shared their different strategies and it was agreed that the answer was 266. Nathan asked Paula aye, so the answer's not 248? Paula's response was No, the answer is 266 but that doesn't matter.

7.2.6 Discussion: Inconsistent individual student positioning

When working with her lowest group Paula positioned Wyatt differently to the rest of the group. Different positioning was also apparent with Nathan in Paula's highest strategy group. With the exception of Nathan, Paula positioned herself to provide opportunities for students to work with peers to correct errors and misconceptions. These students positioned by Paula were expected to share, explain, critique, defend, argue, and record their own and others' mathematical know-how. Theo identified where he went wrong by listening to Liam's correct multiplicative strategy and from this he was able to self-correct his place-value error in Excerpt 7.2.5. As with the lowest group, there were no exceptions to the positionings of the majority of the group across the three lessons. Paula did not position herself as the first or only person to help. Again, students accepted their positionings and sought to assist each other without needing to be asked by Paula.

The positioning by Paula of Nathan in the highest group was similar to her positioning of Wyatt in the lowest group. Paula was the only person positioned to help Nathan across the three lessons with the highest strategy group. When Paula overheard Nathan acknowledging he had made an error, she knelt beside him and quietly questioned him about his error (Excerpt 7.2.6). She was unable to follow Nathan's mathematical reasoning but rather than question him further for clarification she commented *Interesting* and praised him for his excellent (but incorrect) strategy. There were no examples in the three lessons where Paula positioned Nathan to go beyond sharing his mathematical know-how to having to explain or defend it with anyone other than herself.

Paula's positioning decisions regarding students sharing their mathematical know-how differed depending on the student and because of this the storylines and social acts created with her highest group differed, as they did with her lowest group. The first storyline mirrored her lowest group with most students having a duty to work collectively to correct any misconceptions or errors. Students were expected to work through their disagreements and misconceptions and come to a shared agreement. Working together was a significant social act for students in the highest group. In all lessons students organised themselves to work with others without any direction from Paula. In similar ways to the teachers and groups discussed in Chapter Six, students' strategies took on a social force when they were appropriated by Paula and peers.

The contrasting storyline that occurred within the highest group was that Nathan did not have a duty to work with his peers in the capacity of getting or giving help. The storyline pertaining to Nathan was that he would work quietly with Paula and would not be expected to take responsibility for ensuring his explanations are understood by Paula or other students. At the conclusion of the third lesson Paula commented that she had difficulty following Nathan's thinking but *because other boys often mocked him* she did not want to *bring any more attention to him or embarrass him further*. Her intent, as she described, was *to save face for Nathan*. Paula also expressed concern regarding the amount of time Nathan would need individually within the group, and whether this would be fair on the group: *The problem is when the others try to help he tends to confuse them as well and that takes up more time unravelling new confusions*.

To conclude, Paula positioned the majority of students in the highest group to participate in each other's mathematical know-how by expecting them to resolve their mathematical differences, review their thinking in light of peers' contributions, and apply each other's know-how to self-correct errors and misconceptions. Paula did not appear to hold these same expectations for Nathan. Nathan was not held accountable for his explanations making mathematical sense as other students were. Paula's inability to follow Nathan's explanations meant Nathan was not liable for his mathematical know-how and this may have had the effect of further confusing him because incorrect answers and strategies were accepted by Paula.

7.3 Faith, Pacific School, Year 4

Table 7.2: Faith's strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	7	10
Year Level	4	4
Age	7 and 8	7 and 8
Gender	3 boys and 4 girls	2 boys and 8 girls
Strategy Stage	Stage 5: Early additive part-whole thinking.	Stage 6: Advanced additive to early multiplicative part-whole thinking
Achievement	As expected	As expected

This section describes the teaching of Faith and her lowest and highest strategy groups. The positioning decisions Faith made for herself and students in her lowest and highest groups were not consistent. With the lowest group Faith praised students for correct answers, repeated students' answers, and provided explanations for correct and incorrect answers. Students in the highest group were encouraged to work collaboratively to solve their mathematical disagreements and build on each other's strategies. The excerpts that illustrate Faith's teaching and positioning decisions with her lowest group are presented first.

7.3.1 "2 good, 3 good, 4 good"

The first lesson Faith taught her lowest strategy group required students "to solve problems about sharing into equal sets" (MoE, 2007f, p. 17). The following excerpt demonstrates Faith's procedural approach of praising correct answers, repeating answers, and providing explanations for students' answers.

Participant	Dialogue
Faith	*Okay I need you to work in pairs for this. On your pirate ship there are two pirates and each pirate has got two doublets, give each pirate two doublets, how many doublets or how many pieces of gold has each pirate got George?*
George	*2.*
Faith	*2 good. How many doublets have your pirates got Odette?*
Odette	*2.*
Faith	*2 alright! So 2 pirates have got 2 doublets, how many have they got altogether Lana?*
Lana	*4.*
Faith	*4, so 2 times 2 is 4, it's doubling isn't it? How many did your pirates have Henry?*
Henry	*4.*
Faith	*4 good. Now give each pirate 1 more coin, how much money is each pirate going to have now Odette?*
Odette	*3.*
Faith	*Yes 3 and how many pirates are there George?*
George	*2.*
Faith	*2 okay so how many coins altogether Mandy?*
Mandy	*6.*
Faith	*Good, yes, there are 6 altogether.*

Students worked in pairs to determine the total amount two pirates would have if they had 2, then 3 gold coins each. Faith confirmed with George, Odette, and their partners that the pairs had two gold coins each. Lana correctly stated pairs of students had four gold coins between them and Faith provided the explanation so 2 times 2 is 4, it's doubling isn't it then confirmed with Henry that his pair also had 4 coins. Students were instructed to give themselves another coin each and Odette was asked how much money is each pirate going to have now? Faith repeated Odette's correct answer of 3, questioned George about how many pirates there were, repeated his correct answer of 2, asked Mandy how many coins altogether, and repeated her correct answer of 6. This pattern of dialogue continued until students had 6 coins each and 12 between the pairs. At this point Faith told students they would have to share their 12 coins between three pirates and reorganised students to work in groups of three.

Participant	Dialogue
Faith	*Right so now the gold has to be shared between 3 pirates, how are you going to organise that? How many pirates are there now?*
Students	*3.*
Faith	*3 good, and how many pieces of gold to share?*
Students	*12.*
Faith	*Yes 12 pieces of gold shared between 3 pirates. How are you going to organise that Kirsten?*
Kirsten	*It's 4.*
Faith	*It is 4 isn't it because 12 shared by 3 is 4. Well done. Right I'm going to give you another pirate – so now you have how many pirates Henry?*
Henry	*4.*
Faith	*Yes 4 pirates and I want you to share out 20 pieces of gold between your 4 pirates.*

Students responded correctly to Faith's two questions: how many pirates are there and how many pieces of gold to share and Faith revoiced their answers as Yes 12 pieces of gold shared between 3 pirates. Kirsten noted the answer was 4 and Faith explained her answer by saying It is 4 isn't it because 12 shared by 3 is 4. Students were told to imagine a fourth pirate had joined their group and to divide the 12 gold coins between four pirates. The dialogue between Faith and students continued with Faith asking questions, students providing answers, and Faith repeating and at times explaining the answers.

7.3.2 "Don't worry about that at the moment"

The focus for the second lesson with Faith and her lowest strategy group was for students to use their known 2, 5, and 10 multiplication facts to work out unknown multiplication problems. Faith presented students with a Slavonic abacus showing two groups of seven as five blue and two yellow beads on the top and second rows. Students were asked to record what the Slavonic abacus was showing in two different ways. The following excerpt illustrates how Faith praised, repeated, and revoiced correct answers, corrected inaccurate answers, and overlooked Mandy's advanced multiplicative thinking.

Participant	Dialogue
Faith	*Okay George what's one way that you have got?*
George	[reads recording 2 x 7 = 14] *2 times 7 equals 14.*
Faith	*2 times 7 equals 14, good, is there another way?*
Odette	*You could go 7 plus 7.*
Faith	*Yes you could go 7 plus 7.*
Henry	*7 times 2.*
Faith	*No Henry because I can see 2 groups of 7. I cannot see 7 groups of 2 can I?*
Henry	*No.*
Mandy	*You could go 2 times 5 and 2 times 2.*
Faith	*Not at the moment. What about something starting with a d?*
Students	*Division* *Divided by*
Faith	*Dou…*
Students	*Doubling*
Faith	*Doubling isn't it? It is doubling isn't it? You've got 2 times 7, or 7 plus 7, or double 7.*
Mandy	*And also you've got double 5 is 10 and double 2 is 4 and 10 and 4 is 14.*
Faith	[aside to Mandy] *Don't worry about that at the moment.* [to the group] *What if I had, what if I moved over this many beads* [shows 2 sets of 9 on the abacus] *what can you tell me about this?*

As with Excerpt 7.2.1, Faith praised students and repeated their answers for them. Henry suggested 7 times 2; Faith informed him this was incorrect and explained why: No Henry because I can see 2 groups of 7; I cannot see 7 groups of 2. Faith responded with not at the moment to Mandy's suggestion you could go 2 times 5 and 2 times 2 and led students to suggesting doubling. Mandy attempted to share her strategy again, this time using the word double, which was promoted earlier by Faith. She was informed by Faith don't worry about that at the moment. Mandy was suggesting a strategy that aligned to the learning intention shared at the beginning of the lesson: "we are learning to work out multiplication facts from what we know about twos, fives, and tens" (MoE, 200f, p. 21). Solving 2 x 7 as (2 x 5) + (2 x 2) may not be an efficient way but it did appear that Mandy was applying an advanced stage 5 part-whole strategy. The lesson continued with students recording 2, 5, and 10 multiplication facts in different ways.

7.3.3 Discussion: Positioning for student answers and teacher explanations

Faith positioned herself with her lowest group as having the right to ensure students were following the correct steps to solve the problem and getting the correct answer. This was evidenced in her asking questions that focussed on the procedural steps of the tasks and the answer. Faith also positioned herself to provide praise, repeat and revoice answers, and offer explanations for correct and incorrect answers. In Excerpt 7.3.1, Faith repeated each correct answer, praised each student providing the correct answer, and explained why the answers were correct. Repeating, praising, and explaining by Faith also occurred in Excerpt 7.3.2. Faith also told Henry he was wrong and explained why, and dismissed Mandy's advanced multiplicative thinking. Students in Faith's lowest group appeared to be positioned as having a duty to provide answers. They were not positioned by Faith to explain the strategies used to determine the answers. An implication of this was that students could be recalling knowledge rather than applying strategies to new learning. As Faith did not require explanations it is unclear if students were recapping existing knowledge or applying new knowledge.

A prominent storyline occurring in the teaching of Faith and her lowest group was that answers were important and students did not need to explain or monitor their mathematical know-how. This storyline was evidenced by Faith requiring students to provide answers and by her providing the explanations for the correct and incorrect answers. Students were not required to self-regulate as Faith monitored their thinking for them by affirming when they were correct and correcting errors for them. The prominent positioning of Faith in this storyline may result in her mathematical know-how and students' answers becoming the significant social acts for this group. A consequence of this could be that students have limited opportunities to discuss their mathematics and, therefore, become reliant on Faith to do the thinking and regulating for them.

Faith's positioning decisions with her highest strategy group are explored in the following two excerpts. The decisions Faith made with her highest group were not consistent with those she made for herself and her lowest group. As the following excerpts illustrate, Faith positioned students in the highest group to resolve their own disagreements and correct their errors.

7.3.4 "Who can tell us more?"

In the second lesson with Faith and her highest strategy group students were learning to change the order of the factors to make multiplication easier. She presented students with equipment including animal arrays, interlocking blocks, the Slavonic abacus, and counters. Students were asked to individually model 3 multiplied by 4 using equipment, then to discuss their representations with a partner.

Participant	Dialogue
Karen	*Miss we don't agree.*
Faith	*Don't you? What is it you don't agree about?*
Karen	*I have done 3 groups of 4 but I think Vicky did 4 groups of 3.*
Vicky	*But they are still the same.*
Faith	*Are they? Why are they the same? Do we agree with Vicky? What have Vicky and Karen done with their two factors?*
Chase	*They turned them round.*
Faith	*Can anyone add to that?*
Owen	*Well Karen made 3 groups of 4 and Vicky made 4 groups of 3 and they are the same.*
Faith	*Are they the same?*
Students	*Yes.* *No.*
Faith	*Who can tell us more, why are they the same or why aren't they the same?*
Maddie	*They don't look the same, they are the same answer, but they don't look the same.*
Faith	*Well done, they are the same answer but they don't look the same. Why might one person choose to show 3 groups of 4 and another person choose to make 4 groups of 3?*
Maddie	*You might know your 3s better than your 4s. I know more of my 3s than my 4s.*
Jolene	*I know my 4s better than my 3s – double, double!*

Faith	*Very good and part of what we are learning about today is the commutative property; that means you can change the factors around but the product, the answer, will stay the same and you might choose to change the factors to make the problem easier for you to solve.*

Karen and Vicky noticed they had modelled 3 multiplied by 4 differently with Karen showing 3 groups of 4 and Vicky showing 4 groups of 3. Faith brought the disagreement to other students' attention by asking if they agreed with Vicky's claim that 3 x 4 and 4 x 3 are the same. Maddie observed the two equations had the same answer but did not look the same. Faith praised Maddie for her contribution and asked students: Why might one person choose to show 3 groups of 4 and another person choose to make 4 groups of 3? Maddie and Jolene identified that some people might know their multiplication facts of 3 better than their 4. The commutative property was introduced by Faith as a strategy for making multiplication problems easier to solve and students were alerted to the connection between the learning intention and the commutative property. The lesson continued with Faith suggesting problems such as 20 x 4 and 50 x 6 for students to solve. At the end of this lesson Faith asked students to review the commutative property and discuss how it could support their multiplicative problem solving.

7.3.5 Discussion: Positioning for flexible thinking

Faith positioned herself and students in her highest strategy group differently to how she positioned herself and students in her lowest group. With her highest group Faith positioned herself to ask questions that required students to explain, justify, and monitor their own and others' mathematical know-how. In Excerpt 7.3.4, Faith did not sort out the disagreement between Karen and Vicky. Instead, she positioned students to discuss and decide on the similarities and differences of showing 12 as 4 multiplied by 3, or 3 multiplied by 4. Faith provided the opportunity for students to reach an understanding regarding the flexibility of factors in a multiplication problem and how this flexibility could make their problem solving easier. In positioning herself not to solve disagreements, answer students' questions, or provide correct answers, Faith was positioning students to share, explain, and justify their mathematical know-how, resolve their differences, correct errors, and make connections between their learning intentions and the content of the lesson.

Two storylines developed between Faith and her highest strategy group. The first storyline with her highest group was that students had to go beyond simply providing answers to reviewing and defending the strategies behind the answers. In the second storyline Faith encouraged students to approach their mathematics by considering the easier or more efficient way to solve the problem. Students' disagreements, explanations, and strategic choices became important social acts when they were made public by Faith for discussion and clarification. By making their mathematical know-how public, Faith was positioning students as having made interesting or important contributions to the learning.

In conclusion, Faith was not consistent in her approach to positioning students in her lowest and highest strategy group to share their mathematical know-how. Students in the lowest group were predominantly positioned to follow the procedures outlined by Faith and provide correct answers. She rarely positioned students to explain, justify, or review their own or others' mathematical know-how. Instead, she positioned herself to repeat, revoice, and explain correct and incorrect answers for students in the lowest group. With her highest group, Faith expected students to explain, justify, and reconsider their own and others' mathematical know-how. She positioned herself with this group to provide opportunities for students to argue, trial, and compare their know-how. Students' mathematical know-how was shared more in the lessons with Faith's highest group than her lowest group.

7.4 Naomi, Pacific School, Years 2 and 3

Table 7.3: Naomi's strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	4	5
Year Level	2 and 3	2 and 3
Age	6 and 7	6 and 7
Gender	2 boys and 2 girls	4 boys and 1 girl
Strategy Stage	Stage 3: Counting from one by imaging	Stage 5: Early additive part-whole thinking.
Achievement	Cause for concern	As expected

The positioning decisions Naomi made for students to share their mathematical know-how in her lowest and highest strategy groups were not consistent. Students in the lowest group were positioned to listen and watch carefully and to provide answers for themselves and others. Students in the highest group were positioned to share their mathematical know-how and discuss the efficiency of their own and others' know-how.

7.4.1 "Listen carefully and watch carefully"

Students in Naomi's lowest strategy group were learning how to "count objects by creating groups of 10 from materials" (MoE, 2007e, p. 16) in the first and second lessons. In the second lesson students were instructed to work in pairs using their fingers as equipment to show how many more to make 10. Naomi showed students tens frame cards illustrating amounts to 10; one student was asked to indicate (using their fingers) how many dots the card showed and a second student was asked to show (with their fingers) how many more dots were needed to make 10. For example, if shown the tens frame below, one student would show seven fingers and the other would show three fingers.

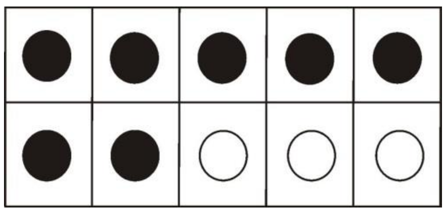

***Figure 7.1.* Tens frame showing groupings of 7 and 3**

Participant	Dialogue
Naomi	*Now listen very carefully to what is going to happen now. I want you with your partner to show me, watching carefully, if you've got that number* [holds a tens frame with 6 dots bolded] *how many more will your partner need to do it on the other side? To make the number 10? So Freddie is showing 6 – how many more would you need to show to make 10 Brian?*
Brian	*4.*
Naomi	*4 good because 6 and 4 makes 10 doesn't it? Okay Freddie how many are you showing?*
Freddie	*6.*
Naomi	*6 yes good and Debbie how many more would you need to show to make 10?*

Debbie	*5. 5 and 5 makes 10.*
Naomi	*No, no, no, I want you to show me how many more to make 10* [turns the tens frame card toward Debbie] *with this number.*
Debbie	*5.*
Naomi	*Listen to Brian.*
Brian	*4.*
Debbie	*4.*
Naomi	*Yes good 4.*

Naomi directed students to watch and listen carefully as she shared the instructions for the lesson. Freddie held up 6 fingers and Brian held up the corresponding 4 fingers to show 10 fingers in total. Naomi praised Brian and explained why he was holding up 4 fingers: because 6 and 4 makes 10. When asked by Naomi how many more would you need to show to make 10? Debbie answered 5 then stated 5 and 5 makes 10. Naomi reiterated that she wanted Debbie to make 10 using the tens frame showing 6 bolded dots. Debbie again answered 5 and Naomi directed her to listen to Brian. Brian stated the correct answer 4, Debbie repeated 4, and Naomi praised her: Yes good 4. By copying Brian's answer Debbie was able to provide the correct answer.

Naomi continued to show tens frame cards with bolded dots from 0 to 10 and students worked in pairs to demonstrate with their fingers how many dots each card showed and how many more were required to make 10. Once all tens frame cards showing 0 to 10 bolded dots had been represented on fingers, Naomi asked students to record the sum the dots were showing.

Participant	Dialogue
Naomi	*Now could you write for me what the dots are showing in a sum? How many dots can you see Freddie?*
Freddie	*8.*
Naomi	*Good, write that down* [Freddie records 8] *and how many* [to other students], *watch Freddie, watch Freddie, how many do you have to add to make your group of 10?*
Freddie	*8.*
Naomi	*How many did you have to add to make your group of 10?*
Freddie	*8.*

Naomi	*No, no how many did you have to add, you had 8 how many did you have to add to make 10?*
Freddie	*2.*
Naomi	*Good so write that, everybody watching, 8 plus 2 equals 10* [Freddie records 8 + 2 = 10]. *Brian what would you write?*
Brian	*8 plus 2 equals 10.*
Naomi	*What have you got there? How many dots have you got there?*
Vienna	*5.*
Naomi	*Look how many dots have you got?*
Vienna	*5.*
Naomi	*How many dots have you got?*
Vienna	*5.*
Naomi	*Listen to Debbie.*
Debbie	*8.*
Vienna	*8.*
Naomi	*8 good, and how many more would you have to add to make 10?*
Vienna	*12.*
Naomi	*How many extras have you got?*
Debbie	*2.*
Vienna	*2.*
Naomi	*Right can you write that for me as a sum, think about what I'm trying to say? How many dots have you got?*
Vienna	*8.*
Naomi	*Good, write it down and how many did you have to add to make a group of 10?*

Freddie correctly identified the 8 bolded dots on the tens frame. Naomi asked him to write that down and directed other students to watch Freddie. When asked how many more to make 10, Freddie incorrectly suggested 8. Naomi repeated her question and Freddie again responded 8. Freddie was told no by Naomi and she repeated the question a third time. If students were watching Freddie, as Naomi suggested earlier, they may have been confused as the correct recording had not been forthcoming. Once Freddie correctly answered 2 Naomi praised him and stated: Good so write that, everybody watching, 8 plus 2 equals 10. Brian was asked by Naomi what would you write and he correctly answered 8 plus 2 equals

10. Brian may have known how to correctly record the equation or he may have copied Naomi. When asked by Naomi how many dots she had, Vienna responded with 5. Naomi repeated the question twice then suggested Vienna listen to Debbie. Debbie provided 8 as the correct answer and Vienna repeated her to confirm the answer. The question of how many more to make ten was repeated twice by Naomi, answered by Debbie, and echoed by Vienna.

7.4.2 Discussion: Positioning for student answers and teacher explanations

Naomi positioned herself with her lowest strategy group to question students until a correct answer was given, assure students when they were correct, and to provide the explanations for correct answers. Naomi affirmed Brian was correct by repeating his correct answer, praising him, and providing the reason he was correct because 6 and 4 makes 10 doesn't it? Students were positioned by Naomi to listen and watch carefully and it appeared from some dialogue in Excerpt 7.4.1 that repeating another student was an acceptable means of providing a correct answer. When Debbie was unable to provide the correct answer of how many more to make 10, Naomi directed her to listen to Brian. Brian stated the correct answer, 4, Debbie repeated 4, and Naomi praised her with yes good 4. When Vienna was unable to correctly identify 8 dots, the question was repeated twice by Naomi and then she was told to listen to Debbie. Debbie and Vienna may have been able to provide the correct answer but there is no evidence to suggest they understood why their answers were correct as Naomi did not question students beyond the answer.

One storyline in the teaching of Naomi and her lowest strategy group was that students were expected to give correct answers but not to provide explanations of why their answers were correct. This resulted in students relying on others to provide the correct answers and limited students' opportunities to discuss why answers were correct or incorrect. A second storyline was that learning was expected to occur through listening and watching. By being positioned to watch and listen, students could have fewer opportunities to participate in their own and others' mathematical know-how. Listening and watching were developing as important social acts for this group. The emphasis given to listening and watching by Naomi resulted in passive positions becoming socially significant and valued by the group.

To summarise, Naomi had a right to question until the correct answer was given and to provide the explanations for the students' answers. Students had the duty to listen and watch, and find the correct answer. The positioning decisions Naomi made for herself and her highest groups were not consistent with those she made for herself and with her lowest group. Excerpts 7.4.3 and 7.4.4 illustrate Naomi's positioning decisions with her highest strategy group and include students sharing and critiquing explanations for efficiency.

7.4.3 "Is there another way you could have done it even faster?"

The focus for the second lesson with Naomi and her highest strategy group was for students to learn how "to use compatible numbers to solve problems like 5 + 3 + 6 – 8, by first adding 5 and 3 to 8 and removing the 8" (MoE, 2007e, p. 26). The lesson began with Naomi reading the following problem to students:

> *You went shopping and you bought 2 marbles and then Mum gave you some more pocket money and you went and bought another 6 and Dad said hey you've done so well at school you can go and buy another 4 and Nana found 3 in her pocket and she gave you 3 and then you were running round in the playground with all these marbles in your pocket and you lost 7 of them.*

As Naomi read the word problem, students recorded the corresponding number story in their books: 2 + 6 + 4 + 3 – 7 =. The following excerpt shows Irvin's less efficient approach where he used a 'make 10' strategy rather than the required compatible numbers strategy. Naomi's conversation with the group to review the efficiency of Irvin's strategy and to elicit a faster, quicker, easier strategy is described.

Participant	Dialogue
Irvin	*Okay 6 plus 4 equals 10.*
Naomi	[records 6 + 4 = 10] *Yeah.*
Irvin	*And 2 plus 3 equals 5.*
Naomi	[records 2 + 3 = 5] *Okay.*
Irvin	*And 10 plus 5 equals 15.*

Naomi	[records 10 + 5 = 15] *Yeah.*
Irvin	*So if you take away 7 – so if you take away 15 from 7 you've got 5 from 15 equals 10 takeaway another 2 equals 8.*
Naomi	[records 15 – 5 – 2 = 8 in the modelling book] *Well done - but can you think of a pattern? That's a good … you've just gone a really really long way round and it's – the answer is correct but is there another way you could have done it even faster?*
Zoe	*He could have gone 3 plus 4 equals 7.*
Naomi	[records 3 + 4 = 7] *Tell us a bit more.*
Zoe	*Yeah 3 plus 4 equals 7 so cross out the 4 and the 3 and the 7 and you are left with 2 and 6 and that's the answer – 8.*
Naomi	*Irvin – can you see what Zoe did* [points to Zoe's recording] *can you see that what she did was quicker, easier?*
Irvin	*Yip the 4 and the 3 is the takeaway 7 so you can cross them off.*

Naomi did not introduce or model the compatible numbers strategy as the learning intention for this lesson so students were not aware of what was expected of them in terms of strategy use. Naomi praised Irvin for his correct strategy and asked students if there was a strategy that was faster. Zoe suggested you could have gone 3 plus 4 equals 7 and Naomi asked her to elaborate on her explanation. As Zoe expanded on her explanation, Naomi recorded Zoe's strategy and asked Irvin to compare his and Zoe's strategies in terms of quickness and ease. Irvin was able to identify the specific point of Zoe's strategy where it provided a more efficient approach than his the 4 and the 3 is the takeaway 7 so you can cross them off.

Later in the same lesson students were asked to solve the following problem:

> *Right here is your next one. You got 3 lollies at a shop – 3 lollies lucky you! The good shop keeper gave you another 9, you found a friend in the shop and that friend gave you another 4, you were so excited that you quickly gobbled up 7. You ate them so quickly because you didn't want to share them when you got home and you thought on the way home oh well I'll just eat another 2 quickly. What is the pattern and what is your answer as quickly as possible.*

Students recorded the number story as the problem was read to them: 3 + 9 + 4 – 7 – 2 =. Hamish and Kase suggested different strategies and Naomi positioned students to discuss the efficiency of the two strategies.

Participant	Dialogue
Hamish	[records 3 + 9 + 4 – 7 - 2] *Okay so 3 plus 4 …*
Naomi	*…yeah?*
Hamish	*Equals 7 takeaway 7* [crosses out the 3, 4, and 7] *and 9 minus 2 is 7.*
Naomi	*Well done Hamish fantastic! This is what Hamish did. He said those two* [points to 3 and 4] *make 7 so we cross them out* [crosses out 3, 4, and 7] *because 7 takeaway 7 is 0 and he went 9 takeaway 2 and got his answer which was what?*
Hamish	*7.*
Kase	*Or 7 and 2 and take off the 9 and that leaves 4 plus 3 is 7.*
Naomi	*Ah yes very good Kase there are two efficient ways to solve our problem. We could have said 3 plus 4 is 7, cross off the 7, and that leaves us with 9 minus 2 is 7 or we could have – tell us again Kase?*
Kase	*The 7 and the 2 makes 9 so cross off the 9 and that leaves 3 plus 4 is 7.*
Naomi	*That's great guys so when we look at the numbers we might have more than one quicker or easier way to solve the problem.*
Paul	*That could help us with checking too if we did it both ways.*
Naomi	*Fantastic idea Paul – excellent.*

Hamish described the strategy to solve 3 + 9 + 4 – 7 – 2 and Naomi recorded, then repeated his thinking. Kase volunteered a different approach using the compatible numbers strategy. After repeating Hamish's approach, Naomi asked Kase to repeat his, then emphasised to students the value of looking at the numbers to determine which way to quickly or easily solve the problem. Paul identified that having two approaches could also help with their checking and Naomi praised his idea.

7.4.4 Discussion: Positioning for efficient mathematical thinking

Naomi positioned herself to support students in her highest strategy group to share their mathematical know-how. As students shared their strategies, Naomi recorded the corresponding number story, thus providing an additional way for students to consider and understand the strategy. Naomi's and the students' recordings were used to review and critique the different strategies and compare them for efficiency. When students in her highest group suggested less efficient strategies, Naomi encouraged them to seek out strategies that would be faster or easier to apply. She positioned students in the highest group to share and explain their own mathematical know-how and to critique and appropriate the mathematical know-how of others. Irvin was able to learn from Zoe's strategy and he applied more efficient strategies throughout the lesson. Students were also encouraged to try to suggest different approaches. Having access to more than one strategy may have helped students to understand there can be more than one acceptable way to solve a problem and that depending on the numbers, one strategy could be more efficient than another.

The first storyline created by Naomi and her highest group was that strategies needed to be correct as well as efficient. Naomi acknowledged that Irvin's strategy was correct in Excerpt 7.4.3, and encouraged him to try other strategies that would be more efficient. A second storyline evident was that there was more than one acceptable and efficient strategy for solving problems. Students' explanations and critiques of each other's thinking became significant to the group as social acts when they were recorded by Naomi and when other students were asked by Naomi to critique and consider them for their own use.

In conclusion, Naomi positioned students in her highest group to participate in each other's mathematical know-how by asking them to share and explain their strategies, and critique and appropriate the know-how of others. It appeared that students in Naomi's highest group had more opportunities to share their mathematical know-how and participate in the know-how of others than students in the lowest group.

7.5 Brooke, Tasman School, New Entrant

Table 7.4: Brooke's strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	6	6
Year Level	New entrant	New entrant
Age	5	5
Gender	5 boys and 1 girl	3 boys and 3 girls
Strategy Stage	Stage 1: One to one counting	Stage 2: Counting from one on materials
Achievement	Below	As expected

Brooke's approach to positioning students in her lowest and highest strategy groups was inconsistent. Students in the lowest group had fewer opportunities to share their mathematical know-how because Brooke emphasised answers more than explanations. Students in the highest group had more opportunities to discuss, explain, and critique their own and others' know-how. Sione was new to Tasman School and Brooke's class at the time of this research. He chose to join each group that Brooke called up to work with her and so he features in the excerpts from the lowest and highest strategy groups.

7.5.1 "No"

During the first lesson students in the lowest strategy group were learning how to "order numeral cards from 1 to 10" (MoE, 2007e, p. 5). Five minutes into the lesson Brooke pulled a card from a bag and asked students to identify the number on the card. As shown in the following excerpt, Brooke responded no to incorrect answers until a correct answer was found.

Participant	Dialogue
Brooke	*Okay watching everybody? What am I going to pull out? What's that number?*
Sione	*6.*
Brooke	*No.*
Quon	*7.*
Brooke	*No*
John	*5.*
Brooke	*No*
Te Ariki	*8.*
Brooke	*No*
Kalepo	*9.*
Brooke	*Yes good thank you Kalepo that is the number 9.*

Four students identified the number 9 incorrectly and Brooke responded No to them. On hearing No, the boys were observed scowling and sitting back in their seats with their arms folded. The boys removed themselves from the actions of others in the group as they sat back beyond the group periphery and stopped contributing verbally to the lesson. Kalepo provided the correct answer, perhaps because he knew it or perhaps through a process of elimination he happened on the correct number. When Brooke thanked Kalepo for his correct answer, the four boys smiled and leaned into the group again. The boys seemed to take Brooke's response of No as a signal they should stop participating in the lesson. Brooke's confirmation of a correct answer appeared to signal to the boys that they could re-join the lesson. This sequence of incorrect answers and withdrawal from the lesson occurred frequently with the boys in Brooke's lowest strategy group.

7.5.2 "Where do I go?"

In each lesson with her lowest group Brooke chose to rephrase and repeat her questions in response to an incorrect answer. Brooke introduced the learning intention at the beginning of the second lesson – "we are learning to identify teen numbers" (MoE, 2007d, p. 3). Kalepo asked what's teen numbers and Brooke told students they were numbers ending with teen and provided examples. Brooke introduced the equipment on which the students would represent different teen numbers – counters, tens frames, strings of beads, and a Slavonic abacus. Each student modelled 10 on a different piece of equipment, then Brooke asked them to model teen numbers. The following excerpt shows how Brooke responded to incorrect answers by rephrasing then repeating her question.

Participant	Dialogue
Brooke	*Kalepo says 10. Show me 10 another way, good boys.* [students are making and showing 10 on the equipment]. *Ranjita can you show me 10?* [Brooke speaks to individual students]. *That's 10 and that's 10.* [waits until all students have shown 10 with their equipment]. *Right I'm going to add 1 more.*
Te Ariki	*Makes 1.*
Sione	*Makes 11.*
Kalepo	*Makes 11.*
Brooke	*1 more onto 10 makes?*
Te Ariki	*100.*
Brooke	*Oh where's my hundreds board? And where is my number line? Ranjita can you grab the hundreds board off the table for me?* [Ranjita gets the hundreds board] *We can use both but we will use this one first – this is 10* [points to 10 on the hundreds board] *and if I move one more step where do I go?*
Kalepo	*8.*
John	*To 8.*

Te Ariki	*8.*
Sione	*What no, stop, stop, stop* [waves arms] 10 and 1 is 11, it's 11 aye Miss?
Brooke	*Watching Te Ariki – no – if I'm on 10 now and I go to the next number what is it going to be?*
Kalepo	*8.*
Brooke	*Is it? What is the next number?*
Te Ariki	*9, 9.*
Brooke	*Not backwards.*
Ranjita	11.
Brooke	*Good girl yes 11 it's like when I read a book* [traces finger backwards 10 to 1 and drops down to 11]. *Kalepo what happens when we're reading we get to the end of the line* [places finger on 10] *what do we do next? We go there* [places finger on 11]. *Right there's 10* [places finger on 10], *where would I go next?*
Ranjita	*11.*
Brooke	*To 11* [traces finger backwards 10 to 1 and drops down to 11]. *Good girl.*
Sione	*But I sayed* [sic] *that, I sayed* [sic] *ages ago it was 11.*

When each student correctly modelled 10 with their equipment Brooke stated right I'm going to add one more. Te Ariki incorrectly answered 1 and Sione and Kalepo correctly answered 11. Brooke rephrased the question and Te Ariki suggested 100. Students were shown 10 on the hundreds board and the question was rephrased again. Kalepo suggested 8 and this was echoed by John and Te Ariki. Sione was observed waving his arms around and asking Brooke and students to stop. Te Ariki's answer of 9 was responded to by Brooke with not backwards, indicating she thought he was taking one away from 10 instead of adding one. Ranjita provided the correct answer but she was not asked to explain how she knew the number after 10 was 11, so this could have been a guess. Brooke explained to the group it's just like reading a book – we get to the end of the line and what do we do next? Then answered her question stating we go there. Sione was observed sitting despondently back on his seat stating But I sayed [sic] that, I sayed [sic] ages ago it was 11. Had Brooke appropriated Sione's correct answer and asked for an explanation he may have been able to assist other students to successfully strategise the answer to 1 more than 10. Noting earlier that reading the hundreds board was like reading a book or suggesting a counting strategy may have enabled students to read for themselves that 11 was 1 more than 10.

7.5.3 Discussion: Positioning for answers

Brooke positioned herself to inform students if their answers were correct or incorrect, to question students until the correct answer was established, and to explain to students why their answers were correct. In Excerpt 7.5.1, Brooke responded no to incorrect answers and students continued to offer suggestions as to what number the card showed. Kalepo suggested 9 and this was accepted by Brooke, and Kalepo was thanked for his contribution. The four boys who were incorrect were not given an opportunity to reconsider their thinking. Brooke did not seek an explanation from the boys and may not have been aware of why they were incorrect, or if they were making wild guesses. Brooke's decision to respond No to incorrect answers did not provide opportunities for mathematical know-how to be shared. She asked the same question regarding 1 more than 10 four different ways in Excerpt 7.5.2. Brooke interpreted incorrect answers as students not understanding the question and she repeated and rephrased the same question four times. Students may not have realised the essence of the question was the same and may have thought Brooke was asking for something new. Brooke positioned herself to keep asking what she believed was the same question until she heard an acceptable answer. Students were not asked to explain their incorrect answers which could have arisen due to misunderstanding the questions rather than the mathematics. Brooke positioned students in her lowest group as having a duty to provide the correct answer; they were not expected to provide the explanations as to why their answers were correct or reconsider incorrect answers.

One storyline evident in Brooke's teaching with her lowest group was that students took turns to provide an answer. If their answer was incorrect they withdrew until the correct answer was established. There seemed to be a game-like quality to this storyline. Students had a turn in the game, but if they were incorrect they did not continue playing and only resumed playing once the correct answer was found.

The second storyline was that answers were important. This storyline developed because Brooke emphasised answers more than the strategising that led to them. An emphasis on answers resulted in students making wild, implausible guesses.

The social acts valued by the group were giving answers and students appeared eager to please Brooke by providing the correct answer.

To summarise, students in Brooke's lowest group were positioned by her to provide correct answers. Correct and incorrect answers were not reviewed or critiqued by students. Brooke positioned herself to elicit the correct answer from students. They did not appear to have opportunities to share their own or participate in others' mathematical know-how. The following two excerpts show how Brooke's positioning decisions with her highest group were different to those with her lowest group. Students in the highest group were positioned to determine correct answers, to review and appropriate peers strategies, and to build on each other's mathematical know-how.

7.5.4 "I hope everybody was listening to Olivia"

Brooke's highest strategy group focused on numbers up to 20 in their third lesson. This included recognising, writing, and creating numbers to 20 and exploring place value of tens and ones. Brooke used different equipment to show amounts up to 20 and students were asked to identify the total amount of the objects they could see. A tens frame showing 7 black dots and 3 dot outlines was shown to students.

Participant	Dialogue
Olivia	*7.*
Ngahuia	*Yip 7.*
Olivia	*It is aye it's 7.*
Brooke	*Tell us why you think it's 7.*
Olivia	*Because there is 2 more.*
Brooke	*2 more than what? I'm not sure I understand what you mean.*
Olivia	*See there is 5* [points to 5 black dots on the tens frame] *and there is 2* [points to 2 black dots on the tens frame]. *It's 2 more than 5, 7 is 2 more than 5.*
Brooke	*Oh I hope everybody was listening to Olivia – what a very clever idea – Olivia knows that 7 is 2 more than 5 so when she saw 2 more black dots she knew there were 7 in total. Okay what about this one?* [shows the tens frames with 9 black dots]
Peter	[singing] *Ah, ah, ah, ah, ah, ah, ah, ah, ah 9!*
Sione	[singing] *La, la, la, la, la, la la, 9!*

Olivia	[pretends to play a trumpet] *Do, do, do, dooooooooo 9!*
Brooke	*Peter tell me why you think there are 9 dots altogether.*
Peter	[holds up one hand] *5* [holds up 4 fingers] *and 4 more is 9. It's 4 more than 5 so it's 9.*
Brooke	*Ah well done now you were listening to Olivia weren't you and you had another very clever idea to use your fingers to show us 5 and 4 more is 9. Do we all agree?*
Students	*Yes.*

Olivia and Ngahuia correctly identified 7 dots on the tens frame. Olivia explained she could see 5 dots and 2 dots and she knew 7 was 2 more than 5. Brooke drew students' attention to Olivia's strategy by describing it as a very clever idea and repeating Olivia's description of what she knew. Students were shown another tens frame and they correctly described it as having 9 black dots. Peter described 9 as 4 more than 5 and modelled this by holding up 5 fingers on one hand and 4 fingers on the other hand. Brooke praised Peter for listening to Olivia and for sharing another clever idea of using his fingers to model 5 and 4 more. Later in this lesson, Brooke showed students a Slavonic abacus with 10 beads on the top row and 3 beads on the second row. Brooke asked students to identify how many beads altogether. Olivia claimed oh that's easy it's everything and 3 more so 13.

Participant	Dialogue
Brooke	*What do you mean everything and 3 more Olivia? I think we should all listen carefully to Olivia, I have a feeling this is going to be another very good idea.*
Olivia	*Well it's everything on the top* [points to top row of the abacus] *so that's 10 and then it's* [points to second row of the abacus] *3 more so everything and 3 more is 11, 12, 13 -13.*
Brooke	*Wow another excellent idea from our Olivia – you are doing some wonderful thinking and really helping us all. Okay Sione your turn how many now – and try and use Olivia's - everything and - strategy* [shows 16 on the Slavonic Abacus]
Sione	*Oh I know it's everything aye Olivia? So that's 10 aye? And then it's 11, 12, 13, 14, 15, 16 – is it 16 Olivia?*
Olivia	*Yes it is but I didn't need to count I just knew there was 6.*
Sione	*Oh me too but it's good to double check aye Miss?*
Brooke	*It does indeed pay to double check Sione. So now we have two wonderful strategies the '5 and' strategy and the 'everything and' strategy.*

Students' attention was drawn to Olivia's explanation by Brooke suggesting they listen carefully to Olivia and promoting her explanation as another very good idea.

Olivia claimed that 13 was everything and 3 more and after being questioned by Brooke she explained everything was the top row of the Slavonic abacus so that's 10 and 3 more is 11, 12, 13. Brooke praised Olivia for her excellent idea and thanked her for doing some wonderful thinking and really helping us all. Sione was directed by Brooke to use Olivia's 'everything and' strategy to determine how many beads were showing on the abacus. He did this successfully and checked in with Olivia to ensure he was using the strategy correctly and had the correct answer. The lesson continued with students applying Olivia's '5 and' and 'everything and' strategies to identify amounts to 10 and 20.

7.5.5 Discussion: Positioning for efficient thinking

Brooke positioned students in her highest strategy group to resolve their own mathematical disagreements by sharing and explaining their mathematical know-how, and critiquing and appropriating peers' know-how. In Excerpt 7.5.5, Olivia provided strategies for recognising amounts to 10 then 20. The first strategy was to notice '5 and' how many more to determine amounts to 10 and the second strategy was 10 or 'everything and' how many more to recognise amounts to 20. Students had a duty to solve their own mathematical dilemmas, to share their own and participate in others' mathematical know-how, to critique their own and others' know-how for efficiency, and to listen to and apply each other's know-how. Students were positioned by Brooke to explain their answers, listen to and utilise others' explanations, and discuss the efficiency of different strategies.

A prevalent storyline created by Brooke and her highest group was that students' mathematical know-how was important and should be listened to and appropriated. Brooke positioned students in her highest group to benefit from the mathematical know-how of others. Students' strategies were emphasised by Brooke when she repeated them and asked students to listen carefully to each other. Listening to and understanding each other was developing as a significant social act. Olivia's '5 and' and 'everything and' strategies became significant social acts for students when Brooke emphasised their value and positioned students to apply them.

The positioning of students in Brooke's lowest and highest strategy groups differed as did the positioning of Brooke. With her lowest group Brooke sought answers whereas students in the highest group were expected to explain the strategies behind their answers. The dialogue in Excerpts 7.5.1 and 7.5.2 had a turn-taking feel with students offering answers in their turn and removing themselves from the game if their answer was incorrect. Sione, who was very new to the class and school, followed the other boys and learned how this game was played. Students in the highest strategy group were positioned by Brooke to go beyond sharing answers to explaining the strategies behind the answers. It appeared students in the highest group were given more opportunities to share their own and experience others' mathematical know-how than students in the lowest group.

7.6 Lisa, Pacific School, Year 1

Table 7.5: Lisa's strategy group data

	Lowest Strategy Group	Highest Strategy Group
Number in Group	5	6
Year Level	1	1
Age	6	6
Gender	5 girls	3 boys and 3 girls
Strategy Stage	Stage 3: Counting from one by imaging	Stage 4: Advanced counting
Achievement	As expected	As expected

The positioning decisions Lisa made regarding herself and students in her lowest and highest strategy groups were inconsistent. Lisa was the only teacher who appeared to provide more opportunities for students in her lowest group to share their mathematical know-how. The importance of students in her lowest group being able to explain their mathematical know-how and consider the mathematical know-how of others was stressed by Lisa. Mathematical know-how was considered less by students in her highest group because the emphasis from Lisa was more on knowing how to behave than mathematical know-how.

7.6.1 "The 'why' is the really important bit"

Learning about the place value of teen numbers was the focus for the first lesson Lisa taught her lowest strategy group. Lisa showed students a Slavonic abacus with ten beads on the first row and one bead on the second row. Students were asked to describe what they could see and Lisa emphasised the importance of students explaining the strategy they used, not just providing the answer.

Participant	Dialogue
Lisa	*Let's hear from Ripeka first and there will be a chance for everybody else to share later. Please tell us your answer and then explain why you think you are correct – the why is the really important bit.*
Ripeka	*It's 11 because 10 plus 1 makes 11.*
Lisa	*Okay you think 10 plus 1 does anyone else think something different?*
Students	*No.*
Lisa	*Okay Emily can you point for us, where did Ripeka get 10 plus 1 from?*
Emily	[points to the abacus] *Here and here.*
Lisa	*Okay so close your eyes and I'm going to show a different teen number* [slides 2 more beads across on the second row] *okay eyes open and Dora what can you see?*
Dora	*There is 10 on that one and 3 on that one.*
Lisa	*Good and how much do you think that might be altogether?*
Dora	*10 and 3.*
Lisa	*Okay would you like to talk to Tui about how many there altogether?*
Tui	[whispers to Dora] *It's 13.*
Lisa	*Thank you Tui but is there a way you could help Dora to understand why there are 13 instead of just telling her there are 13?*
Tui	[points to the first row] *There are 10 on this row Dora and then count up with me.* [points to the second row]
Tui & Dora	*11, 12, 13.*
Lisa	*Very good so Tui knew there were 13 because she used a counting on strategy – is that a strategy you could use next time Dora?*
Dora	*Yes.*

Ripeka provided the correct answer 11, and a strategy 10 plus 1 makes 11. Different ways of strategising were sought from students and when none where forthcoming Lisa asked Emily to explain Ripeka's strategy. Emily indicated the row of 10 beads and the single bean on the abacus. Lisa showed 13 on the abacus and Dora was asked to describe what she could see. Dora correctly described seeing 10 and 3 but when asked how much 10 and 3 would be altogether she again stated 10 and 3. Lisa asked Dora if she would like to talk with Tui, and Tui leaned in and whispered: It's 13 to Dora. Tui was thanked for providing the answer and reminded that it was important that she help Dora understand why the answer was 13. Talking to Dora, Tui stated there are 10 on this row and then invited Dora to count with her to 13. Lisa described the strategy Tui used as a counting-on strategy and checked with Dora if this was a strategy she could use in the future. Throughout this lesson Lisa reminded students to use Tui's counting-on strategy if they needed to check they were correct and Dora effectively applied the strategy to determine the total number of beans on various occasions.

7.6.2 "Did anyone else do anything different?"

The second and third lessons with Lisa's lowest group focussed on students learning how to "image numbers up to 20 to solve addition and subtraction problems" (MoE, 2007e, p. 15). The third lesson included solving subtraction problems and Lisa sought varying strategies from students. She showed students two tens frames, one with 10 counters and the other with 4 counters and presented the problem as follows.

Participant	Dialogue
Lisa	*Okay so imagining that the counters on our tens frames are 14 lollies – Ripeka's lollies! But Ripeka is such a lovely girl that she is going to share her lollies and give Dora 5 of them.*
Dora	*Yay I get 5.*
Lisa	*So imagine that you had 14 lollies and you gave away 5 of them.*
Ripeka	*I already know how much of them there would be left.*
Lisa	*Do you already know?*
Ripeka	*Uh-huh! It's 9.*
Lisa	*Okay so tell us how you are working that out.*
Ripeka	*If you take away 5 it's going to be 9 because you take away the 4 and then 1 more and that's 10 then 9.*
Lisa	*Right – very interesting. Tui can you tell us what Ripeka did?*
Tui	*No.*
Lisa	*Can you tell us what you did?*
Tui	*I did 14,* [holds up 1 finger] *13,* [holds up 2 fingers] *12,* [holds up 3 fingers] *11,* [holds up 4 fingers] *10,* [holds up 5 fingers] *9.*
Lisa	*That's an interesting strategy too. Did anyone else do anything different to Ripeka and Tui?*

Mabel	*Yes I pretended to take 5 off the tens frames and then I looked and it would be 9 because there would be 1 less than the 10 left and 1 less than 10 is 9.*
Lisa	*Oh okay so you imagined what it would look like if you took 5 counters off our tens frames?*
Mabel	*Yes.*
Lisa	*Right I think it would be a really good idea for us to have a closer look at those 3 strategies. We had Ripeka's part-whole strategy, Tui's counting back strategy, and Mabel's imaging strategy. Let's start with counting back – what can you tell me about that strategy?*

The problem 14 minus 5 was solved three different ways by students. Ripeka used a part-whole strategy, Tui used a counting back strategy, and Mabel imagined how the tens frames would look if 5 of the 14 counters were removed. Lisa described the different strategies as interesting and provided the opportunity for students to explore the different strategies. They were supported to make their own discoveries about the strategies and the appropriateness of using them.

7.6.3 Discussion: Positioning for understanding

Lisa positioned herself with her lowest strategy group to elicit different strategies from students and to ensure students had opportunities to explore and understand the strategies they and others used. In Excerpt 7.6.1, Lisa emphasised the importance of students explaining why they thought they were correct. Tui was expected to go beyond telling Dora the correct answer to helping her to understand the strategy. Students suggested three different strategies for solving 14 – 9 in Excerpt 7.6.2. Lisa ensured students had the opportunity to explore and understand each strategy by reviewing them. Students had a duty to share and explain their mathematical know-how and to assist their peers to understand the strategies used. Tui modelled a counting-on strategy which helped Dora identify 13. Ripeka, Tui, and Mabel modelled three different strategies which students were expected to consider, comprehend, and where appropriate, apply. By positioning students to pay attention to each other's strategies, Lisa emphasised the importance of shared mathematical know-how. Sharing mathematical know-how was an important storyline for this group and each different strategy became a significant social act for the group because of the emphasis it was given. A second important storyline was that the 'why' of correct answers was important. Lisa expected students to do more than share correct answers; she expected them to explain why the answers were correct.

In summary, students in Lisa's lowest strategy group were expected to share and explain their own mathematical know-how and to pay attention to, and appropriate, the mathematical know-how of others. Lisa positioned herself to ask questions that required students to go beyond answers and to use students' strategies to progress students' learning. Students in Lisa's highest group were not positioned to share their mathematical know-how in ways consistent with the lowest group. The emphasis with the highest group, as illustrated in the following excerpts, appeared to be more about behaviour management than mathematical know-how.

7.6.4 "Now once everybody is sitting up properly I'll show you something"

The first lesson Lisa taught her highest strategy group required students to "count objects by creating groups of 10 from materials" (MoE, 2007e, p. 17). Lisa began the lesson by writing the number 36 in the group modelling book and asking one student How do we say that number? The next two excerpts illustrate the emphasis Lisa gave to students sitting on their bottom, sitting up properly, putting their hands up, and not calling out.

Participant	Dialogue
Lisa	*I'm going to show you a number and somebody who is sitting on their bottom with their hand up is going to tell me what that number is. How do we say that number Nina?*
Students	*36.*
Lisa	*No! You are not all called Nina and you are not all sitting on your bottoms please be quiet until you are asked. Nina?*
Nina	*36.*
Lisa	*Okay is that what you all think?*
Students	*Yes.*
Lisa	*Now once everybody is sitting up properly I'll show you something. Remember to put your hands up and I'll be able to give you a sticker at the end of this. Each of these tins is always going to hold 10* [shows students a tin] *so let's put 10 in here* [puts 10 beans into a tin].
Students	*1, 2, 3, 4, 5, 6, 7, 8, 9, 10.*
Lisa	*Good – so how many tins of beans would we need for this number?* [Shows 36]
Students	*30.* *3 and 6.* *3.*

	36.
Lisa	*Ah sitting on bottoms thank you, no calling out, and hands up! Right one at a time, Vincent what did you think? No calling out, I don't like it.*

Students in Lisa's highest group were very keen to share their answers and strategies. Lisa was persistent about students sitting on bottoms, putting their hands up, waiting until they were asked, and taking turns. Each student had a turn at stating how many tins of 10 would be needed to show 36 but none were asked how they knew 3 tins of 10 were required to show 36. Lisa summarised students' contributions by stating okay so we seem to be agreed that we need 3 tins of 10 to start with. Students were then asked so how much more do we need?

Participant	Dialogue
Students	*6 more.* *6 tins.* *6 more tins.* *6 more ones.*
Lisa	*And again I am going to remind you – no calling out, hands up, and sitting on bottoms before we continue. What do we need more of? Six tins or 6 beans. Now if you have got your hand up that means that you can say what we need to do to make this 30 into 36. Benji?*
Benji	*We need 3 more tins of 10.*
Lisa	*That is interesting – does anybody else have any ideas?*
Students	*No not 3 tins that would be 60.* *It's 6 ones we need 6 ones.* *No 6 tins.*
Lisa	*Right everybody sitting on bottoms and watching and only answer if I ask you. If I add another tin how many would I have Yasmeen?*
Yasmeen	*40.*
Lisa	*And is 40 more than 36 Benji?*
Benji	*Yes.*
Lisa	*Right so it isn't another tin I need to add. How much would I have if I added 1 bean Quentin?*
Quentin	*31.*
Lisa	*Two beans Nina?*
Nina	*32.*
Lisa	*Three beans Vincent?*
Vincent	*33.*
Lisa	*Four beans Quentin?*

Quentin	*34.*
Lisa	*Five beans Gabby?*
Gabby	*35.*
Lisa	*Six beans Benji?*
Benji	*36.*
Lisa	*Good so how many beans did I need to add onto 30 to make our number 36?*
Students	*6.*
Lisa	*Okay so 36 is 3 lots of 10 and 6 ones. Right now I want you to close your eyes and keep them shut until I tell you to open them. I am going to make a number using the tens and ones and I want you to tell me what I am showing. Okay open your eyes.*
Students	*It's 21.* *No it's 10, 20, 30.* *20 hundred.* *No it isn't.* *It's 21.* *No shush 10, 20, 30.*

Students suggested both 6 more tins and 6 more single beans were needed to show 36. Lisa reminded students no calling out, hands up, and sitting on bottoms before we continue. Benji suggested we need 3 more tins of 10 and Lisa sought different ideas from other students. Students called out their suggestions and Lisa directed them to only answer if I ask you. Lisa modelled adding another tin and asked Yasmeen how many would I have? Yasmeen answered 40, Benji noted 40 was more than 36, and Lisa declared so it isn't another tin I need to add. Lisa added one individual bean to the group of 3 tins and asked students one at a time how much would I have? Each student had a turn counting from 31 to 36 and Lisa declared Okay so 36 is 3 lots of 10 and 6 ones. Students were asked to close their eyes whilst Lisa made another number using the tins of 10 and individual beans. On being asked to open their eyes students resumed their calling out and shushing of each other, in what appeared to be attempts to be heard.

7.6.5 Discussion: Positioning for good behaviour

Lisa positioned herself with her highest strategy group to ensure students behaved appropriately during their maths lessons. Appropriate behaviour included students sitting on their bottoms, putting their hand up when they wanted a turn, and not calling out. Students were rewarded for their behaviour by getting a turn to share their answer. For example, in Excerpt 7.6.4, Lisa stated somebody who is sitting on their bottom with their hand up is going to tell me what that number is. Good behaviour took precedence over mathematical know-how. Students were expected to behave appropriately to get a turn and having their hand-up was interpreted by Lisa that they knew the answer. However, students were not expected to explain the strategies behind their answers. Lisa had a right to expect students to behave appropriately but students did not always accept this as their duty. If Lisa posed a question to the group, rather than an individual, students called out answers and leaned in toward Lisa waving their hands as a means of gaining her attention. When students did not respond with appropriate behaviours Lisa adopted a procedural and turn-taking approach of calling on individual students.

The prevalent storyline created by Lisa for her highest strategy group was that good behaviour was a prerequisite to participating. This limited opportunities for some students to participate because the mathematics may have been easier to accomplish than adhering to the good behaviour expectation. The expectation that students behave appropriately could be interpreted as the most significant social act for this group because of the emphasis it was given by Lisa. However, because students often resisted this expectation, challenging the need to sit properly, put your hand up, and not call out may have become an important social act from the students' perspective.

In conclusion, Lisa did not position students in her highest strategy group in the same way she positioned students in her lowest strategy group. Students in the highest group were positioned to behave appropriately and take turns. Good behaviour appeared to be more important to the learning process than the sharing of mathematical know-how.

7.7 Summary

This chapter illustrated the case study of inconsistent positioning practices of five teachers who differentially enabled students to share their mathematical know-how.

Different positioning decisions were made for one student in Paula's lowest and highest strategy group and for the students in Faith's, Naomi's, Brooke's, and

Lisa's lowest and highest strategy groups. The individual student in Paula's groups, the students in Faith's, Naomi's, and Brooke's lowest strategy groups, and the students in Lisa's highest strategy group appeared to have fewer opportunities to share their mathematical know-how and had less access to others' know-how. Teachers positioned themselves with these students to protect students from embarrassment, ensure they followed the correct steps to solve their problems, ask questions that required answers more than explanations, repeat and rephrase questions until the correct answer was given, and provide explanations for students' correct and incorrect answers. Students had a duty to watch and listen carefully, to have turns responding to the questions, to provide answers but not always explanations, and to behave appropriately.

The majority of the students in Paula's groups, the students in Faith's, Naomi's, and Brooke's highest groups, and the students in Lisa's lowest group had opportunities to share their mathematical know-how. With these students, teachers positioned themselves to provide opportunities for students to resolve their mathematical disagreements, to assist each other through errors or misconceptions, and to consider and apply each other's different or advanced strategies. Students had a duty to share their mathematical know-how by explaining, modelling, and recording their thinking, providing and evaluating explanations for difference, efficiency, and sophistication, and reviewing and critiquing their own and others' work for accuracy.

The potential affording and limiting effects of consistent and inconsistent positioning decisions by teachers for themselves and students in their lowest and highest strategy groups are analysed and explored in Chapter Eight: Discussion.

Chapter Eight: Taxonomy of Teacher Positioning

> By expressing their ideas, students are able to make their mathematical reasoning visible and open for reflection. Not only does the expression of student ideas provide a resource for teachers, informing them about what students already know and what they need to learn; the ideas also become a resource for students themselves—challenging, stimulating, and extending their own thinking. (Walshaw & Anthony, 2008, p. 526)

8.1 Introduction

This study addressed one key research question and three sub-questions:

> How do teachers in New Zealand primary schools position themselves and students in their lowest and highest mathematics strategy groups so that mathematical know-how can be shared?

1. What acts of teacher positioning with the lowest and highest strategy groups afford or constrain the sharing of teacher and student mathematical know-how?
2. What storylines are created by each teacher and group when shared know-how is afforded or constrained?
3. What social acts become significant for each teacher and group when shared know-how is afforded or constrained?
4. What impact could positioning have on student individual and shared learning

Teachers positioned themselves in various ways when working with the students in their lowest and highest groups. Some positions appeared to be affording whilst others seemed constraining. In the sections that follow, I propose an emerging taxonomy of teacher positionings that afforded the sharing of mathematical know-how and the counter-examples where the sharing of know-how was constrained. The emerging taxonomy provides a means of identifying patterns across the data and serves to facilitate the discussion of the findings in relation to literature in the field.

In Section 8.2, I introduce an emerging taxonomy of teacher positioning and explore the affording and constraining positionings of teachers in my study. Patterns of teacher positioning and their potential effects on teaching and learning are described in Section 8.3. In Section 8.4, I discuss recommendations for future research. The possible limitations of my study are acknowledged in Section 8.5. This thesis is concluded in Section 8.6 with a personal reflective comment.

8.2 Emerging Taxonomy of Teacher Positioning

Through my study I have contributed to the growing body of knowledge that focusses on understanding how mathematics education in New Zealand primary schools can be enhanced. Positioning theory provided a unique lens through which to understand how teachers afford or constrain the sharing of mathematical know-how. Evidence within the study suggests that how teachers position themselves and their students greatly influences who gets to collaborate and participate in the mathematics.

The emerging taxonomy of teacher positioning (see Table 8.2) presented in this section has been developed, and the categories identified, from an analysis of the excerpts in Chapters Six and Seven. The taxonomy addresses a critical aspect of mathematics pedagogy, that is, how teachers position themselves and their students in ways that afford and constrain the sharing of mathematical know-how. Teachers' affording and constraining positions have been included because it is important to identify features of productive and unproductive mathematical discussions that may open up or inhibit opportunities for student learning (Cirillo, 2013a). It was important to me to address disparities in education and outcomes for different groups of students. The differences identified in the ways teachers position themselves and students in their lowest and highest strategy groups makes this taxonomy an original and informative contribution with useful practical application for initial teacher education and on-going professional development.

The structure of the emerging taxonomy of teacher positioning comprises the three teacher positions which were found to afford the sharing of mathematical know-how and the three positions which were found to constrain the sharing of mathematical know-how. The terms given within my emerging taxonomy of affording and constraining teacher positionings are shown in Table 8.1.

Table 8.1: Teacher positioning

Teacher Affording Positioning	**Teacher Constraining Positioning**
Teacher as Appropriator	Teacher as Custodian
Teacher as Procurer	Teacher as Proclaimer
Teacher as Provoker	Teacher as Protector

The affording and constraining acts of teacher positioning identified through this study represent moments in time and as such are variable and fluid. The positions as rights and duties, the storylines that developed between the teacher in each position and their students and the social acts that came to be significant are described. Also described are the triggers (such as students' errors and misconceptions or their different or sophisticated explanations) which prompted the teachers' positioning. Teachers' affording positionings are described first, followed by teachers' constraining positionings.

Table 8.2: Taxonomy of teachers' positioning

	Appropriator	Procurer	Provoker	Custodian	Proclaimer	Protector
Positions as rights and duties	With: • actual & potential errors & misconceptions (EMC) • self & peer corrections • disagreements • questions Teachers: • Monitor talk, text, & actions • Notice & appropriate • Elicit elaborations, questions, explanations, & justifications • Elicit a mathematical stand • Revoice for confirmation of understanding • Promote errors, misconceptions & corrections, disagreements, & questions	With: • different, sophisticated, efficient explanations Teachers: • Monitor talk, text, & actions • Notice & elicit • Elicit elaborations, questions, explanations, & justifications • Elicit understanding of difference & sophisticated • Elicit efficiency of different & sophisticated • Elicit connections & generalisations • Revoice for confirmation of understanding • Promote different, sophisticated, efficient	With: • talk, text, and actions Teachers: • Provide and model accurate & inefficient thinking • Withhold thinking • Feign ignorance • Elicit help • Elicit elaborations, questions, explanations, & justifications • Elicit connections & generalisations • Revoice for confirmation of understanding • Promote/model asking for help	With: • actual & potential errors and misconceptions • correct answers and designated strategies Teachers: • Monitor talk, text, & actions • Determine acceptability of answers & explanations • Repeat and praise correct answers & explanations • Provide elaborations, questions, explanations, & justifications • Provide connections & generalisations • Repeat for confirmation of understanding	With: • actual & potential errors and misconceptions Teachers: • Monitor talk, text, & actions • Explain & demonstrate • Check, confirm, repeat, & explain correct and incorrect answers	With: • individual students errors and misconceptions Teachers: • Monitor talk, text, & actions • Shield individuals from social & mathematical embarrassment • Simplify procedural steps

	Appropriator	Procurer	Provoker	Custodian	Proclaimer	Protector
Storylines	• Learning can occur from EMC • EMC must be managed respectfully • Explanations & justifications are important • Monitoring & checking are important • A mathematical stand must be taken • Emphasis should be given to eliciting disagreements & questions	• Different and sophisticated explanations are important • The difference and sophistication must be understood, mathematically sound, and efficient • A mathematical stand must be taken	• Learning can occur from EMC • Answers and explanations are not always correct • Monitoring and checking are important • Connections and generalisations are important	• Teachers sanction answers & explanations • Problem solving occurs quickly & correctly • Correct answers & designated strategies are important • Explanations are not as important • Good behaviour is rewarded with participation in the mathematics	• Teachers provide answers & explanations • Problem solving occurs quickly & correctly • Problem solving strategies are ready made • Explanations are not as important	• Collaborative approach to correct answers • Individualistic approach to incorrect answers
Social acts	• Making EMC, disagreeing, asking questions • Analysing & deciphering the EMC • Appropriating and respecting EMC, disagreements, questions • Revoicing • Asking for and providing help	• Offering different and sophisticated explanations • Appropriating different and sophisticated explanation • Reasoning about efficiency	• Sharing correct and inefficient know-how • Feigning ignorance • Monitoring and checking • Connecting and generalising	• Teachers providing • Students following • Behaving appropriately • Turn-taking	• Teachers providing • Students following • Mimicking, repeating,	• Protecting • Individualising • Students following

8.2.1 Teacher as Appropriator

In this study, the teachers as appropriator positioned themselves as having two specific rights. The first and predominant right was to monitor students' talk, text, and actions, then notice, and appropriate actual and potential errors, misconceptions, and corrections. The second, less prevalent positioning, was the right to monitor, notice, and appropriate students' questions and disagreements. Smith and Stein (2011) contended that by monitoring students' thinking teachers were able to identify next steps, connect the key mathematical ideas in the lesson, and advance the collective understandings of the group.

Students' errors and misconceptions were interpreted and appropriated by the teacher and used to focus and engage students in mathematical discussions (Sections 6.5.3, 6.5.6, 6.7.3, 6.7.6, 7.2.3, & 7.2.6). Teachers appropriated errors, misconceptions, and corrections as being interesting and worth further consideration. They made the error, misconception, or correction public by repeating or revoicing it and encouraged discussion through questions such as, does that sound right? Choppin and Herbel-Eisenmann (2012) found that when teachers made students' contributions public, they prolonged both the social and academic benefits of the contribution. As a social construct of their learning students were expected to explain and justify their mathematical thinking, and the resulting academic benefit was that the mathematical quality of their explanations and justifications improved. Students were positioned to take a mathematical stand regarding the accuracy of answers and explanations. They were expected to decipher the thinking, know if the thinking was correct, or incorrect, and know why. Taking a mathematical stand in this research is similar to Chapin et al.'s (2009) expectations that students should have "a position on the idea" (p. 18). In both studies, teachers and students were expected to have a mathematical opinion and be able to defend that opinion.

Teachers ensured there was space in the lesson for students to explore, explain, defend, and reconsider their own and others' thinking before they, as teacher, offered their own thinking. This was similar to the teacher in Fraivillig et al.'s (1999) research who "conveyed a sense of believing that students could find the correct answer if they thought more carefully" (p. 156). According to Zimmerman

(2000, 2002), when teachers position students to attend to their own and their peers' errors they are positioning them to utilise their self-regulating skills and enhance their mathematical independence. Through appropriation, teachers were concurrently provided with insights into their teaching and students' thinking.

Mathematically-based questions from students and their disagreements have been shown to indicate thoughtful consideration, engagement, and increasing responsibility (Lampert, 1990; Martino & Maher, 1999; Reinhart, 2000). In this study, there were fewer examples of teacher's appropriating students' questions and disagreements because there were fewer examples of students asking questions and voicing their disagreements. The reason for this could be that the NDP model enacted by some teachers in this study did not promote or elicit student questioning or disagreements.

When appropriating questions and disagreements, teachers positioned themselves similarly to when they appropriated errors, misconceptions, and corrections. Teachers publicised the questions and disagreements and gave students the opportunity to discuss them before they offered any assistance. Both were used to increase the difficulty of the task and discussion. Although sparse, students' questions and disagreements were appropriated as valuable teaching and learning tools that could influence the content and direction of the lesson.

The four storylines and related social acts developing between the teacher as appropriator and their students are discussed next. The first storyline was that errors, misconceptions, and corrections were valuable teaching and learning resources. Research from others has shown that when used respectfully, errors, misconceptions, and corrections can increase the productivity of the conversation and the mathematics (Anthony & Hunter, 2005; Stein et al., 2008). A second simultaneous storyline was that to become valuable teaching and learning resources, errors, misconceptions, and corrections needed to be managed respectfully by the group. Hunter (2009) found that when teachers respectfully appropriated errors, students could be encouraged to persevere, re-evaluate their thinking, and grapple with complex ideas. Implicit within this storyline was the understanding that it was okay to make mistakes. As one group of students in Anthony and Walshaw's (2006) research noted, "it was alright to get it wrong. Because the teacher said you can always learn from it; so it doesn't matter if you make a mistake” (p. 24). The third storyline was that mathematics was about more than correct answers. It was also about explanations, justifications, and conjectures. Within this storyline was the expectation that students would monitor their own and others’ thinking and be able to articulate or model why they were correct or incorrect and how they knew to make corrections. The final storyline was less explicit, and that was that students did not ask questions or openly disagree as often as they made errors or corrections.

Four social acts also emerged from the actions of the teacher as appropriator and the students in their groups. The first social act involved students making mistakes and corrections, disagreeing, and asking questions. They came to be significant to the group because of the way teachers valued and made use of them to enhance access to know-how. Teachers noticing, deciphering, and then appropriating mistakes, corrections, disagreements, and questions were the second and third social acts. If teachers had disregarded these contributions from students or corrected them, they would not have come to have the same value for teaching and learning. The shared action of teachers and students asking for and providing assistance was the fourth social act. Teachers and students seeking and providing help enhanced the collaborative nature of the teaching and learning.

8.2.2 Teacher as Procurer

The teachers as procurer in my study positioned themselves as having the right to elicit different, sophisticated, and efficient, explanations and justifications from students and positioned students with the duty to engage with the explanations and justifications of others (Sections 6.3.3, 6.3.5, 6.4.2, 6.4.4, 6.5.6, 6.6.2, 7.3.5, 7.4.4, & 7.5.5). In similar positioning to the teacher as appropriator, the teacher as procurer highlighted explanations that were different or advanced by making them public to the group and encouraging further discussion.

The emphasis of discussions was on student explanation, meaning, and understanding. Yackel and Cobb (1996) claimed that students should be positioned to “attempt to make sense of explanations given by others, to compare others’ solutions to their own, and to make judgements about similarities and differences” (p. 466). In my study, teachers did not appraise the attributes of different or sophisticated explanations. Instead, they pressed students "to refine, revise, or elaborate their explanation" and "elicited comments from other students about the explanation" (Choppin, & Herbel-Eisenmann, 2012, p. 3).

When students have a duty to consider the adequacy of an explanation for others and not just themselves, the explanation itself becomes the "explicit object of discourse" (Yackel & Cobb, 1996, p. 471). The explanation behind the answer and the efficiency of that explanation comes to have more meaning to the mathematics than the answer itself. It was not acceptable in my study for students to only suggest different or sophisticated explanations. Instead, as previous research has shown, students' emphasis had to be on efficient, different, and sophisticated explanations that contributed to each group's shared and enhanced understandings, and progressed the opportunities for learning (Truxaw & DeFranco, 2007; Wagner & Herbel-Eisenmann, 2013). Leinhardt and Schwarz (1997) advocated that a focus on the efficiency of explanations could move students away from offering wild guesses toward more plausible ones because with each explanation came the expectation of meaningful justification. Mathematical difference is considered a construct of mathematical creativity because students are flexibly considering the problem from different perspectives and attempting unanticipated solution strategies. Davies and Hunt (1994) intimated that when teachers treat difference with high regard they are positioning the individuals who contribute the different ideas as valuable to the group's progress.

Research has shown that when teachers promote different and sophisticated explanations they provide different entry points into the discussion, give directionality to the learning, and support the development of flexible reasoning (Cirillo, 2013b; Franke et al., 2007). Teachers in my study had the right to call on students to elaborate on their own and others' initial thinking by repeating, revoicing, and adding on to the explanations. Positioning students to say more, revoice, and add on to others' contributions is advocated by Chapin et al. (2009) as talk moves that enhance students' engagement with their own and others' mathematical thinking and reasoning. Teacher revoicing has been found to be advantageous when clarifying and sequencing students' thinking, reshaping what has been asked to increase accessibility to the question, and pressing students to clarify or revise their own and others' explanations (Hunter, 2005; Stein et al., 2008). Yamakawa and colleagues (2005) identified that when teachers revoiced students' ideas, they sent an implicit message that the contribution was worth listening to and could advance the discussion in productive ways. Teachers in Yamakawa and colleagues' study were also found to significantly increase the participation of other students when they revoiced the mathematics of one individual.

It follows then that student revoicing would be equally advantageous. As observed in my study, for students to revoice or add on to others' thinking they had to understand the mathematical thinking up to that point in the discussion, anticipate the possible directions the discussions could go, and reconsider how to articulate their thinking so that others could better understand.

Whilst the teacher as procurer did not directly share their know-how, they did participate by knowing which student know-how to procure and what questions to ask (Attard, 2013; Smith & Stein, 2011). As Lampert (2001) noted, teachers knew how to get "particular pieces of mathematics on the table" (p. 140). For example, in my study, teachers posed questions shaped from students' ideas that required different or more in-depth thinking. Teacher questioning allowed access to diverse mathematical knowledge and targeted the development of particular mathematical understandings. As research has shown, questions derived from students' ideas acknowledge the value of the student contribution, demonstrate teacher interest in how students think, and enhance the shift in focus from answers to explanations (Anthony & Walshaw, 2009; Irwin & Woodward, 2005; Lampert, 2001).

In my study, the teacher as procurer slowed the pace of the lesson to allow time for students to access the mathematics in different ways, to trial different and advanced ways of reasoning, and on occasion changed the planned direction of the lesson. Chapin and O'Connor (2007) found that such actions communicated to students that they had some autonomy within the lesson and could influence the content of the lesson and direction of the learning.

Students had a duty to make a wide range of mathematical contributions. Positioned similarly to the students in Pape et al.'s (2003) research, students were expected to explain, judge, and justify their own and others' know-how and access and trial multiple representations and strategies. Students had opportunities to develop their mathematical creativity because they were flexibly approaching problems from varied solution strategies (Bolden et al., 2009). To participate in the mathematics, they had to be able to explain and justify their own and peers' thinking. Students had to think metacognitively, that is, they had to think about their thinking (MoE, 2013). They had to know and be able to articulate why their explanation was different, sophisticated, and efficient and reflect on what constituted the difference, sophistication, and efficiency.

Within the position of teacher as procurer, there were three evolving storylines. The first storyline was that mathematical difference was a valuable resource for deepening and strengthening mathematical know-how. Implicit within this storyline was the understanding that it was okay to be different and the expectation that mathematical difference would be purposeful, understood, justified, and lead to increased efficiency with problem solving. However, the use of routine problems may have limited opportunities for students to explore substantially "different mathematical territory'' (Lampert, 2001, p. 44). There were examples of creativity within teacher affording positioning whereby students used more advanced or sophisticated strategies than those advocated by teachers but they were still applying a best-fit strategy to a routine problem. The problems given to students contained numbers designed to match specific strategies and were not designed to elicit substantially different or sophisticated solution strategies. Students learnt that problems were not always solved quickly and that more than one solution strategy could exist in the second storyline. Being correct with answers was important but so too was being considered and efficient with explanations. The third storyline related to student accountability. Students had to understand and be able to articulate what constituted mathematical difference, advancement, and efficiency.

Two social acts were developing between the teacher as procurer and students. Students' different and sophisticated explanations came to be significant because of the emphasis given by the teacher. Teachers promoted explanations and elicited discussions around difference, sophistication, and efficiency. The mathematical reasoning regarding the efficiency of the different and sophisticated explanations was the second social act. Explanations came to have more social force than answers.

8.2.3 Teacher as Provoker

The teachers as provoker positioned themselves as having the right to share and withhold their own know-how in ways that prompted further interactions (Sections 6.2.3, 6.2.6, 6.3.5, 6.6.5, 7.4.4, & 7.6.3). They provided accurate and inefficient mathematical models, highlighted mathematical connections and patterns, and positioned students as having a duty to help the teacher and each other.

Teachers in my study positioned as provoker had the right to share accurate and inaccurate mathematical talk, text, and actions, and to pretend not to know in ways that provoked and generated student talk, text, and actions (Lev-Zamir & Leikin, 2011). They modelled deliberating, contemplating, and reasoning with their knowledge and provided examples of how students could do the same. Lampert (1990) identified that when teachers verbally pondered over the mathematics using words such as maybe and perhaps in their questions and explanations, they elicited more opinions, guesses, explanations, and justifications from students. Fraivillig et al. (1999) contended that examples from teachers such as those suggested by Lampert, provided students with starting points and bridges for discussion.

Accurate explanations, written recordings, and models have been shown to provide scaffolding for students to discuss, trial, and contribute to their developing awareness of themselves as legitimate creators of mathematical knowledge (Anthony & Walshaw, 2007; Attard, 2013). Through teachers' models and scaffolding, students were alerted to know-how that was worth noticing and had time to explore and synthesise their ideas and make connections (Choppin, 2011; Fraivillig, et, al., 1999; Hunter, 2007a). Teachers did not tell students the answers or how to think; instead, as Hunter (2005) and Anderson (2009) identified, they withheld their evaluative authority and provided the space and reason for students to think and the resources with which to think.

Parallel to the findings of Zimmerman and Martinez-Pons (1986), when teachers in my study shared inaccurate and inefficient teacher know-how and feigned ignorance, they provided platforms for students to organise and reorganise their know-how, and models and opportunities for how students could seek and provide help. Misconstrued mathematical models that conflict with students' existing know-how provide them with practice in questioning and sense making, the aim being to "bounce students out of their intellectual numbness by confronting them with misconceptions" (Nansen, 1998, p. 13). Teachers in my study drew on their acting skills, pretended they were approaching the problem for the first time, and modelled indecision, risk-taking, and perseverance (Haylock, 1987; Pólya, 1963). This is one of the few examples of teachers teaching in creative ways in my study. As well as demonstrating a measure of creativity, teachers modelled that mistakes were acceptable, provided valuable material for discussions, and could be used as resources for teaching and learning (Chapin et al., 2009).

In this study, teacher know-how was not, as Lampert (1990) advocated, presented as messy and raw. Some of the discussions could be described as messy and raw when students argued for their novel thinking, but not the problems that initiated the discussions. Teachers tended to present well-considered and developed problems and models that were immediately understood or helpful for the students. There were also no examples of teachers genuinely modelling guesses or pretending to guess (Lakatos, 1976; Lampert, 1990). Kendra (Tasman School) was the only teacher in my study who appeared mathematically challenged to a point where she could have modelled guessing the possibility of 101 and 100. She, instead, chose to defer the discussion until the following day because she was not sure how to support students to answer the question themselves. It seemed that Kendra wanted to be sure she was correct and was not prepared to take a risk with her own mathematical know-how.

Students were concurrently positioned by the teacher as provoker as having a duty to engage with, and attend, to teacher know-how and 'do not know-how' by exploring, critiquing, interpreting, and trialling different approaches to problem solving (Leikin, 2009). As recommended in other research, students had a duty to notice and analyse what the teacher was sharing, anticipate what might come next, make connections with existing knowledge, and construct and apply new learning (Lev-Zamir & Leikin, 2011). Stein et al. (2009) promoted anticipation as a teacher practice that better positioned teachers to understand students' thinking. In my study, teachers also positioned students to anticipate in ways that supported them to engage more with their own and others' thinking. Teachers' scaffolding supported students to participate and provided them with examples of how they could support and provide scaffolding for their peers (Hunter, 2007b). As endorsed by Anthony and Walshaw (2009), each of these behaviours required students to act for themselves and their peers in ways that increased their capacity to purposefully and reflectively think with others. Purposeful and reflective thinking is evidence of students' developing independence and creativity because they are thinking mathematically for themselves in diverse ways (Darr & Fisher, 2005; Lev-Zamir & Leikin, 2011).

The teacher as provoker reiterated similar storylines to that of appropriator and procurer. In a storyline related to the teacher as appropriator, errors provided valuable teaching and learning resources and the monitoring and review of everyone's mathematical thinking was important. By (intentionally) making mistakes, teachers were not positioning themselves as group members who are always correct and were positioning students to attend more closely to their teacher's thinking. In a separate storyline, the teacher as provoker highlighted the importance of mathematical connections and generalisations. Connections and generalisations provided access to increased understandings and strategies for easier problem solving.

The social acts that came to have significance for the group were the teachers' sharing of correct and incorrect answers, and efficient and inefficient explanations. These actions were meaningful because they caused students to pay closer attention to, and review, the mathematics within the discussions. The second social act was the monitoring and checking undertaken by the students. Students were expected to follow and review the accuracy and efficiency of the teachers' contributions. Also significant to the learning was the connecting and generalising between mathematical explanations and ideas undertaken by both teachers and students.

8.2.4 Summary of teacher affording positioning

Three themes have been identified across the positionings of teacher as appropriator, procurer, and provoker. These themes pertain to the collaborative partnership approach to teaching and learning, the focus of the teaching and learning, and the purpose of the mathematics. Each theme is discussed next.

The teacher as appropriator, procurer, and provoker in my study exhibited a pedagogical orientation toward collaborative partnerships. Teachers and students shared their mathematical know-how in ways that aided each other, both made mistakes and learned from them, both explored mathematical difference, and both provided extension or guidance as required. Teachers and students were equally important contributors to the teaching and learning. Students learned mathematics content and how to provide and ask for assistance. The more students participated in the mathematics, the more know-how teachers gained regarding how to facilitate opportunities for more learning. The more teachers were able to facilitate opportunities for learning, the more access students had to learning. This co-positioning created a feeling of collaboration and a shared responsibility for the success of both (Zimmerman & Labuhn, 2012).

Cirillo (2013b) contended that teachers' and students' contributions should deepen and strengthen access to the mathematics, shape instruction, and occasion particular mathematical understandings in the classroom. According to Hunter (2005) and Choppin (2011), teachers and students should have a collective responsibility to purposefully establish, notice, and sustain the mathematical quality of the teaching and learning. In my study, teachers and students individually and collaboratively communicated their ideas, highlighted connections, constructed new learning, influenced the lesson content and direction, and reasoned in mathematically meaningful ways. The storyline arising from the collaboration between teachers and students was that both had something important to learn from the other, and what teachers and students learned and had to learn became significant social acts for the group. Research has shown that when teachers and students believed they had something mathematically important to learn from each other they would also have reason to believe their contributions could make a difference (Boaler, 2003; Ewing, 2007; Frank & Kazemi, 2001). In line with Anderson's (2009) findings, teachers and students in my study who had a right and duty to access resources for meaning making were positioned as the kinds of people who succeed. Positioned as the kinds of people who succeed, teachers and students were situated within the iterative and positive feedback loop identified by Zimmerman and Martinez-Pons (1990) whereby gains in understanding led to gains in achievement, which led to gains in self-efficacy.

The focus of teaching and learning was not teacher-directed or student-centred. Instead, the focus on collaboration was as Goos (2004) advocated, a permutation of both resulting in a community where social endeavour and negotiated mathematical understanding were at the heart of teaching and learning. Stein et al (2009) identified that some teachers in their research believed that for student thinking to be prevalent within discussions, “teachers had to avoid providing any substantive guidance at all” (p. 316). This was definitely not true for affording teacher positioning in my research. In my study, the teacher was positioned as an important and valued member of the group. The teachers who afforded opportunities for mathematical know-how to be shared were effective because they were deliberately in control of their positions as appropriator, procurer, and provoker. They intentionally supported student learning by eliciting students’ ideas and thinking and effectively managed mathematical discussions to as to position the students as co-constructors of the mathematical ideas. These teachers were
open to learning about students’ approaches to mathematics and the discipline itself (R. Hunter, 2006). By positioning themselves as active contributors, teachers provided models of effective explanations, questions, challenges, and justifications. There was a strong partnered feeling about the teaching and learning with teachers and students moving easily within each other’s positions.

Walshaw (2013) has proposed that the purpose of mathematics should be to “allow creative energy to emerge that will facilitate both students’ and teachers’ change” (p. 84). Within affording positioning, teachers in my study created opportunities and made room for the exploration of ideas, explanations, justifications, reasoning, conjectures, and reflection. As such, they could be described as teaching for creativity, that is, teaching in such a way that creativity from students was supported. Less apparent with my students was evidence of teachers teaching with creativity, there were few examples of teachers offering novel or creative problems or solution strategies. Also missing from the teaching was teachers' use of the word guess. Whilst teachers did not prompt students to guess, they did encourage them to trial different ideas and expect them to justify their thinking. Justification is a requirement of plausible guessing and judgement (MoE, 2013) that fosters monitoring, regulating, and generalising.

Teachers and students had opportunities and time to individually and collectively engage with their own and others' know-how and reflect on their learning. From Attard's (2011, 2013) perspective, teachers and students were positioned as being insiders, with both a place and a voice within their group and the mathematics. Time was given for both to explore and respond to their own and others' suggestions, observations, explanations, questions, and reflections and to seek additional information or assistance. However, at some point teachers and students had to make a stand by having and defending a mathematical opinion. By taking a stand, commitment was shown to the discipline of mathematics and to their mathematical thinking. Teachers and students were expected to take mathematical responsibility for their own and others' reasoning and to ensure the mathematics contained within the contribution was explicit (Chapin & O'Connor, 2007). The pace of the lessons, where the purpose of the mathematics was exploratory, appeared to be more relaxed and circuitous. Similar to the findings of Truxaw and DeFranco (2007), mathematical reasoning was presented as cyclic and flexible. Teachers and students used judgement to consider and reconsider their thinking, review and investigate alternatives, generalise, and generate new know-how.

Students readily accepted the positioning of teacher as appropriator, procurer, and provoker. Many students also positioned themselves similarly by asking questions that provoked thought, assisting peers with errors or misconceptions, and appropriating peers' explanations. In the following sections, I describe the constraining teacher positioning and associated storylines and social acts.

8.2.5 Teacher as Custodian

The teachers positioned as custodian in my study had the right to take ownership of the mathematical know-how and gave a commentary from the perspective of "one who could judge which aspects of the students' activity might be mathematically significant" (Cobb, Boufi, McClain, & Whitenack, 1997, p. 262). Teachers in my study positioned themselves to determine the acceptability of students' answers and explanations (Sections 6.8.3, 6.8.6, 7.3.3, 7.4.2, & 7.6.5). In similar ways to the teachers in Lampert's (1990) and Maher and Martino's (1996) research, teachers were quick to judge the appropriateness of student contributions and did not ask students to explain the reasoning behind their thinking. Wild guesses from students were responded to, albeit briefly, as though they were legitimate mathematical contributions. Correct answers were repeated and praised and incorrect answers were responded to with no. Chapin et al. (2009) suggest that by responding no, teachers are not meeting the requirements of respectful discourse and students would not be left feeling that it was "okay to be wrong" (p. 202).

The pace of the lesson where the teacher positioned themselves as custodian was quick. Teachers asked and repeated questions in directive and quick-fire succession and little space was made available for them or students to share and engage with mathematical know-how. Choppin (2011) warned that some students would be unable to sustain the pace of learning and others would concentrate solely on answers. The need for speedy responses was reiterated when teachers asked questions they already knew the answer to and that usually required one word answers rather than explanations (Irwin & Woodward, 2005). Correct answers were repeated, praised, and accepted as evidence of understanding and progress. Teachers' obvious expressions of delight would have left little doubt with students that correct answers were highly valued.

Different or sophisticated student explanations were discouraged by the teacher as custodian. Chapin and O'Connor (2007) claimed that when teachers ignored students' ideas they were disrespectful to the students themselves. Rather than seeking diverse thinking from students, teachers appeared to be "funnelling them toward a particular goal" (Choppin, 2011, p. 185). As Nadjafikhah et al. (2012) predicted, students may have been learning that mathematical difference was not required and not to challenge themselves or extend their thinking.

Students positioned by the teacher as custodian had a duty to take turns, recall facts, provide correct answers, and apply the designated strategy. A focus on rote learning and application rather than experimentation meant students had few opportunities to experience guessing (Maher & Martino, 1996). Students were not required to use any regulating strategies because the answer was more the focus than their explanation. Wong, et al. (2002) also found that when teachers focussed on correct answers they discouraged student use of guesses and justifications. Students did not have the challenge or triumph of grappling with their own or others' mathematical know-how. There was limited know-how available for them to build on, connect from, or learn with. Lev-Zamir and Leikin (2011) contended that student creativity was constrained by a focus on correct answers and the application of specific strategies.

In Lisa's (Pacific, Year 1) highest group, participation was dependent on students meeting Lisa's behavioural expectations and the reward for appropriate behaviour was a turn at the mathematics (Davies & Hunt, 1994). However, as McClain and Cobb (2001) contended once students had their turn they tended to make statements rather than offer ideas for discussion and did not endeavour to be understood or interpret others. The taken-as-shared understanding for students taught in this way could be that correct answers are sufficient evidence of knowledge and "doing mathematics consists of [little] more than producing right answers" (Anthony & Walshaw, 2009, p. 14).

There were three storylines developing between the teacher as custodian and students. The first storyline related to the teacher's positioning as the arbitrator of the mathematical thinking. Students did not need to consider or defend their know-how because the teacher did that for them. This positioning would have lessened the degree to which students were able to participate in the mathematics and correspond with the second storyline. The second storyline, is that maths is completed quickly and correctly and correct answers are evidence of mathematical know-how. The emphasis given by teachers to correct answers and knowledge and the limited opportunities to explore strategies and explanations meant the pace of the lesson was quick. The third storyline, developing more explicitly with Lisa (Pacific, Year 1) and her highest group, was that participating in the mathematics was the reward for good behaviour. Students were expected to sit and respond appropriately before they were able to have a turn sharing their answers. This behavioural expectation seemed to elicit answers more than explanations and there was limited engagement between students and their ideas. These three storylines resulted in teachers' and students' actions taking on a social force.

The teachers' right to arbitrate and judge the acceptability of the mathematics offered was the most significant social act for the teacher as custodian in my study. This is because students could not participate mathematically (or at times socially) without teacher approval. Students' correct answers and their use of designated strategies came to have importance within the groups because teachers emphasised them and praised students who contributed them. This resulted in a correlated social act which was the importance of teacher praise and endorsement. Teachers praised correct answers and endorsed the use of specific strategies. Both social acts could inhibit social and mathematical interactions between students. Behaving appropriately and taking turns were also significant social acts for some students. If students did not appropriately behave, they were prohibited from participating in the mathematics.

8.2.6 Teacher as Proclaimer

The teachers positioned as proclaimer in my study had the right to explain and demonstrate how students should approach the mathematics (Sections 7.3.3, and 7.4.2, 7.5.3). This was often before students had an opportunity to participate. The intent seemed to be to state rather than share, and students appeared positioned to engage with "the activity of following procedural instructions" (Cobb, Wood, Yackel, McNeal, 1992, p. 574).

The teacher as proclaimer had the right to check, confirm, repeat, and explain students' correct answers. Zimmerman and Labuhn (2012) contended that when teachers explain students' incorrect answers they are not supporting students to develop self-regulating skills such as asking for or providing help. Teachers provided one model or explanation and students may have been learning that each problem had one correct explanation (Leikin, 2009). The explanations and corrections were made quickly and presented as statements rather than ideas for discussion; therefore, students were accessing teachers' know-what more than their own or peers'. Schoenfeld (1985 1988) suggested that through such teacher positioning students may have been learning that problem solving occurs quickly and correctly.

The flow of mathematical ideas in this study seemed to be constrained because teachers tended to focus on filling gaps and fixing weaknesses and so students were not able to develop fluency or show flexibility with their know-how (Cirillo, 2013a; Anthony & Walshaw, 2009). Memorising and mimicking appeared to be promoted over thinking and resulted in what Lithner (2008) described as imitative thinkers. Students were able to make few decisions about their learning and did not have opportunities to wrestle with the appropriateness of their own and others' thinking. Research has shown that if students are not able to make decisions about their learning, their opportunities for mathematical independence decrease and their reliance on teachers may increase (Darr & Fisher, 2005; Nadjafikhah et al., 2012).

Students positioned by the teacher as proclaimer had a duty to listen and watch carefully and answers repeated from peers were praised and accepted as evidence of understanding. The danger here, as Murphy (2013) identified, is that students may have been learning how to report ready-made strategies and not developing their mathematical understandings of increasingly complex concepts. There was not an expectation that students would know why their own or peers' answers were correct or incorrect. Opportunities for peer interactions were limited because the teacher as proclaimer contributed most and praised and repeated correct answers. When students did contribute answers, they appeared to be derived without going through any process of regulation or reflection. Anthony and Walshaw (2009) warn that this positioning could have entrenched students' reliance on teachers in terms of knowing how to act and interact in mathematically appropriate ways. Deferring to the teacher could become the prominent regulating strategy of students (Zimmerman & Martinez-Pons, 1986). Students appeared to accept their positioning perhaps because there was little room in the lesson to do otherwise.

Three storylines were occurring between the teacher as proclaimer and students. The most prominent storyline was that it was acceptable for teachers to do the students' mathematical, thinking, talking, recording, and modelling. Teachers had a right to be the first group member to provide answers and explanations. As with the rights of the teacher as custodian, the positioning of teacher as proclaimer limited the opportunity students had to engage with their own and others' know-how. The second storyline was that students had a duty to show they were learning by following, mimicking, and repeating the teacher. The third storyline concurred with one from the teacher as custodian, that is that teaching and learning occurred quickly, and correctness was emphasised. Teachers presented their know-how as fact and students were not expected to connect with the shared mathematics. Explanations were rarely offered by teachers or sought from students. Within each storyline described above, there is an abundance of teacher rights and this influenced the significance teachers' actions came to have. Teachers' actions had a social force; students' mimicry may have come to have importance within the group but it was unlikely that the importance would be mathematically beneficial to the students' opportunities for increased mathematical understanding.

8.2.7 Teacher as Protector

In my study, the teachers positioned as protector had the right to explicitly and implicitly shield students from social or mathematical discomfort when they were incorrect or confused (Sections 6.8.6, 7.2.3, 7.2.6, & 7.3.3). The aim appeared to be two-fold, first to make the mathematics simpler by emphasising procedural steps and transmitting the required knowledge and secondly to protect the student from possible challenges or teasing from peers. Hunter (2006) posited that such actions could support students to withdraw and rethink "without losing face" (p. 312). However, in my study, it appeared students were withdrawn to listen and follow more than rethink.

One teacher, Paula (Pacific School, years 5/6) was explicit in her positioning of herself as protector with one student in her lowest and highest groups. When these two boys' errors or misconceptions were identified, Paula moved them away from the group and discreetly explained the error and how it could be corrected. Peers did not have a duty to assist the boys, nor were they allowed to question or challenge their explanations. From the position of protector, Paula had the right to shield both boys from potential mathematical and social embarrassment and prevent any opportunity for disrespectful behaviour from peers (Chapin & O'Connor, 2007). However, students were also prevented from trialling new strategies, testing justifications, taking risks, and developing resilience. The boys had a duty to share their mathematical thinking, but if that thinking was erroneous, Paula assumed the right to be the only person to assist them. According to Anthony and Walshaw, (2009), Paula was potentially limiting the boys' access to broader interpretations of the mathematical ideas.

Teachers in this study also implicitly protected students in less obvious ways, first, when teachers did the thinking for students by providing the mathematical reasoning behind correct and incorrect answers and explanations, secondly, when teachers did not position students as accountable for reconciling their own or peers' misconceptions, thirdly, when advanced or sophisticated mathematical thinking was dismissed, and fourthly, when good behaviour and turn-taking took precedence over the quality of the mathematics being shared.

The first storyline developing between the teacher as protector and students was that not all students' mathematical know-how was able to be publicly considered by the group. The teacher was the only person who had the right to address certain students' errors and misconceptions. In this situation only the teachers' talk, text, and actions could come to have social significance for the group. The talk, text, and actions of the students being protected did not have an opportunity to contribute to the groups shared knowledge, and so their contributions would not have been able to become socially meaningful.

8.2.8 Summary of teacher constraining positioning

The discussion of teacher constraining positioning in my study has identified three themes. These themes relate to how the teacher positioned as custodian, proclaimer, and protector directed the teaching and learning through an individualised turn-taking approach, focussed on correct answers and specific strategies, and conducting the lessons at a brisk pace.

The mathematical know-how within constraining teacher positioning predominantly belonged to the teacher who, as Wagner and Herbel-Eisenmann (2013) described, positioned herself as the gatekeeper. The sense of collaborative partnership apparent within affording positioning was not evident within constraining acts of positioning. Teachers and students operated more individualistically and did not appear as united in their efforts to collaborate in ways that enhanced the opportunities for both teaching and learning.

Teachers in my study asked and answered questions, modelled and explained, summarised learning, and dismissed opportunities to explore incorrect answers and different or sophisticated explanations. They made authoritative statements and decisions and gave directions that were quick, correct, and one-dimensional. This positioning was similar to the student expert position in Barnes' (2004) research. The indicators for the expert position were: "Makes authoritative mathematical statements, and decides what is correct, or is asked for help by others who accept the answers as authoritative" (p. 6).

Constraining instruction in my study was predominantly teacher-directed and, as Davies and Hunt (1994) found in their study, teachers' mathematical contributions and personal mathematical beliefs and values were dominant within the discussions and developing mathematics. In my study mathematical teaching and learning was often constructed as what Ball (2001) defined as "show and tell". Stein et al. (2009) further described show and tell as teachers and students taking turns, and sharing correct answers and solution strategies with limited intervention or elaboration from either. I identified turn-taking as a constraining position whereby teachers' negative responses and repeated questioning appeared to instigate a turn-taking approach to participation amongst students. The intent may have been to ensure "equitable participation" (Chapin & O'Connor, 2007, p. 125) by giving each student an opportunity to express their ideas. However, in reality on hearing no, some students chose to stop participating in the lesson.

Teachers were more prominent within the group because they positioned themselves to do most of the mathematical talk and tasks, decided what was taught and learned, how, and by whom. This positioning limited the ways in which teachers could come to understand how their students approached mathematics learning. A more traditional view of teaching was evident within these teachers' positioning of themselves and their students (Hunter, 2008). This view was one where teaching meant the transmission of instructional strategies, and students appeared "constructed as the passive recipient[s] of someone else's knowledge" (Ewing, 2004, p. 144). As passive recipients of teacher knowledge, students would have struggled to suggest plausible guesses or develop the right attitude toward plausible guessing within their problem solving (Wong et al., 2002). One reason for this positioning could be, as Hunter (2007) identified, that some teachers felt more confident about their teaching when they positioned themselves to explain rules and procedures. Another possible reason for this is that teachers had confidence in their capacity to know the mathematics content but not to teach it. Students positioned this way had fewer opportunities to self-regulate and develop independence and could become hesitant about individually or collectively persevering without the presence of the teacher.

Constraining teacher positioning in my study did not leave room for students to participate in their own or others' mathematics. Teachers did not position themselves to facilitate opportunities for collaboration. Students had few opportunities to grapple with their own or others' mathematical know-how because teachers steered them towards particular solutions and strategies and smoothed that path for them (Chapin & O'Connor, 2007). In my study, requests for repetition did not seem to be a means for ensuring students understood what was being said. Instead, as Chapin et al. (2009) found, repetition was accepted by teachers as confirmation of hearing correctly and being able to echo the correct answer or strategy. Pólya (1958) contended that students should show a vested interest in their problem solving and that teachers should position students to bring something of themselves to their mathematics. This was not true for the teachers in my study who constrained the sharing of mathematical know-how. Students were not positioned to have, or develop, a mathematical opinion and were not required to give anything of themselves to solving the problems.

Hunter (2005) identified that some teachers in her research may have held ambivalent beliefs about the "value of communication and the length of time mathematical decisions took" (p. 453). This may have also been true for the teachers in my study who constrained the sharing of know-how because communication was less encouraged and the pace of lessons was brisk. Students were not expected to, nor given time to, offer explanations, clarify their reasoning, build connections, or self-correct. According to Schoenfeld (1985), the speed with which teaching and learning occurred may have reiterated for some students that mathematics problems could be solved quickly using a predetermined strategy.

Students' errors indicated a lack of understanding which teachers corrected, gaps indicated a lack of knowledge which teachers provided, correct answers were praised, and original know-how was not appropriated (Askew et al., 1997). The focus on correctness and pace of the teaching and learning limited the opportunities students had to share their know-how and the depth at which they could share. Pape and Wang (2003) identified that when students' opportunities for participation were limited they did not come to understand that their successes and failures were attributable to themselves. The danger, as Choppin (2011) identified, is that the fewer opportunities students had to share their mathematical know-how, the fewer opportunities they had to experience reasoning and act purposefully and reflectively with others.

The use of routine problems from the NDP teacher resource books could have negatively influenced the extent to which teachers or students had opportunities to guess or be creative. The suggested problems in the books are routine problems because they are aligned with the strategy being explored and therefore do not require thinking beyond the specified learning intention of the lesson. Predetermined solution methods and problems that are best solved according to that method are unlikely to prompt students to guess or elicit creative thinking (Askew, 2011). Teachers and students did not need to guess as the best solution strategy had already been determined. Both provided explanations of and justifications for their answers and strategies but these were based on sound mathematical inferences, not guesses. There was nothing surprising or unfamiliar about the problem or solution for students to be intuitive or creative with (Nadjafikhah et al., 2012; Sriraman, 2005).

Most students accepted the positioning of teacher as custodian, proclaimer, and protector. On the rare occasions the positioning was challenged, (Section 6.8.6) other students and the teacher referred to the teacher's status to reinforce the positioning. Students who contested the positioning were deemed not to have the authority within the group to challenge (Davis & Hunt, 1994).

8.3 Potential Implications of Teachers' Positionings

Research has shown that if teachers or students limit themselves or are limited to constrained positions, their rights and duties within that position become restricted (Davis & Hunt, 1994; Yamakawa et al., 2005). The longer the teacher or student is constrained by the positioning, the less likely the positioning could be altered or disrupted (Anderson, 2009; Barnes, 2003; Harré & van Langenhove, 1999).

The positions that afforded and constrained the sharing of mathematical know-how within the emerging taxonomy of teacher positioning have been organised in Table 8.3 according to the teacher and their lowest and highest group. The affording positionings are italicised and the constraining positions are underlined.

Table 8.3: Individual teacher and group positioning

Teacher and (Year Level)	Lowest Group	Highest Group
Jenna (NE)	*Appropriator*	*Appropriator & Procurer*
Brooke (NE)	<u>Custodian</u>	*Procurer*
Lisa (1)	*Appropriator & Procurer*	<u>Custodian</u>
Delphi (1/2)	*Procurer*	*Procurer*
Sheridan (2)	*Appropriator*	*Appropriator & Provoker*
Hannah (2/3)	*Procurer*	*Provoker*
Naomi (2/3)	<u>Proclaimer</u>	*Procurer*
Faith (4)	<u>Custodian & Proclaimer</u>	*Appropriator & Procurer*
Chelsea (4/5)	<u>Custodian & Proclaimer</u>	<u>Custodian & Proclaimer</u>
Greer (4/5)	*Provoker*	*Procurer & Provoker*
Kendra (5/6)	*Appropriator*	*Appropriator & Provoker*
Paula (5/6)	*Appropriator* <u>& Protector</u>	*Appropriator* <u>& Protector</u>

Two patterns of teacher positioning within this study have been identified. The first pattern pertains to the six teachers whose positioning afforded the sharing of mathematical know-how with both groups: Jenna, Delphi, Sheridan, Hannah Greer, and Kendra. Table 8.4 shows that these teachers positioned themselves in ways that provided opportunities for students in both groups for active participation, authentic involvement, and reflection (Attard, 2009, 2011). The difference within the pattern of affording positioning was that four of the six teachers, Jenna, Sheridan, Greer, and Kendra positioned themselves in two different ways with their highest group but in only one position with their lowest group. For example, Jenna predominantly positioned herself as appropriator with her lowest group and appropriator and procurer with her highest group. Whilst positions of appropriator, procurer, and provoker were all identified as promoting students' opportunities for engagement, providing those opportunities through the same acts of positioning may limit the ways students in the lowest group could access and communicate their own and others' know-how. It is also relevant to note that the achievement of three of the groups whose know-how was afforded through one prominent act of teacher positioning was considered to be below expectation (MoE, 2009).

Table 8.4: Affording teacher positioning with the lowest and highest group

Teacher and (Year Level)	Lowest Group	Highest Group
Jenna (NE)	*Appropriator*	*Appropriator & Procurer*
Achievement	Below	As expected
Delphi (1/2)	*Procurer*	*Procurer*
Achievement	As expected / Below	As expected
Sheridan (2)	*Appropriator*	*Appropriator & Provoker*
Achievement	Below	As expected
Hannah (2/3)	*Procurer*	*Provoker*
Achievement	As expected	As expected
Greer (4/5)	*Provoker*	*Procurer & Provoker*
Achievement	As expected	As expected
Kendra (5/6)	*Appropriator*	*Appropriator & Provoker*
Achievement	As expected	As expected

The second pattern relates to the four teachers whose positioning did not afford the sharing of know-how with both groups, Brooke, Lisa, Naomi, and Faith, and Paula who positioned herself as appropriator with all but one student in both groups. As Table 8.5 outlines, Paula, Naomi, Faith, and Brooke afforded opportunities for mathematical engagement with their highest group and Lisa with her lowest group. With their lowest group, Naomi, Faith, and Brooke positioned themselves as proclaimer and custodian. The rights they assumed from these positions were to determine the know-how that was shared when, and by whom and to share, explain, and correct for students. Students in the lowest group with these three teachers did not have the same opportunities as those in the highest group to engage with their own and peers' know-how. They did participate in their teacher's know-how but access was narrow and restrictive. As custodian, Lisa restricted access to the mathematics discussions to those who behaved appropriately. Paula positioned one student in each group differently. Most students in both groups had a duty to notice, monitor, explain, and review their own and others' thinking. In both groups, the duty of one student was to listen to, and follow, Paula's thinking. The intent may have been to shield the two students from social or mathematical embarrassment. However, this position may have also resulted in their being marginalised because they were not able to share their thinking with peers.

Table 8.5: Constraining positions with the lowest or highest group

Teacher and (Year Level)	Lowest Group	Highest Group
Brooke (NE)	<u>Custodian</u>	*Procurer*
Achievement	Below	*As expected*
Lisa (1)	*Appropriator & Procurer*	<u>Custodian</u>
Achievement	*As expected*	*As expected*
Naomi (2/3)	<u>Proclaimer</u>	*Procurer*
Achievement	Below	*As expected*
Faith (4)	<u>Custodian & Proclaimer</u>	*Appropriator & Procurer*
Achievement	*As expected*	*As expected*
Paula (5/6)	*Appropriator* <u>& Protector</u>	*Appropriator* <u>& Protector</u>
Achievement	*As expected*	*As expected*

The positionings of Chelsea did not fit with either pattern as her positioning constrained the sharing of mathematical know-how with both groups. Table 8.6 illustrates how Chelsea positioned herself as custodian and proclaimer with both groups. She determined the know-how that was shared when, and by whom, and positioned herself to direct students to use specific strategies, to explain for students, and to correct their errors and misconceptions. Students in both groups were marginalised from mathematical engagement because of their corresponding imitative duties. By positioning themselves as the dominant participant in the mathematical discussions, these teachers, according to Attard (2011) and Boaler (2011), were limiting their opportunities to connect in mathematically meaningful ways with their students.

It could be argued that 10 of the 12 teachers in this study positioned themselves to teach 10 of the 24 groups of students and two individuals in qualitatively diminished ways. Chelsea's positioning as custodian and proclaimer constrained the sharing of mathematical know-how with both groups. Brooke, Naomi, and Faith positioned themselves in ways that constrained the sharing of mathematical know-how with their lowest group. Evidence shows that the students in these groups received more procedural and simplified instruction from an authoritative teacher (Bartholomew, 2003; Boaler et al., 2000). The interactions occurred mainly between the teacher and an individual student and the goal appeared to be to follow specific strategies and determine correct answers. Paula's positioning constrained the sharing of know-how with one student in each group. The intent appeared to be to protect the boys from mathematical challenges and potential teasing from other students (Nadjafikhah et al., 2012). The interactions between Paula and the two boys were hidden from their peers and the boys may have been left feeling that their mathematical contributions were less valued. Jenna, Sheridan, Greer, and Kendra afforded the sharing of know-how with both groups but through fewer acts of positioning with their lowest group. The teachers of these four groups of students provided fewer opportunities for participation because they positioned themselves one way. By positioning themselves differently with the highest group these four teachers were increasing the opportunities students had to contribute and the ways they could contribute. In a finding contradictory to that of Bartholomew, (2003) and Boaler and colleagues (2000), Lisa's positioning constrained the sharing of mathematical know-how with her highest group. Good behaviour was emphasised over mathematical ideas and Lisa appeared to guard the mathematics for those who met her behavioural requirements.

The positioning practices of 10 teachers in my study may have marginalised the opportunities 10 groups of students and two individuals had to share their own and participate in others' mathematical know-how. Students in this study whose mathematical opportunities were negatively impacted upon by their teachers' positioning may have had their potential for mathematical success marginalised (Davies & Hunt, 1994).

Attard (2013) contended that it is adversative to any community for the dominant practices of one to preclude the engagement and meaning making of others. Naomi, Faith, and Brooke dominated the learning with their lowest group by completing most of the mathematical talk and tasks. This was also true for Paula with one student in each group. Lisa dominated the learning with her highest group by making good behaviour a prerequisite for participation. The single affording positioning of Jenna, Sheridan, Greer, and Kendra with their lowest group may have precluded opportunities for engagement and meaning making for some. Chelsea constrained the sharing of mathematical know-how for both groups of students. The dominant positioning of teachers and qualitatively different positioning of students in this study was not likely to change patterns of underachievement.

8.4 Limitations of the study

My study has contributed new knowledge to understanding the discipline and teaching and learning of mathematics. However, any research has limitations and I acknowledge that there are limitations to this study that may have influenced the findings of the 12 teachers' acts of positioning within their lowest and highest groups. The limitations to the study I have identified include the situatedness of this study within the NZ NDP, researcher bias, data analysis, participant characteristics, theoretical frame choice, data collection sample and processes, and the exclusion of some data. Each limitation will be considered and then recommendations for how each limitation could be overcome in future research follows.

Situating this study within the NDP mathematics programme and numeracy strand may have predetermined the mathematical pedagogies teachers selected and simultaneously predetermined the positionings they would take and give. The NDP could be considered a more structuralist approach to teaching and learning mathematics and as such teachers could have promoted the "direct instruction of explicit mathematical representations and procedures" (Murphy, 2013, p. 108). The structuralist nature of the suggested model lessons and routine problems in the NDP teaching materials may have unintentionally positioned some teachers to direct students towards certain strategies and teach in more traditional ways. When teachers' positionings constrained the sharing of mathematical know-how the goal appeared to be to push students toward the recommended strategy and correct answer (Conner, et al., 2014). An adherence to the NDP teaching materials may have substantiated or exacerbated that goal. Researcher bias, the second limitation of this study could include the gender related, ethnical, ethical, or cultural beliefs of the researcher or the preconceived motivations, interests, assumptions or perspectives they hold (Flyvbjerg, 2011; Lincoln & Guba, 1985). I acknowledge that the risk of researcher bias may have been increased because of the existing relationship I had with both schools as their NDP advisor. Within this study, I have identified potential biases at each stage of the research, addressed the possibility of negative or positive influence, and actively sought to reduce the risk of researcher bias (Creswell, 2003). I have triangulated and presented evidence from data to support my findings, included contradictory findings, and engaged member checking and peer debriefing (Cohen et al., 2007). It is also important to note that whilst I had an existing relationship with the 12 teachers, positioning was a new construct to me and I had not viewed their teaching through the positioning lens that I applied when analysing the data.

The potential for someone else to analyse and interpret the research data differently to myself is a possible limitation of this study. Teaching and learning mathematics is a complex matter and a different researcher may have selected different excerpts from the 72 lesson transcripts to illustrate the teachers' affording and constraining positioning. Similarly, a different theoretical framework such as discourse analysis may have yielded different insights into teacher positioning. For example, some may contest that in Sections 6.2.1 and 6.2.2 Greer positioned one strategy as more efficient than another and positioned herself as the holder of knowledge because she, and not the students, modelled with the concrete materials. My analysis of the selected excerpts claimed that Greer provided the platform for students to share their repeated strategies, notice connections, discuss the efficiency of different strategies, and align their written recording with Greer's model. I am not claiming that my interpretation of this data is the truth. Rather, this is the story I have constructed from the data (Denzin & Lincoln, 2011). The processes I applied to reduce the risk of researcher bias also contributed to increasing the trustworthiness of the research findings. This included a focus on the reliability, credibility, transferability, dependability, and confirmability of the qualitative research methods (Lincoln & Guba, 1985). Ultimately, any final determinations about this research are reliant on the information I have provided and the reader's interpretation of that information (Merriam, 2009; Stake, 2012).

The data collection sample and processes offer a third possible limitation to this study. This study was conducted in two primary schools situated in urban suburbs of one New Zealand city. Including, rural areas, a wider geographical area, diverse school decile ratings, and intermediate schools may have resulted in different findings. The teachers at both schools were invited to volunteer to participate. There are representational limitations associated with research participant volunteers. For example, participants who volunteer are self-selected and their motivations may be clouded because of a vested interest in the outcome of the study. Volunteer participants may not be typical of the general population and may prejudice or exaggerate the outcome of the study (Denzin & Lincoln, 2011). Each teacher was recorded and observed teaching their lowest and highest mathematics groups over three consecutive NDP based lessons. Including the middle mathematics group, observing lessons over a longer period in mathematics programmes other than the NDP or mathematics strands other than numeracy may have resulted in other affording and constraining teacher positions being identified.

Additional data could have provided another lens to analyse and cross-check findings. Interviews with the teachers and/or students may have provided further insights into the relationship between teachers' acts of positioning and students' achievement. Teacher beliefs were not intended as a focus for this study. However, through-out the study it became more apparent that teachers' beliefs about how mathematics should be taught and learned would underpin and influence their positioning choices. In a future study teachers could have been interviewed about their beliefs regarding the nature of mathematics or their goals for effective teaching and learning of mathematics. Students could have been interviewed in regards to their response to their positioning in their ability group and their beliefs about effective teaching and learning of mathematics. Aggregated student achievement data and students work samples may have provided a stronger link between the affording and constraining acts of teacher positioning and students' mathematical achievement. More in-depth analysis of the contexts of individual student participants within the groups such as gender or ethnicity may have added an additional lens through which to consider the relationship between teacher positioning and the achievement of priority learners.

I recognise there are limitations to my study. I also recognise that the information contained in this study reveals a picture of teacher affording and constraining positioning in mathematics teaching that will be available for comparisons with any subsequent studies relating to mathematics teaching. In identifying the limitations of this study I have also identified recommendations for future research, these are discussed in the following section.

8.5 Recommendations for future research

The examples of qualitatively different teacher positionings and their influence on students' opportunities for learning and the limitations of this study discussed in Section 8.4 indicate four recommendations for future research. The first recommendation is that a future study needs to be extended to mathematics programmes less structured by the NDP suggested model lessons and routine problems. The NDP underpins the teaching and learning of numeracy in approximately 95% of New Zealand's primary schools (Higgins & Parsons, 2009). A further study could analyse the teaching and learning of numeracy through the emerging taxonomy of teacher positioning in New Zealand schools where all teachers were less directed by the NDP teaching materials. Another option would be to analyse teacher positioning in different mathematics strands (for example geometry and measurement or statistics) where the teaching and teaching materials may be less structured. A third option would be to review and critique the emerging taxonomy of teacher positioning with mathematics teachers from countries other than New Zealand. Each option could reveal different examples of teacher positioning, particularly with non-routine problems, and the emerging taxonomy of positioning could be further critiqued and extended. The risk of researcher bias could be ameliorated if the researcher did not have an existing relationship with the teacher participants in future research.

The second recommendation would be to more explicitly research the influence and effect of teacher positioning on student achievement. This could be achieved by including all students in the mathematics class in the study and examples of ability and mixed ability groupings, extending the length of the study to a school year, tracking students beginning and end of year achievement data, and analysing their mathematics teachers' positioning through the emerging taxonomy. Contextual factors such as students' age, gender, or ethnicity and teachers' beliefs or professional development experiences could be considered to strengthen the link between the acts of positioning and students' achievement data.

The future use of the emerging taxonomy of teacher positioning is the third recommendation for future research. The taxonomy could provide a means for prompting teachers to reflect on their positioning or to review perceived and actual positionings. For example, as part of practitioner research, teachers could video their teaching then explore their positioning according to the emerging taxonomy. They could also consider the storylines and social acts their positionings develop and the effects these have on teaching and learning. In a future study I would like the analysis of teacher positionings to be more collaborative perhaps between myself and the teacher participant or between teacher participants as critical friends. Professional development programmes could incorporate research on teachers' positionings and the impact of those positionings impact on students' achievement. It would also be of interest to consider the impact of including routine problems in nationally distributed materials such as the numeracy professional development teacher resources.

The fourth recommendation relates to how teachers establish the prerequisite conditions of respectful and collaborative partnerships evidenced within the acts of affording teacher positioning. It is important to understand the pedagogical positions teachers selected that afforded the sharing of know-how and the reasons behind their selections. An increased understanding of the affording teacher positionings could assist all teachers to further define and explore effective teaching practices with priority learners.

8.6 Personal reflective comment

My experiences as a mathematics learner, teacher, facilitator and lecturer were strong motivators for this research. I believe I was predominantly taught and positioned in ways similar to those enacted by the teacher as custodian, proclaimer, and protector in my study. I was constructed as the kind of person who failed (Anderson, 2009) and continued to believe that positioning into my adult years. I also believe that this positioning was enacted by teachers in my personal experience and in this study with the best of intentions.

However, I am still left wondering, how can students become more comfortable and confident with learning mathematics if they do not get to do the mathematics and be mathematical? The parallel question pertaining to teachers would be, how can teachers become more confident in their teaching of mathematics if they do not get to experience students doing mathematics and being mathematical? The teacher as appropriator, procurer, and provoker positioned themselves and their students in ways that supported students to learn from the discipline of mathematics and from each other. From these positions teachers were both an authority and in authority (Herbel-Eisenmann, 2013). They had mathematical knowledge, knowledge of social constructivist pedagogies, and knowledge of how to harmonise both in ways that increased students' opportunities to also be an authority and in authority. It is hoped that this study contributes further to the shared understandings of how we can enhance the mathematical teaching and learning of all.

References

Aljughaiman, A., & Mowrer-Reynolds. E. (2005). Teachers' conceptions of creativity and creative students. *Journal of Creative Behaviour, 39*, 17-34.

Alton-Lee, A. (2003). *Quality teaching for diverse students in schooling: Best evidence synthesis.* Wellington, New Zealand: Ministry of Education.

Anderson, J. (2003). Teachers' choice of tasks: A window into beliefs about the role of problem solving in learning mathematics. In L. Bragg, C. Campbell, G. Herbert, & J. Mousley (Eds.), *Mathematics education research: Innovation, networking, opportunity*. *Proceedings of the 26th annual conference of the Mathematics Education Research Group of Australasia* (pp. 72-79). Sydney, Australia: MERGA.

Anderson, K. T. (2009). Applying positioning theory to the analysis of classroom interactions: Mediating micro-identities, macro-kinds, and ideologies of knowing. *Linguistics and Education 20*, 291-310.

Angrosino, M. V., & Mays de Perez, K. A. (2003). Rethinking observation from method to context. In N. K. Denzin & Y. S. Lincoln (Eds.), *Collecting and interpreting qualitative materials* (pp. 107-154). Thousand Oaks: Sage.

Anthony, G., & Hunter, R. (2005). A window into mathematics classrooms: Traditional to reform. *New Zealand Journal of Educational Studies, 40*(1), 25-43.

Anthony, G., & Walshaw, M. (2006). *Numeracy practices and change*. (Teaching and Learning Research Initiative). Wellington, New Zealand: Ministry of Education.

Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in mathematics/pāngarau: Best evidence synthesis iteration [BES]*. Wellington, New Zealand: Ministry of Education.

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics: Educational practices series, 19.* International Academy of Education, International Bureau of Education & UNESCO.

Askew, M. (1999). It ain't (just) what you do: Effective teachers of numeracy. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools* (pp. 91-102). Buckingham: Open University Press.

Askew, M. (2002). *Making connections: Effective teaching of numeracy*. London: BEAM Education.

Askew, M. (2011). Unscripted maths: Emergence and improvisation. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.). *Mathematics: Traditions and [new] practices*. *Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers* (pp. 56-67). Alice Springs: Australia, MERGA.

Askew, M., Brown, M., Rhodes, V., Wiliam, D., & Johnson, D. (1997). *Effective teachers of numeracy in primary schools*. London: King's College, University of London.

Attard, C. (2009). *Student perspectives of mathematics teaching and learning in the upper primary classroom*. Paper presented at the Third International Conference on Science and Mathematics Education (CoSMED 2009), Penang, Malaysia.

Attard, C. (2011). The influence of teachers on student engagement with mathematics during the middle years. In J. Clark, B. Kissane, J. Mousley, T. *Spencer, & S. Thornton (Eds.). Mathematics: Traditions and [new] practices. Proceedings of the 34th annual conference of the Mathematics Education Research Group of Australasia and the 23rd biennial conference of the Australian Association of Mathematics Teachers* (pp. 68-74). Alice Springs, Australia: MERGA.

Attard, C. (2013). "If I had to pick any subject, it wouldn't be maths": Foundations for engagement with mathematics during the middle years. *Mathematics Education Research Journal, 25*(4), 569-587.

Australian Association of Mathematics Teachers. (1998). *Policy on numeracy education in schools*. Australia: Author.
Babbie, E. (2007). *The practice of social research* (11th ed.). Belmont CA: Thomson.
Bakhtin, M. M. (1984). *Problems of Dostoevsky's poetics*. (Trans and Ed, C. Emerson) Minneapolis: University of Minnesota Press.
Bakhtin, M. M. (1981). *The dialogic imagination: Four essays by M. M. Bakhtin*, M. Holquist (Ed.). Austin, TX: University of Texas Press.
Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal, 93*, 373-397.
Ball, D. L. (2001). Teaching, with respect to mathematics and students. In T. Wood, B.S. Nelson, & J. Warfield (Eds.), *Beyond classical pedagogy: Teaching elementary school mathematics* (pp. 11-22). Mahwah, NJ: Erlbaum.
Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 27-44). Reston, VA: National Council of Teachers of Mathematics.
Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, Fall*, 14-22.
Banfield, T. (2005). Ability grouping for mathematically gifted adolescent boys. *International Education Journal, 6*(2), 141-149.
Barbeau, E., & Taylor, P. (2005). *Challenging mathematics in and beyond the classroom.* (Discussion document of the ICMI Study 16). Retrieved from http://www.amt.edu.au/icmis16.html
Barnes, M. (2003). *Patterns of participation in small-group collaborative work*. Paper presented as part of the symposium Patterns of participation in the classroom at the Annual Meeting of the American Educational Research Association, Chicago.
Barnes, M. (2004). *The use of positioning theory in studying student participation in collaborative learning activities*. Paper presented at the Annual Meeting of the Australian Association for Research in Education. Melbourne, Australia.
Barone, D. (2000). Revisioning: Positioning of a parent, student, and researcher in response to classroom context. *Reading Research and Instruction, 40*(2), 101-119.
Bartholomew, H. (2003). Ability grouping and the construction of different types of learner in mathematics classrooms. In L. Bragg, C. Campbell, G. Herbert, & J. Mousley (Eds.), *Mathematics education research: Innovation, networking, opportunity* (Proceedings of the 26th annual conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 128-135). Sydney: MERGA.
Barton, B. (2009). Being mathematical, holding mathematics: Further steps in mathematical knowledge for teaching. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides. Proceedings of the 32nd Annual Conference of the Mathematics Education Research Group of Australasia* (Vol. 1. pp. 4-10). Palmerston North, NZ: MERGA.
Benson, C. (2003). The unthinkable boundaries of self: The role of negative emotional boundaries in the formation, maintenance, and transformation of identities. In R. Harré, & F. Moghaddam (Eds.), *The self and others: Positioning individuals and groups in personal, political, and cultural contexts* (pp. 61-84). Westport: Praeger.
Billig, M. (2009). Discursive psychology, rhetoric, and the issue of agency. *Critical Discourse Analysis, 27*. Retrieved from http://semen.revues.org/8930
Bisel, R. S., & Barge, J. K. (2011). Discursive positioning and planned change in organisations. *Human Relations, 64,* 257-283.
Black, P. J., & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education, 5*, 7–73.
Blatchford, P., Hallam, S., Kutnick, P., & Creech, A. (2008) Classes, groups and transitions: structures for teaching and learning. *Primary review research survey 9*(2). University of Cambridge: Cambridge.

Boaler, J. (1997). Setting, Social Class and Survival of the Quickest, *British Educational Research Journal, 23*(5), pp. 575-595.
Boaler, J. (2002). The development of disciplinary relationships: Knowledge, practice, and identity in mathematics classrooms. In A. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th conference of the International Group for the Psychology of Mathematics Education* (Vol 2, pp. 113-120). Norwich: PME.
Boaler, J. (2003). Exploring the nature of mathematical activity: Using theory, research and 'working hypotheses' to broaden conceptions of mathematics knowing. *Educational Studies in Mathematics, 51*(1/2), 3-21.
Boaler, J. (2005) The 'Psychological Prison' from which they never escaped: the role of ability grouping in reproducing social class inequalities. *FORUM 47*,(2&3), 135–144.
Boaler, J. (2009). *The elephant in the classroom: Helping children learn and love maths*. London: Souvenir Press.
Boaler, J. (2014). Ability Grouping in Mathematics Classrooms. *Encyclopaedia of Mathematics Education*, 1-5.
Boaler, J., William, D., & Brown, M. (2000). Students' experiences in ability grouping: Disaffection, polarisation and the construction of failure? *British Educational Research Journal, 26*(5), 631-648.
Bobis, J. (2003). *Count me in too: Evaluation of Stage 2*. Sydney: NSW Department of Education and Training. Retrieved from http//www.currriculumsupport.nsw.edu.au/maths
Bobis, J., Mulligan, J. T., & Lowrie, T. (2004). *Mathematics for children: Challenging children to think mathematically* (2nd ed.). Frenchs Forest, NSW: Pearson Education Australia.
Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, B., & Young-Loveridge, J. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal, 16*(3), 27–57.
Boekaerts, M. (2002). Bringing about change in the classroom: Strengths and weaknesses of the self-regulated learning approach: EARLI Presidential address, 2001. *Learning and Instruction, 12*(6), 589-604.
Boerst, T., Sleep, L., Ball, D. L., & Bass, H. (2011). Preparing teachers to lead mathematics discussion. *Teachers College Record*, *113*(12), 2844-2877.
Bolden, D. S., Harries, T. V., & Newton, D. P. (2009). Pre-service primary teachers' conceptions of creativity in mathematics. *Educational Studies in Mathematics,* 73(2), 143-157.
Bowers, J., Cobb, P., & McClain, K. (1999). The evolution of mathematical practices: A case study. *Cognition and Instruction, 17,* 25–64.
Boxer, L. (2003). Using positioning theory to assess quality systems. In R. Harré, & F. Moghaddam (Eds.), *The self and others: Positioning individuals and groups in personal, political, and cultural contexts* (pp. 251-278). Westport: Praeger.
Brinkmann, S. (2010). Guilt in a fluid culture? A view from positioning theory. *Culture Psychology, 16*(2), 253-266.
Brophy, J. (2006). Graham Nuthall and social constructivist teaching: Research-based cautions and qualifications. *Teaching and Teacher Education, 22*, 529-537.
Brophy, J. (Ed.). (2002). *Social constructivist teaching: Affordances and constraints.* Oxford, UK: Elsevier Science.
Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning* (2nd ed.). Portsmouth, NH: Heinemann.
Chamberlain, M., & Caygill, R. (2012). *Key findings from New Zealand's participation in the Progress in International Reading Literacy Study (PIRLS) and Trends in International Mathematics and Science Study (TIMSS) in 2010/2011*. Wellington, New Zealand: Ministry of Education.
Chapin, S. H., & O'Connor, C. (2007). Academically productive talk: Supporting students' learning in mathematics. In W. G. Martin & M. E. Strutchens (Eds.),

The learning of mathematics (pp. 113-128). Reston, VA: The National Council of Teachers of Mathematics.
Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn* (2nd ed.). Sausalito, CA: Math Solutions.
Choppin, J. (2011). The impact of professional noticing on teachers' adaptations of challenging tasks. *Mathematical Thinking and Learning, 13*(3), 175-197.
Choppin, J., & Herbel-Eisenmann, B. (2012). Situating revoicing within broader task and social structures. *Paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Radisson Hotel, Kalamazoo.* Retrieved from http://citation.allacademic.com/meta/p571709_index.html
Cirillo, M. (2013a). What are some strategies for facilitating productive classroom discussions? *Research Brief 20*, National Council of Teachers of Mathematics.
Cirillo, M. (2013b). What does research say the benefits of discussion in mathematics class are? *Research Brief, 19*, National Council of Teachers of Mathematics.
Civil, M. (2002). Everyday mathematics, mathematicians' mathematics, and school mathematics: Can we bring them together? In M. Brenner & J. Moschkovich (Eds.), Everyday and academic mathematics in the classroom. *Journal of Research in Mathematics Education Monograph #11* (pp. 40-62). Reston, VA: NCTM.
Clarke, D. (2003). *Practice, role and position: Whole class patterns of participation*. Paper presented as part of the symposium Patterns of participation in the classroom at the Annual Meeting of the American Educational Research Association, Chicago.
Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23*(7), 13-20.
Cobb, P. (1995). Cultural tools and mathematical learning: A case study. *Journal for Research in Mathematics Education, 26*, 362-385.
Cobb, P. (2000). Accounting for mathematical development in the social context of the classroom. In L. P. Steffe (Ed.), *Radical constructivism in action: Beyond the pioneering work of Ernst von Glasersfeld*. London: Routledge Falmer.
Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 3–38). Charlotte, NC: Information Age.
Cobb, P., & Hodge, L. L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical Thinking and Learning, 4*(2/3), 249-284.
Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics, 28*(3), 258-277.
Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29*(3), 573-604.
Cobb, P., Yackel, E., & Wood, T. (1989). Young children's emotional acts while doing mathematical problem solving. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 117-148). New York: Springer-Verlag.
Cobb, S. C. (2012). *"You use your imagination:" An investigation into how students use 'imaging' during numeracy activities*. (Unpublished Master's Thesis). University of Canterbury, Christchurch.
Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education* (5th ed.). London: Routledge Falmer.
Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education* (6th ed.). London: Routledge.
Conner, A., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how

teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics*, *86*(3), 401–429.
Corbin, J., & Strauss, A. C. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Los Angeles, CA: Sage.
Creswell, J. W. (2003). *Research design. Qualitative, quantitative and mixed methods approaches.* Thousand Oaks, CA: Sage.
Crooks, T., Smith, J., & Flockton, L. (2010). *Mathematics: Assessment results 2009.* Dunedin: University of Otago.
Crotty, M. (1998). *The foundations of social research: Meaning and perspective in the research process*. Crows Nest, NSW: Allen & Unwin.
Darr, C., & Fisher, J. (2005). Self-regulated learning in mathematics classes. *Set: Research information for teachers 2*, 44-49.
Davies, B. (1982). *Life in the classroom and playground. The accounts of primary school children*. London: Routledge & Kegan Paul.
Davies, B., & Harré, R. (1990). Positioning: The discursive production of selves. *Journal for the Theory of Social Behaviour, 20*(1), 43–63.
Davies, B., & Harré, R. (1999). Positioning and personhood. In R. Harré, & L. van Langenhove (Eds.), *Positioning theory: Moral contexts of intentional action* (pp. 32–52). Oxford: Blackwell.
Davies, B., & Hunt, R. (1994). Classroom competencies and marginal positionings. *British Journal of Sociology of Education, 15*(3), 389-408.
Davis, G. (1999). *Creativity is forever* (4th ed.). Dubuque, IA: Kendall/Hunt.
De Corte, E., Vershaffel, L., & Op' T Eynde, P. (2000). Self-regulation: A characteristic and a goal of mathematics education. In M. Boekaerts, P. Pintrich, & M. Zeidner (Eds.), *Handbook of self-regulation*. San Diego: Academic Press.
de Vaus, D. (2001). *Research design in social research*. London: Sage.
Denscombe, M. (2010). *The good research guide for small-scale social research projects* (4th ed.). England: Open University Press.
Denzin, N. K., & Lincoln, Y. S. (Eds.). (2005). *The sage handbook of qualitative research* (3rd ed). Thousand Oaks: Sage.
Denzin, N. K., & Lincoln, Y. S. (Eds.). (2011). *The sage handbook of qualitative research* (4th ed). Thousand Oaks: Sage.
Department of Education and Training, New South Wales. (1998). *Count Me In Too: Learning framework in number.* New South Wales, Australia: Author.
Edwards, D. (1997). *Discourse and cognition*. London: Sage.
Edwards, D., & Potter, J. (1992). *Discursive psychology*. London: Sage.
Ell, F. (2001). *Mathematics in the New Zealand curriculum: A concept map of the curriculum document*. (Technical Report 11, Project asTTle). Auckland: University of Auckland.
Ernest, P. (1994). The dialogical nature of mathematics. In P. Ernest (Ed.), *Mathematics, education and philosophy: An international perspective* (pp. 33-48). London: Falmer.
Ernest, P. (1998). *Social constructivism as a philosophy of mathematics*. New York: State University of New York Press.
Ernest, P. E. (1996). Varieties of constructivism: A framework for comparison. In L. P. Steffe, & P. Nesher (Eds.), *Theories of mathematical learning* (pp. 335-349). Mahwah, NJ: Lawrence Erlbaum.
Ervynck, G. (1991). Mathematical creativity. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 42-53). Dordrecht: Kluwer.
Evans, K. S. (1996). Creating spaces for equity? The role of positioning in peer-led literature discussions. *Language Arts, 73*(3), 194-202.
Even, R., & Tirosh, D. (2008). Teacher knowledge and understanding of students' mathematical learning and thinking. In L. D. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 202-222). New York: Routledge.
Ewing, B. (2004) Teacher communication, student identity and classroom participation, in E. McWilliam, S. Danby, & J. Knight (Eds), *Performing educational research:*

Theories, methods and practices, (pp. 137-150). Flaxton, Queensland: Post Pressed.
Ewing, B. (2007). Participation and non-participation in mathematics classrooms. *Proceedings from Ninth International Conference: The Mathematics Education into the 21st Century Project*. Charlotte, NC: UNCC.
Ewing, B. (2011). Direct instruction in mathematics: Issues for schools with high indigenous enrolments: A literature review. *Australian Journal of Teacher Education, 36*(5), 63-91.
Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Groews (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.
Fennema, E., Franke, M. L., Carpenter, T. P., & Carey, D. A. (1993). Using children's mathematical knowledge in instruction. *American Educational Research Journal, 30*(3), 403-434.
Flyvbjerg, B. (2006). Five misunderstandings about case-study research. *Qualitative Inquiry, 12*(2), 219-245.
Flyvbjerg, B. (2011). Case study. In N. K. Denzin & Y. S. Lincoln (Eds.), *The Sage handbook of qualitative research* (4th ed.). Thousand Oaks: Sage.
Fraivillig, J., Murphy, L., & Fuson, K. (1999). Advancing children's mathematical thinking in everyday mathematics classrooms. *Journal for Research in Mathematics Education, 30*(2), 148-170.
Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). Charlotte, NC: Information Age.
Fried, M. N., & Amit, M. (2004). Time and flow as parameters in international comparisons: A view from an eighth grade algebra lesson. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of PME 28*, 2 (pp. 33-40). Bergen, Norway: Bergen University College.
Garden, R. (1996). *Mathematics performance of New Zealand form 2 and form 3 students. National results from New Zealand's participation in the Third International Mathematics and Science Study.* Wellington, New Zealand: Ministry of Education.
Garden, R. (1997). *Mathematics and science in middle primary school: Results from New Zealand's participation in the Third International Mathematics and Science Study.* Wellington: Research and International Section, Ministry of Education.
Gerring, J. (2004). What is a case study and what is it good for? *American Political Science Review, 98*(2), 341-354.
Gillham, B. (2000). *Case study research methods*. New York: Continuum.
Ginsburg, H. (1996). Toby's math. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 175–202). Mahwah, NJ: Lawrence Erlbaum.
Goffman, E (1974). *Frame analysis*. New York: Harper & Row.
Goffman, E. (1959). *The presentation of self in everyday life*. Garden City, NY: Doubleday.
Goffman, E. (1981). *Forms of talk*. Philadelphia: University of Pennsylvania Press.
Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258-291.
Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 105-117). Thousand Oaks, CA: Sage.
Guba, E. G., & Lincoln, Y. S. (2005). Paradigms and perspectives in contention. In N. K. Denzin & Y. S. Lincoln (Eds.), Handbook of qualitative research (3rd ed., pp. 183-190). Thousand Oaks, CA: Sage.
Guerrero, S. (2010). The value of guess & check. *Mathematics Teaching in the Middle School*, *15*(7), 392-398.

Habibis, D. (2006). Ethics and social research. In M. Walter (Ed.), *Social research methods: An Australian perspective* (pp. 53-82). Melbourne, VA: Oxford University Press.
Hall, R. (2000). Video recording as theory. In D. Lesh & A. Kelley (Eds.), *Handbook of research design in mathematics and science education* (pp. 647-664). Mahweh, NJ: Lawrence Erlbaum.
Hansen, E. (1998). Creating teachable moments … and making them last. *Innovative Higher Education, 23*(1), 7-26.
Harré, R. (1979). *Social being*. Oxford: Blackwell.
Harré, R. (1989). Language and science of psychology. *Journal of Social Behaviour and Personality, 4*, 165-188.
Harré, R. (1991). The discursive production of selves. *Theory Psychology, 1*, 51-63.
Harré, R. (1997). Forward to Aristotle: A case for a hybrid ontology. *Journal for the Theory of Social Behaviour, 27*(2/3), 173-191.
Harré, R. (1998a). *The singular self. An introduction to the psychology of personhood*. London, Sage.
Harré, R. (1998b). The advent of cultural psychology: Review essay. *Culture & Psychology, 4*(2), 235-243.
Harré, R. (2004). *Positioning theory*. Retrieved from http://www. massey.ac.nz/~alock/virtual.
Harré, R., & Moghaddam, F. M. (2003). Introduction: The self and others in traditional psychology and in positioning theory. In R. Harré, & F.M. Moghaddam (Eds.), *The self and others: Positioning individuals and groups in personal, political, and cultural contexts* (pp. 65-78). Westport: Praeger.
Harré, R., & Secord, P.F. (1972). *The explanation of social behaviour*. Oxford: Blackwell.
Harré, R., & Slocum, N. (2003). Disputes as complex social events: On the uses of positioning theory. In R. Harré, & F. Moghaddam (Eds.), *The self and others: Positioning individuals and groups in personal, political, and cultural contexts* (pp. 123-136). Westport: Praeger.
Harré, R., & van Langenhove, L. (1991). Varieties of positioning. *Journal for the Theory of Social Behaviour, 21*(4), 393-407.
Harré, R., & van Langenhove, L. (1999). The dynamics of social episodes. In R. Harré & L. van Langenhove (Eds.), *Positioning theory: Moral contexts of intentional action* (pp. 1-14). Oxford: Blackwell.
Haylock, D. W. (1987): A framework for assessing mathematical creativity in school children. *Educational Studies in Mathematics, 18*(1), 59-74.
Healy, L., & Hoyles, C. (1998). *Justifying and promoting in school mathematics* (Technical Report). London: Institute of Education, University of London.
Herbel-Eisenmann, B., & Wagner, D. (2010). Appraising lexical bundles in mathematics classroom discourse: Obligation and choice. *Educational Studies in Mathematics, 75*, 43-63.
Herbert, S., & Pierce, R. (2007). Video evidence: What gestures tell us about students' understanding of rate of change. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice. Proceedings of the Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 162–371). Hobart, Australia: MERGA.
Hiebert, J., Gallimore, R., Garnier, H., Bogard, K., Hollingsworth, H., Jacobs, J., et al. (2003). Understanding and improving mathematics teaching: Highlights from the TIMMS 1999 Video Study. *The Phi Delta Kappan, 84*(10), 768-775.
Hiebert, J., Gallimore, R., Garnier, H., Givven, K. B., Hollingsworth, H., Jacobs, J., et al. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study* (NCES 2003–013 Revised). Washington, DC: U.S. Department of Education, National Center for Education Statistics.
Higgins, J. (2002). *Exploring Issues in Mathematics Education: An Evaluation of the Advanced Numeracy Project 2001*. Wellington, New Zealand: Ministry of Education.

Higgins, J. (2003). The Numeracy Development Project: Policy to practice. *New Zealand Annual Review of Education, 12,* 157-175.

Higgins, J. (2005). Equipment-in-use in the New Zealand Numeracy Development Project: Its importance to the introduction of mathematical ideas. In J. Higgins, K. C. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge, *Findings from the New Zealand Numeracy Development Projects: 2004* (pp.89-96). Wellington, New Zealand: Ministry of Education.

Higgins, J., Irwin, K., Thomas, G., Trinick, T., & Young-Loveridge, J. (2005). *Findings from the New Zealand Numeracy Development Project 2004.* Wellington, New Zealand: Ministry of Education.

Higgins, J., & Parsons, R. (2009). A successful professional development model in mathematics: A system-wide New Zealand case. *Journal of Teacher Education, 60*(3), 231-242.

Higgins, J., & Parsons, R. (2011). Improving outcomes in mathematics in New Zealand: A dynamic approach to the policy process. *International Journal of Science and Mathematics Education, 9*(2), 503-522.

Higgins, J., Parsons, R., & Hyland, M. (2003). The Numeracy Development Project: Policy to practice. In J. Livingstone (Ed.), *New Zealand annual review of education* (pp. 157–174). Wellington: Victoria University of Wellington.

Higgins, J., Sherley, B.M., & Tait-McCutcheon, S.L. (2007). Leading a curriculum reform from inside a school. *Findings from the New Zealand Numeracy Development Projects 2006*. Wellington: Learning Media.

Higgins, J., & Tait–McCutcheon, S. L. (2006). Contextually responsive facilitation. In F. Ell, J. Higgins, K. C. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge, *Findings from the New Zealand Numeracy Development Projects 2005*. Wellington, New Zealand: Learning Media.

Hitchcock, G., & Hughes, D. (1989). *Research and the teacher: A qualitative introduction to school-based research*. Routledge: London.

Hollway, W. (1984). Gender difference and the production of subjectivity. In J. Henriques, W. Hollway, C. Urwin, L. Venn, & V. Walkerdine (Eds.), *Changing the subject: Psychology, social regulation and subjectivity*. London: Methuen.

Holmes, M., & Tait-McCutcheon, S. (2009), Success for underachievers: How do they get it? In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides. Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia*. Wellington, NZ: MERGA.

Holmes, M., & Tozer, L. (2004). *An effective identification and response to children's learning needs in number*. Paper presented at the Mathematics Education into the 21st Century Project: The Future of Mathematics Education Conference, Pod Tezniami, Ciechocinek, Poland. Retrieved from http://math.unipait/~grim/21_project/ CiechHolmesTozer.pdf.

Holster, T. (2006). *Purposeful questioning in mathematics: A guiding framework. ACE Papers, 17*. Retrieved from http://www.education.auckland.ac.nz

Holton, D., Ahmed, A., Williams, H., & Hill, C. (2001). On the importance of mathematical play. *International Journal of Mathematical Education in Science and Technology, 32*(3), 401-415.

Howie, D., & Peters, M. (1996). Positioning theory: Vygotsky, Wittgenstein and social constructionist psychology. *Journal for the Theory of Social Behaviour, 26*, 51-64.

Hughes, P. (2002). A model for teaching numeracy strategies. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. Thomas (Eds.), *Mathematics education in the South Pacific. Proceedings of the 25th annual conference of the Mathematics Education Group of Australasia* (pp. 350-357). Sydney, Australia: MERGA.

Hughes, P. (2007). *Revisions and extensions of the strategy teaching model.* Unpublished presentation at the proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia. Hobart. Sydney, Australia: MERGA.

Hughes, P., & Petersen, L. (2003). Constructing and using a personal numeracy teaching model in a classroom setting. In L. Bragg, C. Campbell, G. Herbert, &

J. Mousley (Eds.), *MERINO Mathematics education research: Innovation, networking, opportunity. Proceedings of the 26th Annual Conference of Mathematics Education Research Group of Australasia* (Vol. 2, pp. 444-451). Geelong, Australia: MERGA.

Hunter, J. (2006). The Numeracy Project: Foundations and development. *ACE Papers, 17*. Retrieved from http://www.education.auckland.ac.nz

Hunter, J. (2009). Developing a productive discourse community in the mathematics classroom. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides.* (Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia (pp. 249–256). Palmerston North, NZ: MERGA.

Hunter, R. (2005). Reforming communication in the classroom: One teacher's journey of change. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonagh, R. Pierce, & A. Roche (Eds.), *Building connections: Research, theory and practice*. (Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia (pp. 451-458). Sydney, Australia: MERGA.

Hunter, R. (2006). Structuring the talk towards mathematical inquiry. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities, cultures and learning spaces . Proceedings of the 29th annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 309–317). Sydney. MERGA.

Hunter, R. (2007a). *Teachers developing communities of mathematical inquiry.* (PhD Thesis). Massey University, Auckland, New Zealand.

Hunter, R. (2007b). Scaffolding small group interaction. In J. Watson & K. E. Beswick (Eds.), *Mathematics: Essential research, essential practice. Proceedings of the 30th Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 430-439). Adelaide, South Australia: MERGA.

Hunter, R. (2008). Facilitating communities of mathematical inquiry. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting directions Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 31-39). Brisbane: MERGA.

Irwin, K. C., & Woodward, J. (2005). Advancing Pasifika students' mathematical thinking. In F. Ell, J. Higgins, G. Thomas, T. Trinick, & J. Young-Loveridge (Eds.), *Findings from the New Zealand Numeracy Development Projects 2005,* (pp. 80-90). Wellington: Learning Media.

Isaksen, S., Dorval, K., & Treffinger, D. (2000). *Creative approaches to problem solving*. Dubuque, IO: Kendall/Hunt.

Johanning, D. (2007). Is there something to be gained from guessing? Middle school students' use of systematic guess and check. *School Science and Mathematics, 107*(4), 123-133.

Johnston, M., Thomas, G., & Ward, J. (2010). Analysis of the Number Framework. In *Findings from the New Zealand Numeracy Development Projects: 2009* (pp. 36-48). Wellington, New Zealand: Ministry of Education.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper elementary mathematics classrooms. *Elementary School Journal, 102*, 59–80.

Kazemi, E., Franke, M., & Lampert, M. (2009). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides*. *Proceedings of the 32nd annual conference of the Mathematics Education Research group of Australasia* (pp. 11-29). Wellington: MERGA.

Lakatos, I. (1976). *Proofs and refutations: The logic of mathematical discovery*. New York: Cambridge University Press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29-63.

Lampert, M. (2001). *Teaching problems and the problem of teaching*. New Haven, CT: Yale University Press.

Lampert, M., Boerst, T., & Graziani, F. (2011). Using organisational resources in the service of ambitious teaching practice. *Teachers College Record, 113*(7), 1-20.
Lannin, J. (2005) Generalisation and justification: The challenge of introducing algebraic reasoning through patterning activities. *Mathematical Thinking and Learning*, *7*(3), 231-258.
Lee, J. (2009). Universals and specifics of math self-concept, math self-efficacy, and math anxiety across 41 PISA 2003 participating countries. *Learning and Individual Differences, 19*(3), 355-365.
Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129-145). Rotterdam: Sense.
Leinhardt, G., & Schwarz, B. B. (1997). Seeing the problem: An explanation from Pólya. *Cognition and Instruction, 15*(3), 395-434.
Lev-Zamir, H., & Leikin, R. (2011). Creative mathematics teaching in the eye of the beholder: Focusing on teachers' conceptions. *Research in Mathematics Education, 13*(1), 17-32.
Liljedahl, P., & Sriraman, B. (2006). Musings on mathematical creativity. *For The Learning of Mathematics, 26*(1), 17-19.
Lin, Y. S. (2011). Fostering creativity through education: A conceptual framework of creative pedagogy. *Creative Education, 2*(3), 149-155.
Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.
Linehan, C., & McCarthy, J. (2000). Positioning in practice: Understanding participation in the social world. *Journal for the Theory of Social Behaviour, 30,* 435-453.
Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*(3), 255-276.
Lockhurst, D., Wubbels, T., & van Oers, B. (2010) Educational dialogues and the fostering of pupils' independence: The practices of two teachers. *Journal of Curriculum Studies, 42*(1) 99-121.
McClain, K., & Cobb, P. (2001). Supporting students' ability to reason about data. *Educational Studies in Mathematics, 45*, 103-129.
Maher, C. A., & Martino, A. M. (1996). The development of the idea of mathematical proof: A 5-year case study. *Journal for Research in Mathematics Education*, *27*(2), 194-214.
Mamona-Downs, J., & Downs, M. (2005). The identity of problem solving. *Journal of Mathematical Behaviour, 24*(3-4), 385-401.
Martin, L. C., Towers, J., & Pirie, S. E. B. (2000). Growing mathematical understanding: Layered observations. In M. L. Fernandez (Ed.), *Proceedings of the twenty-second annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol 1, pp. 225-230). Columbus, OH: Eric Clearinghouse.
Mason, J. (1998). Asking mathematics questions mathematically. *International Journal of Mathematical Education in Science and Technology, 31*(1), 97–111.
Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics, 38*(1-3), 135-161.
Maykut, P., & Morehouse, R. (1994). *Beginning qualitative research: A philosophic and practical guide*. London: Falmer Press.
Menard-Warwick, J. (2007). "Because she made beds. Every day." Social positioning, classroom discourse, and language learning. *Applied Linguistics, 29*(2), 267-289.
Merriam, S. B. (1998). *Qualitative research and case study applications in education*. (Revised and Expanded from Case Study Research in Education). San Francisco: Jossey-Bass.
Merriam, S. B. (2009). *Qualitative research: A guide to design and implementation* (2nd ed.). San Francisco, CA: Jossey-Bass.
Mertens. D. (2005). *Research and evaluation in psychology: Integrating diversity with quantitative, qualitative, and mixed methods* (2nd ed.). Thousand Oaks, CA: Sage.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis*. Thousand Oaks, CA: Sage.
Miller, T., & Bell, L. (2002). Consenting to what? Issues of access, gate-keeping and 'informed' consent. In M. Mauthner, M. Birch, J. Jessop, & T. Miller, (Eds.), *Ethics in qualitative research* (pp. 53-69). London: Sage.
Millett, A., Brown, M., & Askew, M. (Eds.). (2004). *Primary mathematics and the developing professional: Multiple perspectives on attainment in numeracy* Dordrecht: Kluwer Academic.
Ministry of Education. (1992). *Mathematics in the New Zealand curriculum.* Wellington, New Zealand: Author.
Ministry of Education. (1999). *Literacy experts group: Report to the Secretary of Education.* Wellington, New Zealand: Author.
Ministry of Education. (2001). *Curriculum update No. 45: The numeracy story*. Wellington, New Zealand: Learning Media.
Ministry of Education. (2002). *Curriculum update 50*. Wellington: Ministry of Education.
Ministry of Education. (2007a). *The New Zealand curriculum*. Wellington, New Zealand: Author.
Ministry of Education. (2007b). *Numeracy Professional Development Projects: Book 1: The number framework*. Wellington, New Zealand: Author.
Ministry of Education. (2007c). *Numeracy Professional Development Projects: Book 2: The diagnostic interview*. Wellington, New Zealand: Author.
Ministry of Education. (2007d). *Numeracy Professional Development Projects: Book 3: Getting started*. Wellington, New Zealand: Author.
Ministry of Education. (2007e). *Numeracy Professional Development Projects: Book 4: Teaching number knowledge*. Wellington, New Zealand: Author.
Ministry of Education (2007f). *Numeracy Professional Development Projects: Book 5: Teaching addition and subtraction*. Wellington, New Zealand: Author.
Ministry of Education. (2007g). *Numeracy Professional Development Projects: Book 6: Teaching multiplication and division*. Wellington, New Zealand: Author.
Ministry of Education. (2007h). *Numeracy Professional Development Projects: Book 7: Teaching fractions, decimals, and percentages.* Wellington, New Zealand: Author.
Ministry of Education. (2007i). *Numeracy Professional Development Projects: Book 8: Teaching number sense and algebraic thinking.* Wellington, New Zealand: Author.
Ministry of Education. (2009). *Expectations for number*. Retrieved from http://nzmaths.co.nz/expectations-number
Ministry of Education. (2012). *Curriculum update: No. 17*. Wellington, New Zealand: Learning Media.
Ministry of Education. (2013). *What is problem solving*? NZMaths: Author. Retrieved from http://nzmaths.co.nz/what-problem solving
Ministry of Education (2014). *Assessment.* Retrieved from http://assessment.tki.org.nz/Glossary
Moghaddam, F., & Harré, R. (Eds.). (2010). *Words of conflict, words of war*. Santa Barbara, CA: Praeger.
Moghaddam, F., Harré, R., & Lee, N. (2008). Positioning and conflict: An introduction. In F. Moghaddam, R. Harré, & N. Lee (Eds.), *Global conflict resolution through positioning analysis* (pp. 3-20). New York: Springer.
Muhlhausler, P., & Harré, R. (1990). *Pronouns and people*. Oxford: Blackwell.
Mullis, I. V. S., Martin, M. O., Smith, T. A., Garden, R. A., Gregory, K. D., Gonzalez, E. J., et al. (2003). *TIMSS assessment frameworks and specifications 2003* (2nd ed.). Chestnut Hill, MA: Boston College.
Murphy, J. (1988). Equity as student opportunity to learn. *Theory into Practice, 27*(2), 145–151.
Murphy, C. (2013). Thinkpiece: Making space for mathematics learning to happen in group work: Is this really possible? *Teachers and Curriculum, 13,* 108-111.

Nadjafikhah, M., Yaftian, N., & Bakhshalizadeh, S. (2012). Mathematical creativity: Some definitions and characteristics. *Procedia - Social and Behavioural Sciences, 31,* 285-291.
Nathan, M., & Knuth, E. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction, 21*, 175-207.
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
National Research Council. (2001). *Adding it up: Helping children learn mathematics.* (Report prepared by the Mathematics Learning Study Committee). Washington, DC: National Academy Press.
Neill, A., Fisher, J., & Dingle, R. (2010). *Exploring mathematics interventions: Exploratory evaluation of the accelerating learning in mathematics pilot study*. Wellington: NZCER.
Nuthall, G. (1999). Introduction and background. *International Journal of Educational Research, 31*(3), 141-256.
Nuthall, G. (2004). Relating classroom teaching to student learning: A critical analysis of why research has failed to bridge the theory-practice gap. *Harvard Educational Review, 74*(3), 273-306.
O'Doherty, K. C. & Davidson, H. J. (2010). Subject positioning and deliberate democracy: Understanding social processes underlying deliberation. *Journal for the Theory of Social Behaviour, 40*(2), 224-245.
O'Connor, M. C., & Michaels, S. (1996). Shifting participant frameworks: Orchestrating thinking practices in group discussion. In D. Hicks (Ed.), *Discourse, learning and schooling* (pp. 63-103). New York: Cambridge University Press.
Oakes, J. (1985) *Keeping track: how schools structure inequality*. Yale University Press, New Haven.
OFSTED. (2003). *The National Literacy and Numeracy Strategies and the Primary Curriculum*. London: Author.
Osbeck, L. M., & Nerssesian, N. J. (2010). Forms of positioning in interdisciplinary science practice and their epistemic effects. *Journal for the Theory of Social Behaviour, 40*(2), 136-161.
Otrel-Cass, K., Cowie, B., & Maguire, M. (2010). Taking video cameras into the classroom. *Waikato Journal of Education, 15*(2), 109-118.
Pape, S. & Smith, C. (2002). Self-regulating mathematics skills. *Theory Into Practice, 41*(2), 93-101.
Pape, S. & Wang, C. (2003). Middle school children's strategic behaviour: Classification and relation to academic achievement and mathematical problem solving. *Instructional Science, 31*(6), 419-449.
Pape, S., Bell. C., & Yetkin, I. (2003). Developing mathematical thinking and self-regulated learning: A teaching experiment in a seventh-grade mathematics classroom. *Educational Studies in Mathematics, 53*, 179-202.
Parrott, W. G. (2003). Positioning and the emotions. In R. Harré, & F. Moghaddam (Eds.), *The self and others: Positioning individuals and groups in personal, political, and cultural contexts* (pp. 29-44). Westport: Praeger.
Partington, G. (2001), Qualitative research interviews: Identifying problems in technique. *Issues in Educational Research, 11*(2), 32-44.
Patton, M. Q. (2002). *Qualitative evaluation and research methods* (3rd ed.). Thousand Oaks, CA: Sage.
Pauly, J. (1991). A beginner's guide to doing qualitative research. *Journalism Monographs*, *125*, 1-29.
Phillips, D., & Hayes, B. (2008). Securing the oral tradition: Reflective positioning and professional conversations in midwifery education. *Collegian, 15*(3), 109-114.
Piaget, J. (1964) The development of mental imagery (E. Duckworth, trans.). In R. E. Ripple & V. N. Rockcastle (Eds.), *Piaget rediscovered: A report on the conference on cognitive studies and curriculum development* (pp. 7-20). Ithaca NY: School of Education Cornell University.

Piaget, J. (1970). Piaget's theory. In P. H. Mussen (Ed.), *Carmichael's manual of child psychology* (Vol. 1, 3rd ed.). New York: Wiley.
Pintrich, P. R. (1999). The role of motivation in promoting and sustaining self-regulated learning. *Educational Research, 31*, 459-470.
Pirie, S. (2002). *Problem posing: What can it tell us about students' mathematical thinking?* In Proceeding of the Annual Meeting [of the] North American Chapter of the International Group for the Psychology of Mathematics Education.
Pirie, S. E. B. (1996). *What are the data? An exploration of the use of video-recording as a data gathering tool in the mathematics classroom*. Paper presented at the Sixteenth Annual Meeting of the International Group for the Psychology of Mathematics Education. North America, Florida State University, Panama City.
Pirie, S., & Kieren, T. (1989). A recursive theory of mathematical understanding. *For the Learning of Mathematics*, *9*(3), 7-11.
Pirie, S., & Kieren, T. (1992). Watching Sandy's understanding grow. *The Journal of Mathematical Behaviour, 11*(3), 243-257.
Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics, 26*(2/3), 165-190.
Pólya, G. (1958). On the curriculum for prospective high school teachers. *The American Mathematical Monthly, 65*(2), 101-104.
Pólya, G. (1963). On learning, teaching, and learning teaching. *The American Mathematical Monthly, 70*(6), 605-619.
Pólya, G. (1965). *Mathematical discovery: On understanding, learning, and teaching problem solving* (Volume II). New York: John Wiley.
Pólya, G. (1966a). Let us teach guessing: A demonstration with George Pólya. In K. Simon (Producer), MAA video classics: *Mathematical Association of America.*
Pólya, G. (Writer) (1966). *Let us teach guessing: A demonstration with George Pólya* [Video recording]. In K. Simon (Producer), MAA video classics. Washington, D.C.: Mathematical Association of America.
Pólya, G. (1969). Some mathematicians I have known. *American Mathematical Monthly 76*(7), 746–753.
Pólya, G. (1978). Guessing and proving. *The Two-Year College Mathematics Journal, 9*(1), 21-27.
Pólya, G. (1979). More on guessing and proving. *The Two-Year College Mathematics Journal, 10*(4), 255-258.
Ponterotto, J. G. (2005). Qualitative research in counseling psychology: A primer on research paradigms and philosophy of science. *Journal of Counseling Psychology, 52*(2), 126-136.
Powell, A. B., Francisco, J. M., & Maher, C.A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *Journal of Mathematical Behaviour, 22*(4), 405-435.
Räisänen, C., & Stenberg, A-C. (2011). Using positioning theory to understand strategic management. In H. Wamelink, R. Geradedts, L. Volker, T. Daamen, & K. Aalbers (Eds.), *Proceedings of the International Conference on Management and Innovation for a Sustainable Built Environment (MISBE2011)*, Amsterdam.
Redman, C. (2007). *Towards a dialogical perspective on agency in student learning,* AARE conference paper, 2007 International Educational Research Conference, Fremantle.
Redman, C., & Fawns, R. (2010). How to use pronoun grammar as a methodological tool for understanding the dynamic lived space of people. In S. Rodrigues (Ed.), *Using analytical frameworks for classroom research: Collecting data and analysing narrative* (163-182). New York: Routledge.
Remillard, J. T., & Bryans, M. B. (2004). Teachers' orientations toward mathematics curriculum materials: Implications for teacher learning. *Journal for Research in Mathematics Education, 35*(5), 352-388.

Ritchie, S. (2002). Student positioning within groups during science activities. *Research in Science Education, 32*, 35-54.
Robson, S. (2011). Producing and using video with young children: A case study of ethical questions and practical consequences. In D. Harcourt, B. Perry, & T. Waller (Eds.), *Researching young children's perspectives: Debating the ethics and dilemmas of educational research with children* (pp. 178-192). London: Routledge.
Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context.* New York: Oxford University Press.
Roschelle, J. (2000). Choosing and using video equipment for data collection. In D. Lesh & A. Kelley (Eds.), *Handbook of research design in mathematics and science education* (pp. 709–731). Mahwah, NJ: Lawrence Erlbaum.
Roth, W.-M., Tobin, K., & Ritchie, S., (2001). *Re/constructing elementary science*. New York, NY: Peter Lang.
Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255-281.
Sabat, S. R. (2008). Positioning and conflict involving a person with dementia: A case study. In F.M. Moghaddam, R. Harré, & N. Lee (Eds.), *Global conflict resolution through positioning analysis* (pp 65–78). New York: Springer.
Schoenfeld, A. (1985). *Mathematical problem solving*. New York: Academic Press.
Schoenfeld, A. (1987). What's all the fuss about metacognition? In A. Schoenfeld (Ed.), *Cognitive science and mathematics education*, (pp.189-215). Hillsdale, NJ: Erlbaum.
Schoenfeld, A. (1994). Reflections on doing and teaching mathematics. In A. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 53-72). New Jersey: Lawrence Erlbaum.
Schoenfeld, A. (1983). Beyond the purely cognitive: Belief systems, social cognition, and metacognition as driving forces in intellectual performance. *Cognitive Science, 7*, 329-363.
Schoenfeld, A. (1988). When good teaching leads to bad results: The disasters of "well-taught" mathematics courses. *Educational Psychologist, 23*, 145-166.
Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws, (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: MacMillan.
Schunk, D. (2001). Social cognitive theory and self-regulated learning. In B. Zimmerman, & D. Schunk (Eds.), *Self-regulated learning and academic achievement: Theoretical perspectives* (pp. 125-152). Mahwah, NJ: Erlbaum.
Scouller, D. (2009). Has strategy become the new algorithm? *The New Zealand Mathematics Magazine, 46*(3), 1-11.
Sfard, A. (2000). Symbolizing mathematical reality into being - Or, how mathematical discourse and mathematical objects can create each other. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematical classrooms: Perspectives on discourse, tools and mathematical design* (pp. 37–98). Mahwah, NJ: Erlbaum.
Sfard, A., & Kieran, C. (2001). Cognition as communication: Rethinking learning-by-talking through multi-faceted analysis of students' mathematical interactions. *Mind, Culture, and Activity, 8*(1), 42-76.
Silver, E. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing *ZDM: The International Journal on Mathematics Education, 29*(3), 75-80.
Slocum, N., & Van Langenhove, L. (2004). The meaning of regional integration: Introducing positioning theory in regional integration studies. *Journal of European Integration, 26*(3), 227-252.
Slocum-Bradley, N. (2010). The positioning diamond: A trans-disciplinary framework for discourse analysis. *Journal for the Theory of Social Behaviour, 40*(1), 79-107.

Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA: NCTM.

Sriraman, B. (2004). The characteristics of mathematical creativity. *ZDM: The International Journal on Mathematics Education, 14*(1), 13-27.

Sriraman, B. (2005). Are giftedness & creativity synonyms in mathematics? An analysis of constructs within the professional and school realms. *The Journal of Secondary Gifted Education, 17*, 20–36.

Stake, R. E. (2012). *Applications of case study research* (3rd ed.). Thousand Oaks, CA: Sage.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development.* New York: Teachers College Press.

Stein, M. K., Smith, M.S., Henningsen, M., & Silver, E.A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development. Second Edition.* New York: Teachers College Press.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213-226.

Sullivan, P., Warren, E., & White, P. (1999). Comparing students' responses to content specific open-ended and closed mathematical tasks. In O. Zaslavsky (Ed.), *Proceedings of the 23rd conference of the International Group for the Psychology of Mathematics education* (Vol. 4, pp. 249-256). Haifa: PME.

Tabach, M., & Friedlander, A. (2013). School mathematics and creativity at the elementary and middle-grade levels: How are they related? *ZDM: Mathematics Education, 45,* 227-238.

Tait-McCutcheon, S. (2008). Self-efficacy in mathematics: Affective, cognitive, and conative domains of functioning. In M. Goos, R. Brown, & K. Makar (Eds.), *Proceedings of the 31st annual conference of the Mathematics Education Group of Australasia* (pp. 507-513). Brisbane, QLD: MERGA.

Tait-McCutcheon, S. , Drake, M., & Sherley, B. (2011). From direct instruction to active construction: Teaching and learning basic facts. *Mathematics Education Research Journal, 23*, 321-345.

Tait-McCutcheon, S. , & Sherley, B. (2008). Stepping out of the stream: Reflecting-on-action. *Findings from the New Zealand numeracy development projects 2007*. Learning Media: Wellington, New Zealand.

Tesch, R. (1990). *Qualitative research. Analysis types and software*. London: Falmer Press.

Thomas, G., & Tagg, A. (2004). *An evaluation of the Early Numeracy Project 2003.* Wellington, New Zealand: Ministry of Education.

Thomas, G., & Tagg, A. (2005). Evidence for expectations: Findings from the Numeracy Project Longitudinal Study. In J. Higgins, K. C. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge (Eds.), *Findings from the New Zealand Numeracy Development Project: 2004* (pp. 35-46). Wellington, New Zealand: Ministry of Education.

Thomas, G., & Tagg, A. (2006). Numeracy Development Project longitudinal study: Patterns of achievement. *Findings from the New Zealand Numeracy Development Projects: 2005* (pp. 22-33). Wellington: Learning Media.

Thomas, G., & Ward, J. (2001). *An evaluation of the Count Me in Too pilot project.* Wellington, New Zealand: Ministry of Education.

Thomas, G., & Ward, J. (2002). *An evaluation of the Early Numeracy Project 2001.* Wellington, New Zealand: Ministry of Education.

Thomas, G., Tagg, A., & Ward, J. (2002). Making a difference. The Early Numeracy Project. In B. Barton, K. Irwin, M. Pfannkuch, & M. Thomas (Eds.), *Mathematics education in the South Pacific*. *Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia* (pp. 49-57). Sydney, Australia: MERGA.

Thomas, G., Tagg, A., & Ward, J. (2003). *An evaluation of the Early Numeracy Project 2002: Exploring issues in mathematics education*. Wellington, New Zealand: Ministry of Education.

Torrance, E. P. (1963). The creative personality and the ideal pupil. *Teachers College Record, 65*, 220-226.

Torrance, E. P. (1974). *The Torrance tests of creative thinking: Technical-norms manual*. Bensenville, IL: Scholastic Testing.

Treffinger, D. J., Young, G. C., Selby, E. C., & Shepardson, C. (2002). *Assessing creativity: A guide for educators* (RM02170). Storrs: The National Research Centre on the Gifted and Talented, University of Connecticut.

Truxaw, M. P., & DeFranco, T. C. (2007). Mathematics in the making: Mapping verbal discourse in Pólya's "Let Us Teach Guessing" lesson. *The Journal of Mathematical Behaviour, 26*(2), 96-114.

Vale, I., Pimentel, T., Cabrita, I., Barbosa, A., & Fonseca, L. (2012). Pattern problem solving tasks as a means to foster creativity in mathematics. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, 171-178). Taipei, Taiwan: PME.

van Langenhove, L., & Harré, R. (1999). Introducing positioning theory. In R. Harré, & L. van Langenhove (Eds.), *Positioning theory: Moral contexts of intentional action* (pp. 14-31). Oxford: Blackwell.

Varela, C. R., & Harré, R. (1996). Conflicting varieties of realism: Causal powers and the problems of social structure. *Journal for the Theory of Social Behaviour, 26*(3), 313-325.

von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. In R. B. Davis, C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics* (pp. 19-30). Reston, VA: NCTM.

von Glasersfeld, E. (1992). *Aspects of radical constructivism and its educational recommendations.* Paper presented at the ICME-7, Draft to working group #4, Montreal. Retrieved from www.umass.edu/srri/vonGalsersfeld/onlinePapers/html/195.html

von Glasersfeld, E. (1993). Questions and answers about radical constructivism. In K. Tobin (Ed.), *The practice of constructivism in science education* (pp. 23-28). Washington: AAAS Press.

von Glasersfeld, E. (2000). Problems of constructivism. In L. P. Steffe & P. W. Thompson (Eds.), *Radical constructivism in action: Building on the pioneering work of Ernst von Glasersfeld* (pp. 3-9). London, England: Routledge Falmer.

Vygotsky, L. S. (1962). *Thought and language*. Cambridge, MA: Massachusetts University Press.

Vygotsky, L. S. (1978). Collected writings. In M. Cole, V. John-Steiner, S. Scribner, & E. Souberman (Eds.), *Mind in society.* Cambridge, MA: Harvard University Press.

Vygotsky, L. S. (1987). Thinking and speech (N. Minick, Trans.). (Orig. 1934) In R. W. Rieber & A. S. Carton (Eds.), *The collected works of L.S. Vygotsky*. *Volume 1: Problems of General Psychology* (pp. 37-285). New York: Plenum Press.

Wagner, D., & Herbel-Eisenmann, B. (2013). Disbursing authority among mathematics students. *Proceedings of the Seventh International Mathematics Education and Society Conference* (pp. 483-491), Cape Town, South Africa.

Wagner, D., & Herbel-Eisenmann, B. (2009). Re-mythologizing mathematics through attention to classroom positioning. *Educational Studies in Mathematics, 72*(1), 1-15.

Walshaw, M. (2013). Explorations into pedagogy within mathematics classrooms: Insights from contemporary inquiries. *Curriculum Inquiry, 43*(1), 71-94.

Walshaw, M., & Anthony, G. (2008). The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78*(3), 516-551.

Watson, A. (2008). School Mathematics as a Special Kind of Mathematics. Paper presented to Working Group 1 the Symposium on the Occasion of the 100th Anniversary of ICMI, Rome, March, 2008. Retrieved from <http://www.unige.ch/math/EnsMath/Rome2008/partWG1.html>.

Weber, K., Radu, I., Mueller, M., Powell, A., & Maher, C. (2010). Expanding participation in problem solving in a diverse middle school mathematics classroom. *Mathematics Education Research Journal*, *22*(1), 91-118.

Whitenack, J., Knipping, N., & Kim, O-K. (2001). The teacher's and students' important roles in sustaining and enabling classroom mathematical practices: A case for realistic mathematics education. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of the 25th conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 415-422). Utrecht: PME.

Wiersma, W., & Jurs, S. G. (2009). *Research methods in education: An introduction* (9th ed.). Boston: Pearson/Allyn & Bacon.

Willis, K. (2006). Analysing qualitative data. In M. Walter (Ed.), *Social research methods: An Australian perspective* (pp. 257-280). Melbourne: Oxford University Press.

Willoughby, S. (2000). Perspectives on mathematics education. In M. J. Burke, & F. Curcio (Eds.), *Learning mathematics for a new century* (pp. 1-15). Reston, VA: National Council of Teachers of Mathematics.

Wink, J. (2000). *Critical pedagogy: Notes from the real world*. Boston, MA: Allyn & Bacon.

Wittgenstein, L. (1969). *On certainty*. (Trans. D. Paul & G. E. M. Anscombe). Oxford: Blackwell.

Wittgenstein, L. (1953). *Philosophical investigations*. Oxford, England: Blackwell.

Wittgenstein, L. (1980) *Culture and value*. (Trans. P. Winch). Oxford: Blackwell.

Wong, N. Y., Marton, F., Wong, K. M., & Lamb, C. C. (2002). The lived space of mathematics learning. *The Journal of Mathematical Behaviour, 21*(1), 25-47.

Wood, D., Bruner, J., & Ross, G. (1976).The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, 17, 89-100.

Wood, T. (2002). What does it mean to teach mathematics differently? In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.), *Mathematics education in the South Pacific*. *Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia* (pp. 61- 67). Auckland, New Zealand: MERGA.

Woods, D. K., & Fassnacht, C. (2007). *Transana* (Version 2.20). Madison: Wisconsin Center for Education Research.

Woodward, J., & Irwin, K. (2005). Language appropriate for the New Zealand numeracy project. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, et al. (Eds.), *Building connections: Theory, research and practice. Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia* (Vol. 2, pp. 799–806). Sydney, Australia: MERGA.

Wright, R. (1991). An application of the epistemology of radical constructivism to the study of learning. *The Australian Educational Researcher, 18*(1), 75-95.

Wright, R. (1998). An overview of a research-based framework for assessing and teaching early number. In C. Kanes, M. Goos, & E. Warren (Eds.), *Proceedings of the 21st Annual Conference of the Mathematics Education Group of Australasia* (Vol. 2, pp. 701-708). Brisbane, Australia: MERGA.

Wright, R. J. (2000). Professional development in recovery education. In L. P. Steffe & P. W. Thompson (Eds.), *Radical constructivism in action: Building on the pioneering work of Ernst von Glasersfeld* (pp. 134-151). London: Routledge Falmer.

Wright, R. J., Stanger, G., Cowper, M., & Dyson, R. (1996). First-graders' progress in an experimental mathematics recovery program. In J. Mulligan & M. Mitchelmore, (Eds.), *Research in early number learning: An Australian* *perspective* (pp. 55-72). Adelaide, Australia: Australian Association of Mathematics Teachers.

Wright, R., Martland, J., & Stafford, A. (2000). *Early numeracy: Assessment for teaching and intervention*. London: Paul Chapman.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Yamakawa, Y., Forman, E., & Ansell, E. (2005). Role of positioning in constructing an identity in a third grade mathematics classroom. In K. Kumpulainen, C. E. Hmelo-Silver, & M. Cesar (Eds.), *Investigating classroom interaction: Methodologies in action* (pp. 179-202). Rotterdam: Sense.

Yin, R. K. (2003). *Case study research, design and methods* (3rd ed., vol. 5). Thousand Oaks: Sage.

Young-Loveridge, J. (2005). Patterns of performance and progress: Analysis of 2004 data. In J. Higgins, K. C. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge (Eds.), *Findings from the New Zealand Numeracy Development Project 2004* (pp. 5-20). Wellington: Ministry of Education.

Young-Loveridge, J. (2006). Patterns of performance and progress on the Numeracy Development Project: Looking back from 2005. In F. Ell, J. Higgins, K. C. Irwin, G. Thomas, T. Trinick, & J. Young-Loveridge (Eds.), *Findings from the Numeracy Development Projects: 2005*. Wellington, New Zealand: Ministry of Education.

Young-Loveridge, J. (2001). *The use of part-whole strategies by year 6 children at a low SES and a medium SES school.* Paper presented at the Twenty-fourth Annual Conference of the Mathematics Education Research Group of Australasia. Sydney, Australia: MERGA.

Young-Loveridge, J. (2010). A decade of reform in mathematics education: Results for 2009 and earlier years. In *Findings from the New Zealand Numeracy Development Projects: 2009* (pp. 15-35). Wellington, New Zealand: Learning Media.

Zelle, G. (2009). *Exploring the application of positioning theory to the analysis of organisational change*. Australian and New Zealand Academy of Management Conference, Melbourne, Victoria.

Zevenbergen, R. (2003). Grouping by ability: A self-fulfilling prophecy? *Australian Mathematics Teacher, 59*(4), 2-7.

Zevenbergen, R. (2005). The construction of a mathematical habitus: implications of ability grouping in the middle years. *Journal of Curriculum Studies, 37*(5), 607–619.

Zevenbergen, R., & Ortiz-Franco, L. (2002). Equity in mathematics. *Mathematics Education Research Journal, 13*(3), 151-153.

Zimmerman, B. J. (1989). A social cognitive view of self-regulated academic learning. *Journal of Educational Psychology*, *81*(3), 329-339.

Zimmerman, B. J. (1994). Dimensions of academic self-regulation: A conceptual framework for education. In D. H. Schunk, & B. J. Zimmerman (Eds.), *Self-regulation of learning and performance: Issues and educational applications* (pp. 3-21). Hillsdale, NJ: Lawrence Erlbaum.

Zimmerman, B. J. (2000). Attaining self-regulation: A social cognitive perspective. In M. Boekaerts, P. Pintrich, & M. Ziedner (Eds.), *Handbook of self-regulation* (pp. 13-39). Orlando, FL: Academic Press.

Zimmerman, B. J. (2002). Becoming a self-regulated learner: An overview. *Theory into Practice, 41*(2), 64-70.

Zimmerman, B. J., & Labuhn, A. S. (2012). Self-regulation of learning: Process approaches to personal development. In K. R. Harris, S. Graham, & T. Urdan (Eds.), *APA Educational psychology handbook.* (Vol. 1, pp. 399-425). Washington DC: American Psychology Association.

Zimmerman, B. J., & Martinez-Pons, M. (1986). Development of a structured interview for assessing student use of self-regulated learning strategies. *American Educational Research Journal, 23*, 614-628.

Zimmerman, B. J., & Martinez-Pons, M. (1988). Construct validation of a strategy model of student self-regulated learning. *Journal of Educational Psychology, 80*, 284-290.

Zimmerman, B. J., & Martinez-Pons, M. (1990). Student differences in self-regulated learning: Relating grade, sex, and giftedness to self-efficacy and strategy use. *Journal of Educational Psychology, 82*, 51-55.

Appendix A: VUW Ethics Approval Form

TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI
VICTORIA
UNIVERSITY OF WELLINGTON

1 May 2007

Sandi McCutcheon
C/- School of Primary and Secondary Teacher Education
Victoria University of Wellington College of Education
Karori
Wellington

Dear Sandi

RE: Ethics application AARP SPSTE/2007/17

I am pleased to advise you that your ethics application, **'Differentiated Teacher Practice: Grouping Students by Number Strategy Stages',** with the amendments as required by the Ethics Committee, has been approved.

Yours Sincerely

J. A. Loveridge

Dr Judith Loveridge
Convenor
Faculty of Education Ethics Committee

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Appendix B: Principal/Teacher Information Sheet

Differentiated Teacher Practice:
Grouping Students by Number Strategy Stages
Individual PhD Research Project

My name is Sandi McCutcheon and I am a PhD student at Victoria University of Wellington and a Numeracy Lecturer. My PhD thesis title is Differentiated Teacher Practice: Grouping Students by Number Strategy Stages. I have a deep interest in mathematics education and believe that it is important to develop more knowledge about teacher practices and their role in building student capability in Mathematics Education and the impact this has on raising student achievement. This PhD research project will be supervised by Dr. Joanna Higgins. A time-line for this research is attached.

The purpose of this letter is to seek your permission for the involvement of staff and students from your school in my PhD research. The purpose of this research is to examine the strategies that teachers use when teaching students operating at different strategy stages of the Numeracy Project. Teachers will be videoed as they undertake three consecutive lessons with groups of students. The videos will be transcribed by myself.

The research objectives are:

1. To explore the research and New Zealand Ministry of Education documents that underpin and support effective numeracy practice.
2. To examine teacher perceptions and beliefs about effective numeracy practice with able and less able strategy groups
3. To observe teacher numeracy practice with able and less able strategy groups
4. To describe current practices in numeracy within groups of able and less able students.

This research project has been approved by the Victoria University of Wellington Human Ethics Committee. All data collected for this research will be kept confidential, with access limited to the researcher, supervisor and the appropriate participants. No participant will be named, and individual results will not be made available or reported. I do not intend to gain financially from this study.

At the end of the 2011, all paper copies of the data will be destroyed, with audiotapes, videotapes and computer files wiped. This information gained through this study may be used for national and international conference presentations and papers. Participants and parents have the right to ownership of the data as relates to them.

Your written permission is sought to conduct my research study at your school.

As a participant in this research you can:

- refuse permission for your school to participate in this study
- withdraw your school from this study at any time
- ask any questions about the research study at any time during the participation
- be given access to the summary of the findings when the study is completed

If you have any questions regarding the Information Sheet please feel free to contact me 04 463 9647 or by e-mail sandi.mccutcheon@vuw.ac.nz

Yours sincerely
Sandi McCutcheon

Appendix C: Principal Consent Form

Differentiated Teacher Practice:
Grouping Students by Number Strategy Stages.

I have been provided with enough information about the nature and purpose of this research project. I have understood that information and have been given the opportunity to seek further clarification or explanations. I understand that I may ask questions at any time.

I understand that that no participant will be named in the research and that individual results will not be made available or reported. I also understand that the information I give will be used only for the stated purposes.

I understand that I have the right to withdraw my school (participating teachers and students) from this research at any point before the final analysis of the data by contacting the researchers. No reason for withdrawal from the research needs to be given. If for any reason I choose to withdraw from the research project, any data already collected will be destroyed and not used in the project.

I understand that research data will be kept secure and access to the data will be limited. At the end of the research, all paper copies of the data will destroyed.

A summary sheet on the results of this research will be provided to me if I tick the box on the consent form (below). The full results of the research may also be published or presented at national and/or international conferences.

I consent to my school being involved in this research project. Yes/No

Name: ____________________________________

Signature ____________________________________

☐ I would like written feedback about the outcomes of this research.

Address to which the research findings should be sent.

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Please retain this form and I will collect it.

Appendix D: Teacher Consent Form

Differentiated Teacher Practice:
Grouping Students by Number Strategy Stages.

I have been provided with enough information about the nature and purpose of this research project. I have understood that information and have been given the opportunity to seek further clarification or explanations. I understand that I may ask questions at any time.

I understand that any information or opinions I provide will be kept confidential, that no participant will be named in the research, and that individual results will not be made available or reported. I also understand that the information I give will be used only for the stated purposes.

I understand that I have the right to withdraw from this research at any point before the final analysis of the data by contacting the researchers. No reason for withdrawal from the research needs to be given. If for any reason I choose to withdraw from the research project, any data already collected will be destroyed and not used in the project.

I understand that research data will be kept secure and access to the data will be limited. At the end of the research, all paper copies of the data will destroyed.

A summary sheet on the results of this research will be provided to me if I tick the box on the consent form (below). The full results of the research may also be published or presented at national and/or international conferences.

I consent to my teaching being videoed for this research project during Term 1 and 2 2007. Yes/No

I consent to the use of my teaching action plan Term 1 2008 Yes/No

Signature ______________________________

☐ I would like written feedback about the outcomes of this research.

Address to which the research findings should be sent.

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Please retain this form and I will collect it.

Appendix E: Parent/Student Information Sheet

Differentiated Teacher Practice:
Grouping Students by Number Strategy Stages.
Individual PhD Research Project

Dear Parents and Student,

My name is Sandi McCutcheon and I am a PhD student at Victoria University of Wellington and a Numeracy Lecturer. My PhD thesis title is Differentiated Teacher Practice: Grouping Students by Number Strategy Stages. I have a deep interest in mathematics education and believe that it is important to develop more knowledge about teacher practices and their role in building student capability in Mathematics Education and the impact this has on raising student achievement. This PhD research project will be supervised by Dr. Joanna Higgins and a timeline for the research is attached.

The purpose of this letter is to seek your and your child's permission for their involvement in my PhD research. The purpose of this research is to examine the strategies that teachers use when teaching students operating at different strategy stages of the Numeracy Project. The teachers will be videoed as they undertake three consecutive lessons with groups of students. The videos will be viewed and transcribed only by myself and will be destroyed in 2011.

This research project has been approved by the Victoria University of Wellington Human Ethics Committee. All data collected for this research will be kept confidential, with access limited to the researcher, supervisor and the appropriate participants. No participant will be named, and individual results will not be made available or reported. I do not intend to gain financially from this study.

At the end of 2011, all paper copies of the data will be destroyed, with audiotapes, videotapes and computer files wiped. While complete anonymity cannot be guaranteed, complete confidentiality will be assured to both the participants and the school. This information gained through this study may be used for national and international conference presentations and papers. Participants and parents have the right to ownership of the data that relates to them.

Your written permission is sought from you and your child to permit them to participate in this study

As the Parent/Student you have the right;
- To refuse permission for your child/ yourself to participate in this study
- To withdraw your child/yourself from this study at any time
- To ask any questions about the research study at any time during the participation
- To be given access to the summary of the findings when the study is completed

If you have any questions regarding the Information Sheet please feel free to contact me 04 463 9647 or by e-mail sandi.mccutcheon@vuw.ac.nz

Yours sincerely
Sandi McCutcheon

Appendix F: Parent/Student Consent Form

Differentiated Teacher Practice:
Grouping Students by Number Strategy Stages.

I have been provided with enough information about the nature and purpose of this research project. I have understood that information and have been given the opportunity to seek further clarification or explanations. I understand that I may ask questions at any time. I understand that no participant will be named in the research, and that individual results will not be made available or reported.

I understand that I have the right to withdraw my child/myself from this research at any point before the final analysis of the data by contacting the researchers. No reason for withdrawal from the research needs to be given. If for any reason I choose to withdraw my child/myself from the research project, any data already collected will be destroyed and not used in the project. I understand that research data will be kept secure and access to the data will be limited. At the end of 2011, all paper, audio and video copies of the data will destroyed.

I understand that if I do not give permission for my child/myself to participate in this research that they will not be excluded from any learning opportunities because of this.

A summary sheet on the results of this research will be provided to me if I tick the box on the consent form (below). The full results of the research may also be published or presented at national and/or international conferences.

Parent:
I consent to my child being videoed during their mathematics lessons during Term Two of 2007.
Yes/No

Signature ________________________________

Child:
I consent to being videoed during my mathematics lessons during Term Two of 2007.
Yes/No

Signature ________________________________

☐ I would like written feedback about the outcomes of this research.

Address to which the research findings should be sent.

____

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Please return this form to your child's classroom teacher by Friday 18th May