**Table 1.** Maximum groundwater level measurement points ([Boronina et al. 2015](#_ENREF_7))

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Type of Data** | Boreholes | Compensation Basins | Drains | Rivers/ Lakes | Other Points | Total |
| **Sample size** | 104 | 4 | 87 | 11 | 17 | 223 |

**Table 2.** Overview of interpolation methods applied in this comparative study

| **Interpolation Methods (IPMs)** | **Abbreviation** | **Description** |
| --- | --- | --- |
| Topo to Raster | T2R | In ArcGIS, T2R is based on ANUDEM program version 5.3 which uses an iterative finite difference interpolation technique. “The program interpolates topographic data onto a regular elevation grid by minimising a user-specified roughness penalty on the fitted grid values and by imposing constraints that ensure connected drainage structure and sensible representation of ridges and streams” (Hutchinson 2011). A terrain specific roughness penalty is defined as first and second order of partial derivation ⨍ of the interpolation method. (Hutchinson 1988, 1989, 1996, 2000, 2011; Šiljeg et al. 2015) |
| Natural Neighbour | NaN | Natural neighbour interpolation is a weighted average interpolation technique which applies weights to a subset of input samples to the point of interest based on proportionate areas to interpolate a value. The neighbours are selected based on the configuration of the data. Initially, Thiessen polygons are constructed of all the sample points, and then a new Thiessen polygon is built around the point of interest. The proportion of overlap between the new polygon and the initial polygons is used as the weights. (Ledoux and Gold 2005; Li and Heap 2008; Liang and Hale 2010) |
| Ordinary Kriging | OK | Kriging is an advanced geo-statistical procedure that generates an estimated surface from a set of points by assuming a Gaussian process. It uses the semi-variograms and covariance functions to quantify the spatial autocorrelation values. The type of semi-variogram model used and trend removal can influence the estimation of certain kriging methods. In ordinary kriging, the weights depend on a fitted model, the distance to the point of interest, and the spatial relationships between the measured points around the point of interest. OK requires neither knowledge nor stationarity of the mean over the region of interest and OK with local search neighbourhood already accounts for trends (varying mean) in the values of the primary variable. However, OK requires a stationary mean of the local search window and it requires decisions on transformation (e.g., log-transformation for lognormal data), trend models, neighbourhoods, etc. (GISResources 2014; Johnston et al. 2001; Li and Heap 2008; Li and Heap 2014; Nikroo et al. 2010; Varouchakis et al. 2012; Varouchakis and Hristopulos 2013a,b)  |
| Empirical Bayesian Kriging | EBK | This method reduces error by automating the semi-variogram modelling process to select the best model from randomly generated models. Empirical Bayesian kriging combines Bayes’ theorem and kriging interpolation and accounts for the error in estimating the true semi-variogram through iterative simulations. The EBK automatically calculates parameters through a process of sub-setting and simulation to build a valid kriging model. The semi-variogram parameters in EBK are estimated using restricted maximum likelihood estimation. This method allows using moderately non-stationary data and outperforms other kriging methods for small datasets. (Bhunia et al. 2018; ESRI ; Krivoruchko 2012; Pilz and Spöck 2008; Zou et al. 2016). |
| Inverse Distance Weighted | IDW | The inverse distance weighted (IDW) method estimates the values at the point of interest by using an inverse function of the distance from the point of interest to the sampled points for calculating weights of sample points. (GISResources 2014; Johnston et al. 2001; Li and Heap 2008) |
| Radial Basis Functions | RBF | RBFs is the process of fitting a flexible membrane to the data points for minimizing the total curvature of the surface. The selected basis function specifies the way that the membrane fit between the values. It is a form of artificial neural networks.(Chen 1991; Johnston et al. 2001; Peralvo and Maidment 2004; Rusu and Rusu 2006; Szypuła 2016; Xie et al. 2011) |
| Local polynomial | LPI | Local polynomial interpolation fits many polynomials using all points only within the defined localized windows(neighbourhood). “The neighbourhoods’ overlaps and the value used for each prediction is the value of the fitted polynomial at the centre of the neighbourhood” (Johnston et al. 2001). |
| Universal Kriging | UK | The Universal Kriging is Kriging with a smoothly varying and nonstationary trend (average or expected value of the regionalized variable). “It is an extension of OK by incorporating the local trend within the neighbourhood search window as a smoothly varying function of the coordinates” (Li and Heap 2008). In Universal Kriging, the mean is assumed to have a functional dependence on spatial location and can be a function of the coordinates in a linear, quadratic or higher form. (Johnston et al. 2001; Kis 2016; Kumar 2007; Li and Heap 2008)  |
| Spline  | SI | Minimizing the total curvature of the surface is best for gently varying surfaces, such as elevation, water table heights, or pollution concentrations. It creates a smooth surface effect and cliffs and fault lines are not well presented because of the smoothing effect. (GISResources 2014) |
| Trend surface analysis | TSA | TSA is a modified version of Thin Plate Spline and it minimizes surface variance in relation to the input values. Trend surfaces are good for identifying coarse scale patterns in data and the interpolated surface rarely passes through the sample points. (GISResources 2014)  |

**Table 3.** Statistical measures used in this study for accuracy assessment of spatial interpolation methods (kevin Johnston 2001; Li and Heap 2008; Ohmer et al. 2017)

|  |  |  |  |
| --- | --- | --- | --- |
| Measurement Error | Abbreviation | Equation | Description  |
| Mean prediction error | ME | $$\frac{1}{n}\sum\_{i=1}^{n}\left(P\_{i}-O\_{i}\right)$$ | ME is conditionally suitable as a measure of prediction error because negative and positive predicted values offset each other. ME is helpful in highlighting bias in prediction.  |
| Mean absolute prediction error | MAE | $$\frac{1}{n}\sum\_{i=1}^{n}\left|P\_{i}-O\_{i}\right|$$ | MAE is the average of the absolute error values. It uses the magnitude of the error as a measure of the accuracy of the method. |
| Root-mean-squared prediction error | RMSE | $$\sqrt{\frac{1}{n}\sum\_{i=1}^{n}\left(P\_{i}-O\_{i}\right)^{2}}$$ | The RMSE is the square root of the statistical variance of predictions, using a smooth function (an order two polynomial) to approximate the magnitude of the error. As large errors are magnified, it is less robust to outliers than MAE. It possesses the same units as the original data so provides a more readily understood estimate of prediction error.  |
| Coefficient of determination | R2 | $$\frac{\left[\sum\_{i=1}^{n}\left(P\_{i}-P\_{ave}\right)\left(O\_{i}-O\_{ave}\right)\right]^{2}}{\sum\_{i=1}^{n}\left(P\_{i}-P\_{ave}\right)^{2}\sum\_{i=1}^{n}\left(O\_{i}-O\_{ave}\right)^{2}}$$ | R2 measures the square of the correlation between the predicted value and the measured value. |
| *Note: n= number of observations or samples;*$ P\_{i}$*= Predicted value;* $O\_{i}$*=Observed(measured) value;* $P\_{ave}$*= mean of predicted value;*$O\_{ave}$*= mean of observed value* |

**Table 4.** Comparison of the prediction accuracy of different interpolation methods derived from 10 replicates of the cross-validation routine applied to the full dataset

|  |  |
| --- | --- |
| **Error measure** | **Interpolation Methods** |
| **T2R** | **NaN** | **OK** | **EBK** | **IDW** | **RBF** | **LPI** | **UK** | **SI** | **TSA** |
| **ME** | Max. | 0.11 | 0.08 | 0.23 | 0.17 | 0.32 | 0.41 | 0.16 | 0.17 | 0.63 | 0.70 |
| Mean | -0.01 | -0.08 | 0.03 | -0.04 | 0.03 | 0.11 | -0.17 | -0.15 | 0.20 | 0.07 |
| Min. | -0.18 | -0.31 | -0.20 | -0.18 | -0.27 | -0.17 | -0.38 | -0.41 | -0.11 | -0.56 |
| **MAE** | Max. | 0.50 | 0.51 | 0.61 | 0.68 | 0.74 | 0.75 | 0.92 | 1.01 | 1.37 | 3.15 |
| Mean | 0.43 | 0.44 | 0.46 | 0.46 | 0.58 | 0.63 | 0.66 | 0.65 | 0.88 | 2.91 |
| Min. | 0.29 | 0.30 | 0.33 | 0.30 | 0.46 | 0.48 | 0.48 | 0.49 | 0.54 | 2.36 |
| Std. Dev. | 0.07 | 0.07 | 0.09 | 0.11 | 0.09 | 0.10 | 0.12 | 0.15 | 0.30 | 0.22 |
| **RMSE** | Max. | 0.76 | 0.86 | 0.95 | 1.07 | 1.06 | 1.19 | 1.56 | 1.83 | 3.80 | 3.96 |
| Mean | 0.65 | 0.67 | 0.72 | 0.73 | 0.87 | 0.98 | 1.01 | 1.02 | 1.72 | 3.60 |
| Min. | 0.46 | 0.41 | 0.49 | 0.44 | 0.67 | 0.73 | 0.63 | 0.65 | 0.82 | 2.81 |
| Std. Dev. | 0.09 | 0.14 | 0.16 | 0.18 | 0.13 | 0.16 | 0.28 | 0.38 | 0.87 | 0.34 |
| **R2** | Max. | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.98 | 0.75 |
| Mean | 0.99 | 0.99 | 0.99 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.91 | 0.64 |
| Min. | 0.98 | 0.98 | 0.97 | 0.96 | 0.97 | 0.95 | 0.93 | 0.91 | 0.66 | 0.58 |
| Std. Dev. | 0.00 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.10 | 0.05 |

**Table 5.** Performance of Natural Neighbour (NaN) interpolation method relative to other interpolation methods in terms of percentage change in MAE and RMSE relative to NaN and percentage of predictions that are within 0.5m of the observed value

|  |  |
| --- | --- |
| **Error Assessment**  | **Interpolation Methods**  |
| **T2R** | **NaN** | **OK** | **EBK** | **IDW** | **RBF** | **LPI** | **UK** | **SI** | **TSA** |
| **% Change MAE** | Mean | -2.2 | - | 4.4 | 4.9 | 33.1 | 44.0 | 50.1 | 50.6 | 103.4 | 570.4 |
| Std. Dev. | 12.0 | - | 16.2 | 19.6 | 22.7 | 23.0 | 27.2 | 37.4 | 79.4 | 93.8 |
| *p*-value | 1.00 | - | 1.00 | 1.00 | 0.04 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |
|  **% Change****RMSE** | Mean | -1.3 | - | 8.6 | 11.1 | 33.7 | 50.3 | 55.0 | 58.0 | 178.0 | 454.6 |
| Std. Dev. | 15.8 | - | 23.6 | 28.4 | 33.1 | 36.7 | 50.0 | 69.5 | 186.1 | 119.1 |
| *p*-value | 1.00 | - | 1.00 | 1.00 | 0.05 | 0.02 | 0.04 | 0.04 | 0.04 | 0.02 |
| **% Errors < 0.5m** | Mean | 70.3 | 67.8 | 69.3 | 70.9 | 62.1 | 63.5 | 56.0 | 56.0 | 60.5 | 7.5 |
| Std. Dev. | 7.6 | 6.8 | 7.1 | 8.6 | 6.7 | 6.2 | 6.7 | 7.6 | 7.0 | 2.9 |
| *p­*-value | 1.00 | - | 1.00 | 1.00 | 0.44 | 1.00 | 0.09 | 0.26 | 0.58 | 0.09 |
| Note: *Percentage change in MAE and RMSE relative to NaN are calculated as* [MAE (IPMs)- MAE (NaN)]/ MAE (NaN) *and* [RMSE (IPMs)- RMSE (NaN)]/ RSME (NaN) respectively |

**Table 6.** Comparison of the prediction accuracy of different interpolation methods derived from 10 replicates of the cross-validation routine applied to data comprising each of the three spatial patterns

|  |  |
| --- | --- |
| **Error measure** | **Intrpolation method** |
| **T2R** | **NaN**  | **OK**  | **EBK**  | **IDW**  | **RBF** | **LPI** | **UK**  | **SI** | **TSA** |
| **Pattern A** **(Data along the road)** |  **ME** | 0.18 | 0.14 | 0.27 | 0.22 | 0.43 | 0.65 | 0.12 | 0.18 | 0.30 | 1.40 |
| **MAE** | 0.39 | 0.49 | 0.45 | 0.42 | 0.67 | 0.88 | 0.57 | 0.71 | 0.51 | 2.98 |
| **RMSE** | 0.54 | 0.66 | 0.64 | 0.60 | 0.89 | 1.16 | 0.79 | 0.95 | 0.78 | 3.66 |
| **R2** | 0.99 | 0.97 | 0.99 | 0.99 | 0.98 | 0.97 | 0.99 | 0.98 | 0.98 | 0.66 |
| Accuracy order | T2R > EBK> OK> NaN > SI > LPI > IDW> UK> RBF > TSA |
| **Pattern B****(Up-gradient points)** |  **ME** | 0.31 | 0.32 | 0.42 | 0.33 | 0.59 | 0.91 | 0.19 | 0.33 | 0.35 | 1.86 |
| **MAE** | 0.46 | 0.51 | 0.53 | 0.48 | 0.79 | 1.08 | 0.62 | 0.74 | 0.52 | 3.00 |
| **RMSE** | 0.61 | 0.67 | 0.71 | 0.66 | 0.98 | 1.31 | 0.85 | 1.00 | 0.72 | 3.66 |
| **R2** | 0.99 | 0.98 | 0.99 | 0.99 | 0.99 | 0.98 | 0.99 | 0.99 | 0.99 | 0.77 |
| Accuracy order | T2R > EBK> NaN > OK> SI > LPI > IDW> UK> RBF > TSA |
| **Pattern C****(Two-flow directions with** **a common turning point)** |  **ME** | 0.02 | -0.12 | 0.05 | -0.01 | 0.17 | 0.59 | -0.20 | -0.18 | 0.13 | 0.72 |
| **MAE** | 0.29 | 0.43 | 0.30 | 0.27 | 0.50 | 0.93 | 0.39 | 0.48 | 0.41 | 1.43 |
| **RMSE** | 0.35 | 0.55 | 0.37 | 0.35 | 0.65 | 1.39 | 0.56 | 0.64 | 0.62 | 2.14 |
| **R2** | 1.00 | 0.98 | 1.00 | 1.00 | 0.99 | 0.96 | 0.99 | 0.99 | 0.99 | 0.92 |
|  | Accuracy order | EBK>T2R > OK> NaN > LPI > SI > UK>IDW> RBF > TSA |

**Table 7.** Performance of Natural Neighbour (NaN) interpolation method relative to other interpolation methods in terms of percentage change in RMSE relative to NaN for different spatial patterns, and percentage of predictions that are within 0.5m of the observed value for each interpolation method for different spatial patterns

|  |  |
| --- | --- |
| **Error Assessment**  | **Interpolation Method** |
| **T2R** | **NaN** | **OK** | **EBK** | **IDW** | **RBF** | **LPI** | **UK** | **SI** | **TSA** |
| **% Change RMSE** | **Pattern A** | -17.7 | - | -2.5 | -8.6 | 36.0 | 76.5 | 20.7 | 43.9 | 18.4 | 456.7 |
| **Pattern B** | -7.9 | - | 6.5 | -0.7 | 47.4 | 96.7 | 27.4 | 49.3 | 8.0 | 447.2 |
| **Pattern C**  | -36.9 | - | -32.8 | -36.9 | 17.8 | 152.1 | 0.6 | 15.2 | 12.4 | 286.8 |
| **% Errors < 0.5m** | **Pattern A** | 73.5 | 67.9 | 64.7 | 67.6 | 50 | 44.1 | 64.7 | 50 | 73.5 | 5.9 |
| **Pattern B**  | 65.4 | 64 | 57.7 | 61.5 | 38 | 30.8 | 61.5 | 50 | 69.2 | 0 |
| **Pattern C**  | 88 | 70 | 80 | 84 | 60 | 44 | 76 | 68 | 76 | 44 |

**Table 8.** Maximum and Minimum amount of error derived from analysis of total available data and three spatial patterns

|  |  |
| --- | --- |
| **Error Assessment**  | **Interpolation Method** |
| **T2R** | **NaN** | **OK** | **EBK** | **IDW** | **RBF** | **LPI** | **UK** | **SI** | **TSA** |
|  a: (P-O)[m](full data set) | Max. | 2.42 | 2.80 | 4.47 | 3.08 | 3.87 | 4.48 | 2.60 | 4.06 | 23.36 | 12.18 |
| Min. | -3.01 | -3.52 | -3.71 | -4.18 | -3.50 | -4.29 | -7.84 | -8.65 | -12.53 | -12.40 |
|  b: (P-O)[m](Pattern A) | Max. | 1.99 | 1.48 | 1.97 | 1.97 | 2.15 | 2.89 | 2.01 | 2.31 | 2.94 | 7.48 |
| Min. | -0.80 | -1.21 | -0.82 | -0.84 | -0.69 | -1.02 | -1.89 | -1.78 | -1.21 | -3.14 |
|  c: (P-O)[m](Pattern B) | Max. | 2.01 | 1.63 | 1.97 | 1.97 | 2.15 | 2.89 | 2.09 | 2.31 | 2.18 | 6.98 |
| Min. | -0.68 | -0.84 | -0.64 | -0.65 | -0.57 | -1.02 | -2.07 | -1.78 | -0.56 | -2.58 |
|  d: (P-O)[m](Pattern C) | Max. | 0.72 | 0.70 | 0.75 | 0.72 | 1.75 | 4.20 | 0.56 | 0.80 | 2.17 | 4.62 |
| Min. | -0.81 | -1.21 | -0.82 | -0.84 | -0.69 | -1.00 | -1.89 | -1.78 | -1.18 | -3.70 |
| *Note: Maximum and minimum difference between observed and predicted values;* ***a:*** *in all replicates using full data set;* ***b:*** *along the road data points;* ***c:*** *up-gradient data points;* ***d:*** *two flow directions with a common turning point data.* |