**THE EMPIRICAL CAPM:
ESTIMATION AND
IMPLICATIONS FOR THE
REGULATORY COST OF
CAPITAL**

by

Wei Zhang

A thesis
submitted to the Victoria University of Wellington
in fulfilment of the
requirements for the degree of
Master of Commerce and Administration
in Finance
Victoria University of Wellington
2008

Abstract

In calculating the cost of capital for regulated businesses, the New Zealand Commerce Commission uses the Capital Asset Pricing Model to estimate the cost of the equity component of capital, a procedure that involves assuming particular values for unobservable key parameters. This thesis proposes, instead, to estimate these parameters from market data. The principal result is that estimates of these parameters differ significantly from the values assumed by the Commerce Commission. Applying these estimates to two recent cases involving the electricity line and gas pipeline businesses, the estimated costs of capital for the entities involved are 3.5% to 5.5% more than those obtained by the Commission, but the associated confidence intervals are wider. One implication of these findings is that the Commissions approach systematically understates the uncertainty surrounding cost of capital estimates.

Acknowledgement

I am greatly indebted to my supervisors *Prof* Glenn Boyle and *Prof* Graeme Guthrie for their consistent guiding and encouraging throughout the entire master year.

I thank Ian Smith from Investment Research Group Ltd for providing the New Zealand stock price database, and thank Charlotte Clements and Melanie Mills from the library of the Victoria University of Wellington for facilitating the transferring of the database.

Contents

1	Introduction	1
2	Review and Methodology	10
2.1	A review on the methodology for estimating the empirical form of the CAPM	10
2.1.1	The FM method using the estimated <i>stock</i> betas as the independent variable (FM SB)	12
2.1.2	The FM method using the estimated <i>portfolio</i> betas as the independent variable (FM PB)	12
2.2	Methodology in detail	15
2.2.1	A detailed description of the implementation of the FM method using stock betas as the independent variable	15
2.2.2	A detailed description of the implementation of the FM method using portfolio betas as the independent variable	18

<i>CONTENTS</i>	iii
3 Data for estimating the empirical CAPM	22
3.1 Stock return data	22
3.2 Proxies for the market portfolio and the risk free rate	26
3.3 Market capitalization data	32
4 Estimation Results	35
4.1 Estimation results: the FM method with the stock betas (SB) as the independent variable (FM SB)	35
4.2 Estimation results: the FM method with the portfolio betas (PB) as the independent variable (FM PB)	48
4.2.1 The baseline case result: monthly data with the OLS beta-estimation method	48
4.2.2 Robust check I: monthly data with the SW beta-estimation method	56
4.2.3 Robust check II: the FM method with portfolio betas as the independent variable with the weekly data	65
4.3 A discussion on all estimation results	73
5 The implication of the empirical CAPM	77
5.1 Estimating the WACC using the empirical CAPM	77
5.2 Two case studies	79
6 Conclusion	92
7 Appendix	94

List of Figures

3.1	NZSE-40, NZX-50 and NZX All Index from 1990 to 2006 . . .	27
3.2	The 10-year government bond yield and the approximate monthly capital gain returns on the bond from 1990 to 2006 at the monthly frequency	30
4.1	The Histogram of $\hat{\gamma}_{i,t}$ estimated where the OLS method is used to estimate stock betas	39
4.2	The plot of the empirical and theoretical CAPM – FM SB: OLS and SW	47
4.3	The Histogram of $\hat{\gamma}_{i,t}$ – FM PB: OLS+OLS	50
4.4	The plot of the empirical and theoretical CAPM – FM PB: OLS+OLS	54
4.5	The plot of the empirical and theoretical forms of the CAPM – FM PB: SW+SW and SW+OLS	64
4.6	The plot of the empirical and theoretical CAPM – a sum- mary (weekly data)	72

5.1	The distribution of the WACC calculated using the theoretical and empirical CAPM	89
7.1	The Histogram of $\hat{\gamma}_{i,t}$ estimated where the SW method is used to estimate stock betas	96
7.2	The plot of the empirical and theoretical forms of the CAPM – a summary (with an alternative risk free rate)	102
7.3	The distribution of the WACC calculated using the theoretical and empirical CAPM (with an alternative risk free rate)	107

List of Tables

2.1	An illustration of the work mechanism of the FM method using stock betas as the independent variable	17
2.2	An illustration of how to calculated portfolio returns	19
2.3	An illustration of the data panel for estimating $\gamma_{i,t}$	21
3.1	The Number of stocks with returns available at each month from Jan-1990 to Dec-2006	23
3.2	Thin trading problem illustration from 1990 to 2006	25
3.3	Summary statistics of proxies for R_m and R_f	31
3.4	Market capitalization data: large companies dominate the New Zealand Stock Exchange	34
4.1	Stock betas estimated using the OLS method with monthly data	37
4.2	The summary of γ_i^* – FM SB: OLS and SW	41

4.3	The 95% confidence interval of the estimated annual tax-adjusted excess returns for two special securities: $\beta = 0$ and 1 – FM SB: OLS and SW	44
4.4	The summary of portfolio betas estimated using the OLS method	49
4.5	The summary of γ_i^* – FM PB: OLS+OLS	51
4.6	The 95% confidence interval of the estimated annual tax-adjusted excess returns for two special securities: $\beta = 0$ and 1 – FM PB: OLS+OLS	53
4.7	The summary of γ_i^* – FM SB: OLS and SW; FM PB: OLS+OLS, SW+SW and SW+OLS	58
4.8	The 95% confidence interval of the estimated annual tax-adjusted excess returns for two special securities: $\beta = 0$ and 1 – a summary	61
4.9	The summary of γ_i^* – FM PB: OLS+OLS, SW+SW and SW+OLS (weekly data)	68
4.10	The 95% confidence interval of estimated annual tax-adjusted excess returns for two special securities: $\beta = 0$ and 1 – a summary (weekly data)	70
4.11	The summary of all methodologies and data used for estimating the empirical CAPM	74
5.1	The summary of the annual cost of equity and its standard errors esitmated using the empirical CAPM	81
5.2	The WACC and its 95% confidence interval for the electric-ity line business estimated using the empirical CAPM	84

5.3	The probability of the underestimation of the Commission's WACC: case study 1	85
5.4	The WACC and its 95% confidence interval for gas pipeline businesses estimated using the empirical CAPM	87
5.5	The probability of the underestimation of the Commission's WACC: case study 2	90
7.1	Stock betas estimated using the SW method with monthly data	95
7.2	The summary of portfolio betas estimated using the combination of the SW and OLS method	97
7.3	Stock betas estimated using the OLS and SW methods with weekly data	98
7.4	The summary of portfolio betas estimated using combinations of the SW and OLS methods with weekly data	99
7.5	The summary of γ_i^* – FM SB: OLS and SW; FM PB: OLS+OLS, SW+SW and SW+OLS (with the alternative risk free rate)	100
7.6	The 95% confidence interval of the estimated annual tax-adjusted excess returns for two special securities: $\beta = 0$ and 1 – a summary (with the alternative risk free rate)	101
7.7	The WACC of electricity line businesses and gas pipeline businesses estimated using the empirical CAPM – a summary (with an alternative risk free rate)	105
7.8	The probability of the underestimation of the Commission's WACC for both studies – a summary (with alternative risk free rate)	106

Chapter 1

Introduction

The New Zealand Commerce Commission (the Commission) is a competition regulatory agency, whose main duty is to promote a competitive market environment for businesses, so that consumers can benefit from lower prices, better quality products and a greater range of choices. One of the Commission's ways of achieving this goal is to impose controls over the supply of specified goods and services. Under *Commerce Act 1986*, the "specified goods and services" refer to electricity lines and gas pipelines, while under *Dairy Industry Restructuring Act 2001* and *Telecommunications Act 2001*, they refer to raw milk and telecommunication services.

The cost of capital is one of the key parameters that the Commission uses to impose the control on regulated firms and industries. For electricity line businesses, the cost of capital is used as a screening mechanism identifier for businesses' future performance and a tool for assessing returns (Commerce Commission (2005a)). For gas pipeline businesses and telecommunication service providers, it assists with the calculation of the authorised

prices and revenues. For raw milk provider(s) such as Fonterra, it is used as a discount rate to determine the company's share price and, the supply and pricing of the raw milk.

The Commission regards firms' cost of capital as an important multifunctional decision making tool for regulating businesses. Setting an appropriate cost of capital for regulated businesses is closely related to the development of the New Zealand economy. If the cost of capital is set too high for a business, it is very likely that this business will attract excess funds, which may reduce investments for other types of businesses, hence, results an unevenly developed economy. Moreover, as the cost of capital is an important price setting parameter, a high value will increase the price of products, and consequently harms consumers. On the other hand, if the cost of capital is set too low, the reverse may happen. A low allocated cost of capital will discourage investments, which may lead to the capital shortage in the long-run. The supply side may shrink due to the capital shortage. With the demand side remaining, the price will increase and consumers will be worse off.

Given the importance of the role played by the cost of capital, the Commission has released a draft guideline which specifically explains the method of its calculation (Commerce Commission (2005a)). The "capital", according to the guideline, refers to the financial resources invested in a business or a project with a delayed payback and there are two formats, which are the debt capital and the equity capital. In order to incorporate the cost of both capitals to determine the overall cost of capital for a firm, the Commission uses the weighted average cost of capital (WACC), and it is for-

ulated as

$$\text{WACC} = R_e(1 - L) + R_d(1 - T_c)L, \quad (1.1)$$

where

L is the financial leverage ratio

R_d is the cost of debt

R_e is the cost of equity capital

T_c is the corporate tax rate

There are two types of components in (1.1) – observable components and unobservable ones. The observable ones are the financial leverage ratio, the corporate tax rate and the cost of debt¹. The cost of equity is, on the other hand, unobservable, for whose estimation the Commission uses the capital asset pricing model (CAPM). The original version of the model is developed by Sharpe (1964) and Lintner (1965), which is formulated as

$$E(R_e) = R_f + \beta_e[E(R_m) - R_f], \quad (1.2)$$

where

β_e is the equity beta

R_f is the risk free rate

$E(R_m)$ is the expect return on the market portfolio

¹Strictly speaking, the cost of debt is not directly observable, i.e, it is the expected rate of return (which is unobservable), not the promised rate of return on bonds (which is what is used in practice). However, the difference between the two is likely to be small given normal default probabilities.

To better suit the tax regime in New Zealand, the Commission adopts a simplified tax adjusted version of CAPM, known as the “Brennan-Lally model”, based on Brennan (1970) and developed by Lally (1992) and Cliffe and Marsden (1992)², which is presented as

$$E(R_e) = R_f(1 - T_I) + \beta_e[E(R_m) - R_f(1 - T_I)] , \quad (1.3)$$

where T_I is the average (across equity investors) of their marginal tax rates on ordinary income while other components remain.

In essence, the “Brennan-Lally model” employs all the standard CAPM assumptions – such as one-period mean-variance investors having rational expectations, – but takes account of personal tax rates that differ across both investors and sources of income, and adjusts for the effect of any imputation credits attached to dividends. Most importantly, both models assert that the company beta is the only risk that is priced.

The Commission calculates the WACC in a straightforward way. First, it observes, approximates or estimates the parameters appearing in (1.1) – R_f , β_e , T_I , and the market risk premium. Second, it substitutes these estimates into (1.3) to yield the estimated cost of equity. Thirdly, it substitutes this cost of equity together with other relevant variables into (1.1) in order to arrive at an estimate of the cost of capital³. Although a standard error

²This simplified version assumes that dividends are fully imputed and investors have the ability to fully utilise them; the average investor faces a marginal tax rate on interest (currently of 33%); and that capital gains are not taxed. The model also assumes that domestic equity markets are closed to foreign investors

³For example, in the Commission’s report on estimating the WACC for electricity line businesses (Commerce Commission (2005b)), the relevant parameter values are $R_f =$

for this estimate is reported, this only attempts to take into account of possible errors in the input variables and ignores the potential for specification error in the model used to estimate the cost of equity.

In estimating the cost of equity, the Commission uses a “strict” version of the CAPM. That is, it assumes not only that beta is the only priced risk and that the relationship between beta and expected return is linear, but also that the intercept of this relationship is equal to the riskfree rate of interest (or zero if the expected return variable is expressed in excess terms) and the slope equal to the market risk premium. Clearly, any or all of these assumptions may be violated in practice.

Many studies have directly tested the “strict” CAPM (e.g., Black et al. (1972), Fama and MacBeth (1973), Banz (1981), BASU (1983), Fama and French (1992), Kan and Zhang (1999) and Bryant and Eleswarapu (1997)). Testing the CAPM involves estimating the regression model

$$\tilde{R}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + e_i, \quad (1.4)$$

where

$$\tilde{R}_i = R_i - R_f \text{ is the excess return of security } i$$

6.3%, $MRP_{\text{tax-adjusted}} = 7\%$, $\beta_e = 0.67$, $T_I = 33\%$, $T_c = 33\%$, $L = 40\%$ and $k_d = 7.3\%$. By applying (1.3), the cost of equity is calculated as

$$\hat{k}_e = 6.3\%(1 - 33\%) + 0.67 \times 7\% = 8.9\%$$

The cost of capital is then computed applying (1.1)

$$\widehat{WACC} = 8.9\%(1 - 40\%) + 7.3\%(1 - 33\%)40\% = 7.29\%$$

$\hat{\beta}_i$ is the estimated beta of security i

and testing the following hypotheses:

$$H_0 : \gamma_0 = 0$$

$$H_0 : \gamma_1 = E(R_m) - R_f$$

These hypotheses have been consistently rejected. Black et al. (1972) and Fama and MacBeth (1973) found no significantly positive empirical relationship between systematic risks and returns. Fama and French (1992) went further and found that returns are unrelated to beta once other firm-specific features, such as size and market-to-book, are controlled for. Using New Zealand data, Bryant and Eleswarapu (1997) obtain similar results. Overall, both hypotheses above appear to be systematically violated.

There are several possible reasons for the rejection of the null hypotheses. It could be due to the failure of the CAPM itself for some of the unrealistic assumptions (e.g., perfectly efficient capital market, existence of risk free rate and investors homogeneous and rational expectations) are unable to stand up in a real world situation. It also could be that the CAPM may not be testable. As Roll (1977) pointed out, given the true market portfolio is not observable, tests on the CAPM are actually assessing whether the proxy for the market portfolio is mean-variance efficient or not.

Whatever the exact reason or reasons for the failure of the strict form of the CAPM to hold empirically, its rejection must cast some doubt on its usefulness for cost of capital calculations. In this thesis, I investigate this issue in the following way: rather than impose the requirements that $\gamma_0 = 0$ and $\gamma_1 = E(R_m) - R_f$, I instead estimate γ_0 and γ_1 from market data, thus

obtaining an “empirical” form of the CAPM (the terminology I use to distinguish my approach from the theoretical, or “strict”, form). That is, I estimate:

$$\tilde{R}_i = \gamma_0^* + \gamma_1^* \beta_i, \quad (1.5)$$

where, in order to maintain comparability with the Commission

$$\tilde{R}_i = R_i - (1 - T_I)R_f$$

γ_0^* and γ_1^* are the respective market estimates of γ_0 and γ_1 . I then use this empirical form to obtain a cost of equity estimate (and subsequently a WACC) and compare this with the corresponding estimate obtained from the strict form. That is, I calculate

$$\hat{R}_e = (1 - T_I)R_f + \gamma_0^* + \gamma_1^* \beta_e$$

$$\widehat{WACC} = \hat{R}_e(1 - L) + R_d(1 - T_c)L$$

When calculating the $\widehat{WACC}$, I assume all components except for $\hat{R}_e$ can be estimated without error. The reason of doing so is for better examination of the effect solely arising from the use of the “empirical” form of the CAPM.

Note that this approach is not fundamentally different to that followed by the Commission, but is instead simply an alternative, and more general, way of estimating parameters that the insights of the CAPM suggest are important. For example, the Commission sets γ_1 equal to the market risk premium and then proceeds to estimate this latter variable from historical data; my approach essentially cuts out the intermediate step - I estimate γ_1 directly from data without first requiring it to be equal to the market risk

premium. Of course, if $\gamma_0^* = 0$ and $\gamma_1^* = E(R_m) - R_f(1 - T_I)$, the empirical CAPM collapses to its theoretical “Brennan-Lally” version, but not otherwise.

Using the empirical CAPM provides an alternative mean of estimating the cost of capital and its standard error. Even if the CAPM actually holds, applying the empirical model does not contradict with the Commission’s approach which has to estimate parameter values regardless. Moreover, it proposes a conventional econometric way of calculating the standard error of the cost of equity. The WACC of regulated businesses calculated using this approach will reflect the rate of return required by the market since the market price on systematic risks is estimated instead of being appointed.

The contents in this thesis are organised as follows. Chapter 2 reviews methods for estimating the empirical CAPM, followed by detailed descriptions of how these methods are implemented. Chapter 3 describes the dataset used, which are daily stock price data, the NZX All (ordinary shares) Index as a proxy for the market portfolio, the 10 year government bond yield from Reserve Bank of New Zealand as a proxy for the risk free rate and the market capitalization data; the duration for all data is from Jan-1990 to Dec-2006.

Chapter 4 presents all estimation results of the empirical CAPM from different methods. Chapter 5 applies the estimated empirical model to recalculating the cost of capital for two Commission’s reports – Commerce Commission (2005b) and Commerce Commission (2007). By assuming all components in the WACC formula (1.1) can be estimated without errors

except for the cost of equity, I find that the cost of capital reported from the Commission's studies for electricity line and gas pipeline businesses are underestimated by approximately 3.5% and 5.5% respectively, and the probabilities of such underestimation are, considerably high, 95% and 90% respectively. The last chapter concludes key findings and suggests that using the CAPM based WACC to calculate firms' cost of capital should be a starting point and a bench mark, but not the final answer for decision makers.

Review and Methodology

2.1 A review on the methodology for estimating the empirical form of the CAPM

The cross-sectional regression developed by Fama and MacBeth (1973) has become a standard approach for dealing with financial panel data, which is adopted in this paper to estimate the empirical form of the CAPM (1.5). The fundamental idea of their method is to regress returns on the corresponding systematic risks for each cross section, then aggregate estimates for all periods. The regression model at the time period t of N securities is

$$\tilde{R}_t = \gamma_{0,t} + \gamma_{1,t}\beta + e_t, \quad (2.1)$$

where

$\tilde{R}_t$ is the $(N \times 1)$ vector of tax-adjusted excess returns at period t

β is the $(N \times 1)$ vector of systematic risks

There are two steps to execute the Fama-MacBeth (FM, hereafter) method. The first step is, given T periods of data, to use the ordinary least square (OLS) method to estimate (2.1) for each t from 1 to T , which will yield T estimates of $\gamma_{i,t}$. The second step is to aggregate them to yield the intercept and slope coefficients of the empirical form of the CAPM. That is

$$\gamma_i^* = \frac{1}{T} \sum_{t=1}^T \hat{\gamma}_{i,t} \quad (2.2)$$

The variance is given by

$$\hat{\sigma}^2(\gamma_i^*) = \frac{1}{T(T-1)} \sum_{t=1}^T (\hat{\gamma}_{i,t} - \gamma_i^*)^2 \quad (2.3)$$

Finally, the empirical form of the CAPM can be expressed as

$$\tilde{R}_i = \gamma_0^* + \gamma_1^* \beta_i, \quad (2.4)$$

where $\tilde{R}_i$ is the estimated tax-adjusted excess return for security i and β_i is the systematic risk. Assuming the systematic risk can be estimated without error, the standard error of $\tilde{R}_i$ can be calculated by

$$\hat{\sigma}(\tilde{R}_i) = \sqrt{\hat{\sigma}^2(\gamma_0^*) + \beta_i^2 \hat{\sigma}^2(\gamma_1^*) + 2\beta_i \hat{cov}(\gamma_0^*, \gamma_1^*)}, \quad (2.5)$$

where $\hat{cov}(\gamma_0^*, \gamma_1^*) = \frac{\hat{cov}(\hat{\gamma}_{0,t}, \hat{\gamma}_{1,t})}{T}$.

An unavoidable problem of the FM method is that the independent variable, systematic risks, in (2.1) is not observable, which has to be estimated. One approach is to use the estimated stock betas as the independent variable. However, the estimated stock betas are likely to possess large estimation errors. For example, the thin trading problem will bias the OLS estimators of the stock betas downwards (Scholes and Williams (1977)).

To minimize errors in beta estimates, the method adopted by Fama and MacBeth (1973) is applied. They grouped stocks into portfolios in order to increase the precision of beta estimates. As they pointed out, given an estimated portfolio beta can be expressed as a weighted sum of estimated stock betas, as long as the errors in stock beta estimates are less than perfectly positively correlated, portfolio betas can be more precisely estimated than stock betas.

2.1.1 The FM method using the estimated *stock* betas as the independent variable (FM SB)

Using stock betas as the independent variable to estimate γ_i^* involves three steps. The first step is to estimate stock betas. The second step is to use the OLS method on (2.1) to produce time series estimates of $\gamma_{i,t}$. The last step is to calculate the intercept and slope coefficients, and their variances of the empirical CAPM using (2.2) and (2.3) respectively. Since implementing the FM method with stock betas as the independent variable can be seen as a subset of the FM with portfolio betas, any technological discussion is left to the next section.

2.1.2 The FM method using the estimated *portfolio* betas as the independent variable (FM PB)

In practise, the implementation of the FM method (where portfolio betas are the independent variable) can be divided into three parts: the first part is to estimate the systematic risk of individual stocks; group corresponding stocks into a certain number of portfolios based on the rank of stock

betas and calculate portfolio returns by averaging stock returns in each portfolio. The second part is to estimate the systematic risk of portfolios by regressing their returns, calculated in the first part, on returns of the market portfolio. The last part is to regress portfolio returns on the estimated portfolio betas to estimate $\gamma_{i,t}$ (2.1), then γ_i^* (2.2). There are several difficulties to be overcome.

How to form portfolios to prevent the loss of information

There is inevitable loss of information when using portfolios instead of individual stocks in estimating the empirical CAPM. To reduce the information loss, Fama and MacBeth (1973) proposed to form portfolios on the basis of ranked values of estimated stock betas. By doing so, the dispersion of estimated portfolio betas are maximized.

Which method to estimate stock betas

Another problem is which method should be used to estimate stock betas so that one can obtain the precise rank and form appropriate portfolios. There are two candidates capable of this job.

The OLS method is the most well-known and the most frequently used method to estimate stock and portfolio betas (among others, Black et al. (1972), Fama and MacBeth (1973) and Fama and French (1992)). Bartholdy and Riding (1994) reported that there is no significant efficiency gain in estimating betas using other methods, than using the OLS method regardless of data frequency. Hence, the OLS method is adopted to estimate stock as a baseline case.

In a contrast view, as most stocks in the New Zealand Stock Market are traded very infrequently, the OLS estimator of stock betas tends to be biased downwards (Scholes and Williams (1977)). To obtain unbiased estimators, Scholes and Williams (1977) proposed an alternative beta estimation model – the SW model. By assuming non-trading periods for a security are distributed independently and identically over time, Scholes and Williams derived a relationship between the true beta and the measured beta¹

$$\beta^s = \beta - (\beta^{-s} + \beta^{+s} - 2\beta\rho_m^s), \quad (2.6)$$

where

^s stands for anything that is related to the measured returns

β is the true beta

$$\beta^s \equiv \frac{\text{cov}(R_t^s, R_{m,t}^s)}{\text{var}(R_{m,t}^s)}$$

$$\beta^{-s} \equiv \frac{\text{cov}(R_t^s, R_{m,t-1}^s)}{\text{var}(R_{m,t-1}^s)}$$

$$\beta^{+s} \equiv \frac{\text{cov}(R_t^s, R_{m,t+1}^s)}{\text{var}(R_{m,t+1}^s)}$$

$$\rho_m^s \equiv \frac{\text{cov}(R_{m,t}^s, R_{m,t-1}^s)}{\text{var}(R_{m,t-1}^s)}$$

After rearranging (2.6), the estimated true beta, $\hat{\beta}_{SW}$, can be presented as

$$\hat{\beta}_{SW} = \frac{\hat{\beta}_{i,-1} + \hat{\beta}_i + \hat{\beta}_{i,+1}}{1 + 2\hat{\rho}_m}, \quad (2.7)$$

¹In practise, prices for (infrequently traded) securities are recorded only at distant random intervals. Hence, returns measured over any fixed sequence of periods are mere proxies for the true returns. The true beta, therefore, refers to the one derived from the true returns whereas the measured beta refers to the one from the proxy of the true returns.

where

$\hat{\beta}_{i,-1}$ is estimated by $\frac{cov(R_i, R_{m,t-1})}{var(R_{m,t-1})}$

$\hat{\beta}_{i,+1}$ is estimated by $\frac{cov(R_i, R_{m,t+1})}{var(R_{m,t+1})}$

$\hat{\beta}_i$ is estimated by $\frac{cov(R_i, R_m)}{var(R_m)}$

$\hat{\rho}_m$ is estimated by $\frac{cov(R_{m,t}, R_{m,t-1})}{var(R_{m,t-1})}$

Bartholdy et al. (1996) has reviewed several beta-estimation methods dealing with thin trading problems². As reported, the SW model performed over other methods. Therefore, the SW model is applied as a robustness check.

2.2 Methodology in detail

2.2.1 A detailed description of the implementation of the FM method using stock betas as the independent variable

The implementation of the FM method using stock betas as the independent variable involves three steps. The first step is to estimate stock betas,

²Among others, two methods were reported to perform outstandingly, which are Scholes-William model by Scholes and Williams (1977) and Aggregate Coefficient model by Dimson (1979). However, the later method has been seriously questioned by Cohen et al. (1983) and Fowler and Rorke (1983). Especially, Fowler and Rorke (1983) has shown that Aggregate Coefficient method is incorrect and the correct version is identical to the model propose by Scholes and Williams (1977).

the second step is to estimate $\gamma_{i,t}$ using the OLS method, and the last step is to calculate γ_i^* and $\hat{\sigma}^2(\gamma_i^*)$. With monthly data (from Jan-90 to Dec-06), the OLS and SW method are adopted to estimate stock betas.

The preliminary step is to estimate stock betas. Fama and MacBeth (1973) used 5 years of monthly data because of their large dataset (from 1926 to 1968, 43 years of monthly data). As only 17 years of monthly (Jan-90 to Dec-06) stock returns are available for this study, I have chosen to use 3 years of data to estimate stock betas instead. More specifically, for the first 3-year period (Jan-90 to Dec-92), stocks with returns available in at least in the latest two years (Jan-91 to Dec-92) and all returns available in next year (Jan-93 to Dec-93), are selected. Then, stock betas are estimated by regressing all available stock returns in the period Jan-90 to Dec-92 on market portfolio returns.

The reasons for selecting stocks in such ways are two-fold. The first condition that stocks must have at least the latest two year returns available, guarantees that there are enough observations to consistently estimate stock betas. The second condition that stocks must have all returns available in next year is to make sure that the cross-sectional regressions (2.1) can be performed.

The second step is to perform the cross-sectional regression (2.1). For each month from Jan-93 to Dec-93, I use the OLS method to regress tax-adjusted stock returns on their estimated betas. That is there are 12 regressions performed from

$$\tilde{R}_{\text{Jan-93}} = \gamma_{0,\text{Jan-93}} + \gamma_{1,\text{Jan-93}}\hat{\beta} + e_{\text{Jan-93}}$$

to

$$\tilde{R}_{\text{Dec-93}} = \gamma_{0,\text{Dec-93}} + \gamma_{1,\text{Dec-93}}\hat{\beta} + e_{\text{Dec-93}}$$

which yields 12 time series estimates of $\gamma_{i,\text{Jan-93 to Dec-93}}$.

The first step and the second step are then repeated on data with the period interval being moved forward for one year until the last period interval – Jan-2003 to Dec-2006. Table 2.1 illustrates the way of repeating the first and the second step for calculating $\hat{\gamma}_{i,t}$. In total, there will be 168 (14 years $\times$ 12 months) time series estimates of $\gamma_{i,t}$ obtained respectively.

Finally, the intercept and slope coefficients of the empirical CAPM are cal-

Table 2.1: An illustration of the work mechanism of the FM method using stock betas as the independent variable

1st loop:	1990 1991 1992	1993	Output
	Estimating stock betas	12 regressions	$\hat{\gamma}_{i,\text{Jan-93 to Dec-93}}$
2nd loop:	1991 1992 1993	1994	
	Estimating stock betas	12 regressions	$\hat{\gamma}_{i,\text{Jan-94 to Dec-94}}$
3rd loop:	1992 1993 1994	1995	
	Estimating stock betas	12 regressions	$\hat{\gamma}_{i,\text{Jan-95 to Dec-95}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
Last loop:	2003 2004 2005	2006	
	Estimating stock betas	12 regressions	$\hat{\gamma}_{i,\text{Jan-06 to Dec-06}}$

Note: There are 14 loops in total, executed to calculate $\gamma_{i,t}$ from Jan-93 to Dec-06.

culated as:

$$\gamma_i^* = \frac{1}{168} \sum_{t=\text{Jan-93}}^{\text{Dec-06}} \hat{\gamma}_{i,t} \quad (2.8)$$

The variance is calculated by:

$$\hat{\sigma}^2(\gamma_i^*) = \frac{1}{168(168-1)} \sum_{t=\text{Jan-93}}^{\text{Dec-06}} (\hat{\gamma}_{i,t} - \gamma_i^*)^2 \quad (2.9)$$

2.2.2 A detailed description of the implementation of the FM method using portfolio betas as the independent variable

For the baseline case, the monthly data is used along with the OLS method to estimate stock and portfolio betas.

The preliminary step is to estimate stock betas. The details of estimation are exactly the same as ones outlined in previous contents (the FM method with stock betas).

The next step is to rank the estimated stock betas in an ascending order and group them into 10 portfolios, e.g. 10% of stocks with lowest beta are grouped into the first portfolio; another 10% with second lowest beta are grouped into the second portfolio, until the last 10% with the highest beta are grouped into the tenth portfolio. Then portfolio returns in the next period (Jan-93 to Dec-93) are computed by *value*-weighting stock returns, e.g., the return of the first portfolio at Jan-93 is calculated by value-weighting stock returns in the first portfolio at Jan-93, and so forth until Dec-93.

By repeating the process above, except for moving one year forward, the

Table 2.2: An illustration of how to calculated portfolio returns

1st loop:	1990	1991	1992	1993	Output
	Estimating stock betas			Calculating R_p	$R_{p,Jan-Dec-93}$
2nd loop:	1991	1992	1993	1994	
	Estimating stock betas			Calculating R_p	$R_{p,Jan-Dec-94}$
3rd loop:	1992	1993	1994	1995	
	Estimating stock betas			Calculating R_p	$R_{p,Jan-Dec-95}$
$\vdots$	$\vdots$			$\vdots$	$\vdots$
Last loop:	2003	2004	2005	2006	
	Estimating stock betas			Calculating R_p	$R_{p,Jan-Dec-06}$

Note: There are 14 loops in total, executed to calculate 10 portfolio returns from Jan-93 to Dec-06

portfolio returns from Jan-94 to Dec-94 are computed. That is: for the period of Jan-91 to Dec-93, stocks with returns at least available from Jan-92 to Dec-93 and all returns available for Jan-94 to Dec-94 are selected; then using all available stock returns from Jan-91 to Dec-93 to estimate stock betas; 10 portfolios are formed and portfolio returns are calculated from Jan-94 to Dec-94 by value-weighting stock returns at the same period. By continuing to repeat these calculations, 10 portfolio returns from Jan-93 to Dec-06 are calculated. Table 2.2 offers an intuitive illustration of how portfolio returns are calculated for the entire period.

Next, 10 portfolio betas are estimated by using all portfolio returns from

Jan-93 to Dec-06 for each portfolio. There are 10 time series regressions executed. For each regression, the OLS method is used to regress the portfolio returns from Jan-93 to Dec-06 on the market portfolio returns from the same period. 10 portfolio betas are estimated.

These portfolio betas are incorporated into the next step – estimating $\gamma_{i,t}$. However, they cannot be estimated by regressing portfolio returns on the estimated portfolio betas directly as this would give a single regression with just ten observations. On the other hand, Fama and MacBeth (1973) regressed stock returns on the portfolio betas. This alteration was also carried out by Bryant and Eleswarapu (1997).

More specifically, for each month from Jan-93 to Dec-06, stocks in each portfolio are assumed to have their betas equal to the portfolio beta. Table 2.3 illustrates this idea. There are two panels. The first is the stock beta panel. For each month from Jan-93 to Dec-06, all stocks used to form 10 portfolios are assumed to have their betas equal to corresponding portfolio betas. For example, as the table shows, stock A is in the fourth portfolio in 1993 so its beta is assumed to equal the fourth portfolio's beta during that year. As the composition of the 10 portfolios changes yearly, the beta may change for the same stock. For instance, stock A's beta is assumed to equal the third portfolio's beta in 1994. The second panel is the stock return data, which consists of all the corresponding stock returns for each month from Jan-93 to Dec-06. For each month, stock returns are regressed on the assumed stock betas (the portfolio betas) to estimate $\hat{\gamma}_{i,t}$.

Table 2.3: An illustration of the data panel for estimating $\gamma_{i,t}$

Stock:	β				Excess Return			
	A	B	C	...	A	B	C	...
Jan-93	β_{p4}	β_{p2}	β_{p7}	...	R_{Jan-93}^A	R_{Jan-93}^B	R_{Jan-93}^C	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
Dec-93	β_{p4}	β_{p2}	β_{p7}	...	R_{Dec-93}^A	R_{Dec-93}^B	R_{Dec-93}^C	...
Jan-94	β_{p3}	β_{p5}	β_{p6}	...	R_{Jan-94}^A	R_{Jan-94}^B	R_{Jan-94}^C	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
Dec-94	β_{p3}	β_{p5}	β_{p6}	...	R_{Dec-94}^A	R_{Dec-94}^B	R_{Dec-94}^C	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
Jan-06	β_{p3}	β_{p5}	β_{p8}	...	R_{Jan-06}^A	R_{Jan-06}^B	R_{Jan-06}^C	...
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	
Dec-06	β_{p3}	β_{p5}	β_{p8}	...	R_{Dec-06}^A	R_{Dec-06}^B	R_{Dec-06}^C	...

Note: This table is provided simply to illustrate the full version of the FM method. For each year, there are not necessarily the same stocks in both panels. For example, the stock A may be delisted in 1994. If that is the case, it will no longer be in the stock beta and return panel in 1994.

There are in total 168 regressions (14 years $\times$ 12 months) performed from Jan-93 to Dec-06, which yield time series estimates of $\gamma_{i,t}$ from Jan-93 to Dec-06. Finally, the γ_i^* and their variances are calculated in the same way as discussed previously.

Data for estimating the empirical CAPM

3.1 Stock return data

The stock price data has been provided by Investment Research Group Ltd (IRG Ltd) which is known as the IRG “Price Histories” Database. The IRG’s stock database includes adjusted price data for all ordinary stocks (listed and delisted) from 3-Jan-1990 to 29-Dec-2006 on a daily basis. Daily stock returns are calculated by

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1} + D_{i,t}}{P_{i,t-1}}, \quad (3.1)$$

where $D_{i,t}$ denotes for a dividend paid on a security i at date t . Weekly and monthly stock returns are calculated by accumulating daily returns

$$R_{i,T} = \prod_{t=1}^N (1 + R_{i,t}) - 1, \quad (3.2)$$

where N is the number of days in a week and a month respectively.

Table 3.1: The Number of stocks with returns available at each month from Jan-1990 to Dec-2006

The number of stocks with returns available at each month is presented in this table, however, these numbers may not reflect the exact number of stocks listed in the New Zealand Stock Exchange at the corresponding time since there are a small number of stocks missing from the IRG database.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1990	27	74	80	81	82	83	83	86	86	86	86	86
1991	86	86	86	86	86	86	87	89	89	89	89	89
1992	90	90	90	90	92	95	96	97	98	99	102	102
1993	102	102	102	102	102	103	104	105	106	110	116	116
1994	122	122	122	125	126	127	128	131	131	131	133	134
1995	137	137	137	137	137	138	138	138	138	138	138	137
1996	137	137	135	138	135	133	133	133	132	132	132	131
1997	132	131	131	132	132	133	134	135	136	136	136	136
1998	139	138	137	137	137	136	139	142	142	142	142	141
1999	141	142	141	141	141	140	141	139	136	136	137	137
2000	135	135	135	137	140	141	142	139	139	139	139	144
2001	148	146	146	146	146	146	144	142	140	141	139	138
2002	139	139	139	138	139	140	140	141	139	138	136	135
2003	137	137	138	137	137	138	137	138	137	139	140	143
2004	144	144	144	146	146	146	148	151	150	152	153	154
2005	157	157	156	155	154	150	148	149	149	147	147	147
2006	147	147	144	143	143	142	141	141	138	138	139	138

The New Zealand Stock Exchange is far smaller than many others, e.g., the New York Stock Exchange. Table 3.1 shows that the average number of stocks in more recent years (2004-2006) is approximately 140, and only roughly 90 in the early period (1990-1992). In Fama and MacBeth (1973), the stock data from the New York Stock Exchange was used. The number of stocks in 1926 was 710 while it grew to 1261 in 1968.

Thin trading is another feature of the New Zealand Stock Market, which is illustrated in Table 3.2. Each cell in the table represents the percentage of stocks that has been traded more than a given percentage of business days (>50%, etc) in a given year (1990 to 2006). On average, only 51.4% of stocks from 1990 to 2006 has been traded on more than 50% of trading days while no more than 3% of stocks has been traded on a rough daily basis (> 90%). In other words, 50% of stocks from 1990 to 2006 has been traded for no more than 3 days each week.

Table 3.2: Thin trading problem illustration from 1990 to 2006

For each cell in this table, the number is calculated by dividing the number of stocks that have been traded on more than a certain percentage of business days by the total number of stocks in a given year. For example, 18.6% in the first row and the first column reads 18.6% of all stocks has been traded on more than 50% of business days in the year 1990.

Year	Percentage of business days				
	>50%	>60%	>70%	>80%	>90%
1990	18.6%	14%	9.3%	7%	2.3%
1991	20%	15.6%	12.2%	6.7%	0%
1992	49%	39.2%	23.5%	5.9%	0%
1993	64.8%	54.1%	39.3%	10.7%	3.3%
1994	57.7%	40.1%	23.4%	8.8%	3.6%
1995	42.6%	27%	14.2%	5.7%	1.4%
1996	47.7%	32.2%	18.1%	7.4%	2.7%
1997	54.5%	38.6%	26.9%	10.3%	4.1%
1998	52.7%	37.8%	26.4%	12.8%	6.8%
1999	45%	33.8%	13.9%	4.6%	0%
2000	54.1%	38.2%	17.8%	4.5%	1.3%
2001	62%	43.7%	32.9%	15.2%	4.4%
2002	56.1%	42.6%	30.4%	15.5%	2.7%
2003	61.5%	43.9%	29.7%	18.2%	4.7%
2004	63.2%	49.1%	28.8%	16%	4.3%
2005	66.5%	51.6%	32.9%	19.3%	3.1%
2006	58.6%	48%	33.6%	22.4%	5.3%
Average	51.4%	38.2%	24.3%	11.2%	2.9%

3.2 Proxies for the market portfolio and the risk free rate

This paper uses the NZX All Index¹ as a proxy for the market portfolio. This index, according to New Zealand Exchange Limited, comprises all domestic equity securities listed on the New Zealand Stock Exchange Market (NZSX), and its constituents are weighted by free float market capitalisation, where “free float market capitalization” refers to the product of the number of shares available to public and the price of a share.

Using the NZX All Index as the proxy for the market portfolio is different from previous studies – Bryant and Eleswarapu (1997) chose the NZSE-40 Index as a proxy for the market portfolio. Reasons for using the NZX All Index are three-fold. First, it is the only index that covers the entire data period (1990 to 2006). The NZSE-40 Index ceased publishing in 2003 while the NZX-50 Index started to publish after 2002. Second, if a combined index of the NZSE-40 and NZX-50 could be used instead, a sudden change in the composition of this index (from 40 to 50 equity securities at a time point) can be problematic. Finally, the NZX All Index includes all equity securities. As Roll (1977) and Roll and Ross (1994) argued that a proxy for the market portfolio should be as comprehensive as possible. For example, Jagannathan and Wang (1996) includes the human capital along with stock data. Although the NZX All Index is far from being a comprehensive proxy for the market portfolio, it is at least not inferior to the NZSE-40, NZX-50 or any combination of them.

¹For more information: http://www.nzx.com/market/price_by_index/ALL

Figure 3.1: NZSE-40, NZX-50 and NZX All Index from 1990 to 2006

The NZX All Index data is also provided by IRG Ltd, which is on a daily basis from 3-Jan-1990 to 29-Dec-2006. Monthly and weekly returns on the NZX All Index are calculated in the same way as for calculating stock returns (3.2). The NZSE-40, NZX-50 and NZX All Index are plotted in Figure 3.1. The solid black curve shows the NZX All Index which is also the only index to cover the entire period from 1990 to 2006. The NZX All Index grew gradually before 2002, and followed by a rapid growth from 2002 onwards.

The 10 year government bond yield is used as a proxy for the risk free rate, which is obtained from the Reserve Bank of New Zealand. The raw

data is provided in two formats. One is the annualized yield in monthly frequency from Jan-1990 to Dec-2006, while the other is in daily frequency from 03-Jan-1990 to 29-Dec-2006. To obtain monthly yields, the equation below is applied to the yield data in the monthly frequency.

$$R_{f,t}^m = (1 + R_{f,t}^y)^{1/12} - 1, \quad (3.3)$$

where $R_{f,t}^m$ is the monthly yield at the month t and $R_{f,t}^y$ is the annal yield at the same month.

The yield data in the daily frequency is used to obtain the weekly yield. Annual returns on the last day of each week are picked, and weekly returns are calculated using the following equation

$$R_{f,t}^w = (1 + R_{f,t}^y)^{1/52} - 1, \quad (3.4)$$

where $R_{f,t}^w$ is the weekly yield at the week t and $R_{f,t}^y$ is the annal yield at the same week.

The bond yield is different from the return on that bond in the sense that the yield is the rate of return for holding a bond to the maturity while the return is what has been realized after selling the bond. Therefore, an alternative way of obtaining a proxy for the risk free rate is to calculate the one-period (monthly) return on the bond. As the coupon payment for the 10 year government bond is on the annual basis, the capital gain rate can be a good approximation of the one-period return (which it will be unless coupons are paid monthly).

In order to calculate the monthly capital gain return on the 10 year government bond, the bond price at each month has to be obtained. Without

introducing large distortions, it is not inappropriate to assume that the 10 year government bond is a perpetuity. Then, the bond price at the month t can be calculated by

$$P_t = \frac{A}{r_t}, \quad (3.5)$$

where A is the coupon payment and r_t is the yield at the month t . Then, the monthly capital gain return of a bond being purchased at the month $t - 1$ and sold at t can then be calculated by

$$\begin{aligned} R_{f,t} &= \frac{P_t - P_{t-1}}{P_{t-1}} \\ &= \frac{\frac{A}{r_t} - \frac{A}{r_{t-1}}}{\frac{A}{r_{t-1}}} \\ &= \frac{r_{t-1}}{r_t} - 1, \end{aligned} \quad (3.6)$$

Figure 3.2 plots two proxies for the risk free rate. The solid black curve is the 10-year government bond yield (LHS) while the gray curve is the approximate annualized monthly capital gain returns on the bond (RHS). Because of the simplifying perpetuity assumption for deriving the capital gain return, a small change in the yield will result in large variations in the capital gain returns.

Table 3.3 provides summary statistics of proxies for the annual market portfolio returns, 10-year government bond yield (R_{f1}) and annualized monthly capital gain returns (R_{f2}). The bond yield decreases gradually from 1990 to 2006 with an average of 7.34% while the return on the bond has a lower average of 5.38%. On the other hand, the return on the NZX All Index varies strongly. Tax-adjusted excess returns on the market portfolio were positive and significant in more recent years (2001 to 2006)

Figure 3.2: The 10-year government bond yield and the approximate monthly capital gain returns on the bond from 1990 to 2006 at the monthly frequency

which signifies the recent bull market. On average, the tax-adjusted excess return on the market portfolio calculated using R_{f1} ($\tilde{R}_{m1} = R_{m,t} - R_{f1,t-1}(1 - T_I)^2$ where T_I is set equal to 33%) equals 6.3% per year and $\tilde{R}_{m2}$ ($\tilde{R}_{m2} = R_{m,t} - R_{f2,t}(1 - T_I)$) averages to 7.6%.

²When calculating the excess returns using the yield data, I have to shift it one period backward. That is because: the yield is forward looking (e.g., $R_{f,t}$ covers the period from t to $t + 1$); on the other hand, the way of calculating the return for securities ($R_{i,t}$) makes it to cover the period from $t - 1$ to t ; therefore, in order to match the period, $R_{m,t} - R_{f1,t-1}(1 - T_I)$ is used.

Table 3.3: Summary statistics of proxies for R_m and R_f

This table shows the summary statistics of proxies for the risk free rate and the return on the market portfolio. R_{f1} and R_{f2} denote the two proxies for the risk free rate, which are the 10-year government bond yield and the annualized monthly capital gain returns on the bond. R_m denotes the proxy for the return on the market portfolio return.

$$\tilde{R}_{m1} = R_{m,t} - R_{f1,t-1}(1 - T_I)$$

$$\tilde{R}_{m2} = R_{m,t} - R_{f2,t}(1 - T_I)$$

T_I is the average (across equity investors) of their marginal tax rates on ordinary income, whose value is assumed to be 33% in order to be in accordance with the Commission's reports.

	R_{f1}		R_{f2}		R_m		$\tilde{R}_{m1}$	$\tilde{R}_{m2}$
	Mean	Std	Mean	Std	Mean	Std		
1990	12.45 %	0.33 %	1.03 %	8.09 %	-37.12 %	18.82 %	-45.46 %	-37.81 %
1991	10.11 %	1.08 %	38.39 %	10.92 %	32 %	15.17 %	25.23 %	6.27 %
1992	8.4 %	0.6 %	15.2 %	9.82 %	15.5 %	20.57 %	9.87 %	5.32 %
1993	6.93 %	0.66 %	29.07 %	8.92 %	53.98 %	18.67 %	49.33 %	34.5 %
1994	7.63 %	1.25 %	-28.63 %	18.47 %	-5.45 %	18.35 %	-10.56 %	13.73 %
1995	7.78 %	0.51 %	20.98 %	11.49 %	20.14 %	10.02 %	14.93 %	6.08 %
1996	7.89 %	0.62 %	1.07 %	17.15 %	19.23 %	10.31 %	13.94 %	18.51 %
1997	7.19 %	0.46 %	4.81 %	15.06 %	3.89 %	14.32 %	-0.93 %	0.67 %
1998	6.29 %	0.57 %	30.16 %	11.89 %	-0.41 %	25.08 %	-4.62 %	-20.62 %
1999	6.41 %	0.62 %	-24.64 %	9.69 %	18.38 %	16.14 %	14.09 %	34.89 %
2000	6.85 %	0.35 %	17.24 %	10.92 %	-8.57 %	11.08 %	-13.16 %	-20.12 %
2001	6.39 %	0.27 %	-6.68 %	10.18 %	18.18 %	16.55 %	13.9 %	22.66 %
2002	6.53 %	0.25 %	5.74 %	7.2 %	4.68 %	10.16 %	0.31 %	0.84 %
2003	5.87 %	0.28 %	4.85 %	15.04 %	27.01 %	9.42 %	23.08 %	23.76 %
2004	6.07 %	0.16 %	1.91 %	7.9 %	28.55 %	9.48 %	24.49 %	27.27 %
2005	5.88 %	0.14 %	2.34 %	7.3 %	9.65 %	11.81 %	5.72 %	8.08 %
2006	5.78 %	0.06 %	1.17 %	3.82 %	19.59 %	12.89 %	15.72 %	18.81 %
Average	7.32 %	1.77 %	5.38 %	12.01 %	11.2 %	15.67 %	6.3 %	7.6 %

3.3 Market capitalization data

The market capitalization data is needed when calculating portfolio returns (value weighting stock returns). The data is provided in the monthly frequency by IRG Ltd, which covers all ordinary stocks (listed and delisted) from Jan-1990 to Dec-2006. To obtain the weekly data, it is assumed that market capitalization for a stock is constant within a given month; then, breaking each month into a certain number of trading days in accordance with the date of stock returns; finally, choosing market values on the last day of each week to form the weekly market capitalization data.

Another feature of the New Zealand Stock Exchange is that large companies dominate the market, which is illustrated in Table 3.4. Each cell of the table represents a percentage of the total market value dominated by a given percentage of largest companies listed on the New Zealand Stock Market in a certain year. It shows that 5% of companies take up approximately 53% of the market on average from 1990 to 2006. Although the percentage of the market dominated by large companies has dropped in more recent years, there is still approximately 90% of the market that are dominated by only 30% of trading companies each year from 1990 to 2006.

Because of two features of the New Zealand Stock Exchange – the small number of stocks and the dominance of large companies – forming portfolios on a value-weighted base, instead of on an equally-weighted base, is required. Portfolio returns, calculated by equally weighted averaging stock returns, tend to bias towards stock returns with small market capitalization, which will result in portfolio betas to be bias towards small-cap-

stock betas especially when the proxy of the market portfolio is a value-weighted index. Such bias will refrain from the precise estimation of portfolio betas (the independent variable in the FM cross sectional regression), and hence the empirical CAPM estimates.

Table 3.4: Market capitalization data: large companies dominate the New Zealand Stock Exchange

In this table, each cell represents a percentage of the total market value dominated by a given percentage of the largest companies listed on the New Zealand Stock Market at a certain year. It is calculated by adding up the market capitalization of a certain percentage (5%, etc) of the largest companies in a certain year (1990 to 2006), then dividing it by the sum of the market capitalization of all companies. For example, 70.7% in the first cell suggests that 5% of the largest companies accounts for 70.7% of the market value in the year 1990.

	Percentage of the company					
	5%	10%	15%	20%	25%	30%
1990	70.7%	82.9%	87.6%	90.9%	92.7%	94.4%
1991	74.3%	83.2%	88.1%	90.7%	92.7%	94.2%
1992	65.9%	76.1%	82.1%	86.4%	89.3%	91.4%
1993	55.4%	68.3%	77.6%	83%	87.5%	90.5%
1994	51.2%	66.8%	75.9%	82%	86.4%	89.6%
1995	50.7%	65.5%	75.8%	81.7%	86.7%	89.7%
1996	48.4%	64.4%	74.5%	81.2%	86%	89.3%
1997	47.7%	63%	73.6%	80.1%	85.7%	89%
1998	49.1%	64%	74.3%	80.7%	86.3%	89.4%
1999	46.2%	62.5%	72.5%	79.8%	84.6%	88.3%
2000	47%	64%	74.6%	81.1%	85.7%	89.1%
2001	47.5%	64.2%	74%	80.2%	85.6%	88.9%
2002	46.1%	64.5%	74.1%	80.5%	85.3%	88.9%
2003	49%	64.9%	75%	80.7%	85.6%	88.8%
2004	49.4%	65.9%	75.8%	81.9%	86.3%	89.6%
2005	49%	67.2%	75.2%	81.6%	86.5%	89.4%
2006	49%	66.3%	75.8%	82%	86.5%	89.7%
Average	52.7%	67.9%	76.8%	82.6%	87%	90%

Estimation Results

This chapter provides all estimation results of the empirical CAPM. As there are two proxies derived for the risk free rate in Chapter 3, we choose to present results estimated using the 10-year government bond yield as the risk free rate in this chapter while all other key results estimated using the capital gain rate as the risk free rate are presented in the Appendix (There are no significant changes in the final estimates of the empirical CAPM).

4.1 Estimation results: the FM method with the stock betas (SB) as the independent variable (FM SB)

As described in Chapter 2, stock betas are firstly estimated. Table 4.1 presents the summary statistics of stock beta estimates in each period. The first column shows all estimation periods. The second column presents

the number of stocks whose beta is estimated and following columns provide the summary statistics. Stock betas are estimated using both OLS and SW methods. As using the SW method does not alter results significantly, they are placed in the Appendix (Table 7.1).

One way to check if stock betas are appropriately estimated is to examine whether the value-weighted stock beta is (approximately) equal to 1. As the market portfolio is proxied by the NZX All Index and a large part of the index consists of the selected stocks for a given period, a value weighted stock beta should be approximately equal to one. That is:

$$\bar{\beta}^V = \sum_{i=1}^N w_i \hat{\beta}_i \approx 1, \quad (4.1)$$

where $w_i = \frac{MV_i}{MV_1 + MV_2 + \dots + MV_N}$; MV_i denotes the market capitalization of stock i , which is calculated by averaging the market capitalization for each stock over the beta-estimation period; N is the total number of stocks to form portfolios and $\hat{\beta}_i$ is the estimated beta of stock i . For example, for the period of 1990 to 1992, stock betas are estimated. Then, for each stock I take the average of their market capitalization from 1990 to 1992, and divide them by the sum of all stocks' average market capitalization to yield w_i . By applying (4.1), the value-weighted stock beta is computed. Table 4.1 shows that value-weighted stock betas are approximately equal to 1 for all beta estimation periods, which reflects the reliability of the quality of stock betas.

Table 4.1: Stock betas estimated using the OLS method with monthly data

This table shows stock beta estimates. Stocks are selected based on 2 criteria: 1.) a selected stock must have at least two latest years of returns available (out of 3 years of beta estimation period) in order to estimate stock betas. 2.) a selected stock must also have all returns available in the following year in order to perform the cross-sectional regression. The second column shows the number of stocks meeting both conditions. Following columns present the summary statistics of estimated stock betas. The last column shows the value-weight stock betas for each estimation period, which is calculated by taking the average of market capitalization of each stock at each period (90-92, 91-93 etc), dividing it by the sum of all stocks' average market capitalization at the same period to yield w_i , and finally applying Equation (4.1).

	No. of stocks	Min	Median	Max	Mean ^V
1990-1992	86	-1.31	0.75	3.65	0.99
1991-1993	90	-1.29	0.70	4.29	0.98
1992-1994	99	-0.87	0.78	3.59	1.02
1993-1995	104	-0.15	0.81	3.70	1.00
1994-1996	112	-0.75	0.85	2.78	1.03
1995-1997	110	-0.49	0.89	5.80	1.05
1996-1998	107	-0.03	0.83	3.28	1.06
1997-1999	113	-0.77	0.75	3.10	0.97
1998-2000	104	-0.83	0.68	2.71	0.92
1999-2001	105	-0.57	0.61	2.63	0.98
2000-2002	116	-2.68	0.64	3.81	1.00
2001-2003	119	-2.59	0.72	4.74	1.00
2002-2004	114	-1.49	0.69	3.56	0.96
2003-2005	113	-2.14	0.90	3.88	1.00

The next step is to estimate $\gamma_{0,t}$ and $\gamma_{1,t}$. As described in Chapter 2, for each month from Jan-93 to Dec-06, tax-adjusted excess stock returns are regressed on their betas estimated using the previous 3-year data, which yields 168 time series estimates of $\gamma_{i,t}$ respectively. Figure 4.1 shows their histograms (Figure 7.1 shows histograms for $\gamma_{0,t}$ and $\gamma_{1,t}$ estimated when the SW method is applied to estimate stock betas).

Figure 4.1: The Histogram of $\hat{\gamma}_{i,t}$ estimated where the OLS method is used to estimate stock betas

(a) $\gamma_{0,t}$

(b) $\gamma_{1,t}$

With $\hat{\gamma}_{i,t}$, (2.2) and (2.3) are applied to calculate γ_i^* and their standard errors respectively. Results are shown in Table 4.2, where “FM SB: OLS” means stock betas used as the independent variable in the FM cross sectional regressions are estimated by the OLS method while “FM SB: SW” means ones are estimated by the SW method. As noted, $\hat{\gamma}_i$ and their estimated standard errors, estimated using both methods, are with great similarities, which may contradict to the expectation that the SW model will generate rather different results. The use of monthly data may be the culprit resulting such a contradiction. As the thin trading problem is much alleviated in monthly data, the difference between stock betas estimated from the OLS and SW method may also be narrowed.

To avoid repetition, only one set of γ_i^* (FM SB: OLS) is used to illustrate how the following results are derived (corresponding results for (FM SB: SW) are derived following the same processes). Given the information provided in Table 4.2, the empirical CAPM is then presented by

$$\tilde{R}_i = 1.16\% - 0.23\%\beta_i \quad (4.2)$$

The standard error of $\tilde{R}_i$ is calculated by

$$\hat{\sigma}(\tilde{R}_i) = \sqrt{(0.37\%)^2 + (0.26\%)^2\beta_i^2 + 2(-3.98 \times 10^{-6})\beta_i} \quad (4.3)$$

As γ_i^* are estimated using monthly data, the tax-adjusted excess return and the standard error predicted from the empirical CAPM are also in monthly frequency, which are transferred to the annual frequency to facilitate the comparison with results from the Commission’s reports.

For the tax-adjusted excess return, it is compounded up by 12 to yield the

Table 4.2: The summary of γ_i^* – FM SB: OLS and SW

This table shows the summary of γ_i^* calculated when the independent variable (stock betas) in the cross sectional regressions are estimated using the OLS and SW method. “FM SB: OLS” means stock betas used as the independent variable in the FM cross sectional regressions are estimated by the OLS method while “FM SB: SW” means ones estimated by the SW method. (2.2) and (2.3) are applied to calculate γ_i^* and their standard errors. The “P-value” are calculated under the alternative hypothesis of $\gamma_i^* \neq 0$. The covariance of γ_i^* is given in this table as well because it will be used in calculating the standard error of the cost of capital in the later chapter. The covariance is calculated by

$$\text{cov}(\gamma_0^*, \gamma_1^*) = \frac{\text{cov}(\hat{\gamma}_{0,t}, \hat{\gamma}_{1,t})}{168}$$

FM SB: OLS				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$\text{cov}(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	1.16%	0.37%	0.002	-3.98×10^{-6}
$\gamma_{1,t}$	-0.23%	0.26%	0.38	-3.98×10^{-6}
FM SB: SW				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$\text{cov}(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	1.20%	0.36%	0.001	-1.88×10^{-6}
$\gamma_{1,t}$	-0.20%	0.16%	0.21	-1.88×10^{-6}

annual return. That is

$$\tilde{R}_i^y = (1 + \tilde{R}_i)^{12} - 1, \quad (4.4)$$

where $\tilde{R}_i^y$ is the estimated annual tax-adjusted excess return for security i . The “Delta method”¹ (Casella and Berger (2002)) is applied to transfer the

¹Given

$$\sqrt{n}(X - x) \rightarrow N(0, \text{Sd}(X)),$$

monthly standard error to the annual one, which implies

$$\hat{\sigma}(\tilde{R}_i^y) \approx 12\hat{\sigma}(\tilde{R}_i) , \quad (4.5)$$

where $\hat{\sigma}(\tilde{R}_i^y)$ is the annualized standard error.

In following steps, the 95% confidence interval for an estimated annual tax-adjusted excess return is calculated by

$$\begin{aligned} \tilde{R}_i^{y, \text{lower}} &= \tilde{R}_i^y - 1.96\hat{\sigma}(\tilde{R}_i^y) \\ \tilde{R}_i^{y, \text{upper}} &= \tilde{R}_i^y + 1.96\hat{\sigma}(\tilde{R}_i^y) , \end{aligned} \quad (4.6)$$

where $\tilde{R}_i^{y, \text{lower}}$ and $\tilde{R}_i^{y, \text{upper}}$ are the lower and upper bounds for $\tilde{R}_i^y$ respectively.

In order to have a general idea of the difference between the theoretical and empirical CAPM, the latter is used to predict the return for two special securities – the zero systematic risk security and the unit systematic risk security. $\beta_i = 0$ and 1 are substituted into (4.2) and (4.3) to yield monthly $\tilde{R}_i$ where $\text{Sd}(X)$ is the standard deviation of X . The “Delta method” implies

$$\sqrt{n}(f(X) - f(x)) \rightarrow N(0, \text{Sd}(X)f'(x)) ,$$

where $f(\cdot)$ is a function.

In this paper,

$$\sqrt{n}(\tilde{R}_i - \tilde{r}_i) \rightarrow N(0, \hat{\sigma}(\tilde{R}_i)) ,$$

where $\tilde{r}_i$ is the true value of $\tilde{R}_i$. Then,

$$\sqrt{n}(f(\tilde{R}_i) - f(\tilde{r}_i)) \rightarrow N(0, \hat{\sigma}(\tilde{R}_i)f'(\tilde{r}_i)) ,$$

where $f(\tilde{r}_i) = (1 + \tilde{r}_i)^{12} - 1$ and $f'(\tilde{r}_i) = 12(1 + \tilde{r}_i)^{11}$. Assuming $(1 + \tilde{r}_i)$ is very close to 1, $f'(\tilde{r}_i) \approx 12$. As noticed, this approximation will understate the annualized standard error.

estimates

$$\begin{aligned}
 \tilde{R}_i | (\beta_i = 0) &= 1.16\% \\
 \hat{\sigma}(\tilde{R}_i) | (\beta_i = 0) &= 0.37\% \\
 \tilde{R}_i | (\beta_i = 1) &= 0.93\% \\
 \hat{\sigma}(\tilde{R}_i) | (\beta_i = 1) &= \sqrt{(0.37\%)^2 + (0.26\%)^2 + 2(-3.98 \times 10^{-6})} \\
 &= 0.36\%
 \end{aligned} \tag{4.7}$$

Then, these monthly estimates are transferred to annual estimates applying (4.4) and (4.5)

$$\begin{aligned}
 \tilde{R}_i^y | (\beta_i = 0) &= (1 + 1.16\%)^{12} - 1 = 14.9\% \\
 \hat{\sigma}(\tilde{R}_i^y) | (\beta_i = 0) &\approx 12 \times 0.37\% = 4.41\% \\
 \tilde{R}_i^y | (\beta_i = 1) &= (1 + 0.93\%)^{12} - 1 = 11.8\% \\
 \hat{\sigma}(\tilde{R}_i^y) | (\beta_i = 1) &\approx 12 \times 0.36\% = 4.25\%
 \end{aligned} \tag{4.8}$$

Finally, the 95% confidence interval is calculated applying (4.6)

$$\begin{aligned}
 \tilde{R}_i^{y, \text{lower}} | (\beta_i = 0) &= 14.9\% - 1.96 \times 4.41\% = 6.23\% \\
 \tilde{R}_i^{y, \text{upper}} | (\beta_i = 0) &= 14.9\% + 1.96 \times 4.41\% = 23.5\% \\
 \tilde{R}_i^{y, \text{lower}} | (\beta_i = 1) &= 11.8\% - 1.96 \times 4.25\% = 3.5\% \\
 \tilde{R}_i^{y, \text{upper}} | (\beta_i = 1) &= 11.8\% + 1.96 \times 4.25\% = 20.2\%
 \end{aligned} \tag{4.9}$$

Table 4.3 summarizes final outputs from the above processes – annualized tax-adjusted excess returns and 95% confidence intervals for the zero and unit systematic risk securities. Returns for both securities are predicted with very wide confidence intervals and the security with $\beta_i = 0$ is predicted to have a greater return than the one with $\beta_i = 1$. The empirical

CAPM seems to contradict to the theoretical one (1.3). In order to carry on more sophisticated investigation on both theoretical and empirical CAPM, they are plotted in the same graph.

Table 4.3: The 95% confidence interval of the estimated annual tax-adjusted excess returns for two special securities: $\beta = 0$ and 1 – FM SB: OLS and SW

This table shows the 95% confidence interval constructed for the estimated annual tax-adjusted excess return of 2 special securities with $\beta = 0$ and 1 respectively. As outputs from (4.2) and (4.3) are in monthly frequency, they have to be annualized by applying (4.4) and (4.5). Finally, the 95% confidence interval is calculated applying (4.6)

FM SB: OLS				
	Lower bound	Mean	Upper bound	Std Err
$\beta=0$	6.23%	14.9%	23.5%	4.41%
$\beta=1$	3.5%	11.8%	20.2%	4.25%
FM SB: SW				
	Lower bound	Mean	Upper bound	Std Err
$\beta=0$	7.02%	15.4%	23.7%	4.27%
$\beta=1$	4.71%	12.7%	20.7%	4.10%

The graph is plotted in the space, where the x axis shows the systematic risk ranging from 0 to 1 and the y axis shows the annual tax-adjusted excess return. For each systematic risk from 0 to 1, the corresponding annualized tax-adjusted excess return is calculated by

$$\tilde{R}_i^y = (1 + 1.16\% - 0.23\%\beta_i)^{12} - 1 \quad (4.10)$$

(4.10) is obtained by substituting (4.2) into (4.4). The 95% confidence interval is calculated by

$$\begin{aligned}\tilde{R}_i^{y,\text{lower}} &= ((1 + 1.16\% - 0.23\%\beta_i)^{12} - 1) \\ &\quad - 1.96 \times 12 \sqrt{(0.37\%)^2 + (0.26\%)^2 \beta_i^2 + 2(-3.98 \times 10^{-6})\beta_i} \\ \tilde{R}_i^{y,\text{upper}} &= ((1 + 1.16\% - 0.23\%\beta_i)^{12} - 1) \\ &\quad + 1.96 \times 12 \sqrt{(0.37\%)^2 + (0.26\%)^2 \beta_i^2 + 2(-3.98 \times 10^{-6})\beta_i}\end{aligned}\tag{4.11}$$

(4.11) is obtained by substituting (4.3) and (4.5) into (4.6).

(4.10) and (4.11) are functions of the systematic risk (β_i). As the value of β_i varies from 0 to 1, the corresponding returns and the confidence interval are produced. (4.10) presents an approximate linear relationship between the annualized return and the systematic risk as the small coefficient value in front of the β_i will make its square, cube and high power values negligible.

On the other hand, the theoretical form of the CAPM is presented by

$$\tilde{R}_i = \widehat{MRP} \beta_i ,\tag{4.12}$$

where $\widehat{MRP}$ is the tax-adjusted market risk premium. In order to plot the theoretical CAPM, $\widehat{MRP}$ is assumed to be 7%, which is taken from one of the Commission's reports (Commerce Commission (2005b)).

Figure (4.2) plots the empirical and the theoretical form of the CAPM. The black line shows annual tax-adjusted excess returns predicted from (4.10) for any given systematic risk from 0 to 1. The dotted curves are the 95% confidence intervals calculated from (4.11). The gray line represents the CAPM calculated from (4.12).

The empirical CAPM is virtually a flat line compared to the theoretical one, and the confidence interval is remarkably wide. When $\beta_i = 0$, the lower bound of the empirical CAPM's prediction is even higher than the mean value predicted from the theoretical CAPM. Most notably, returns predicted from the empirical CAPM are consistently greater than ones predicted from the theoretical one.

The virtually flat line with the wide confidence interval imply that the empirical relationship between the expected return and the systematic risk in New Zealand might not be what have been stated in the CAPM theory or at least the return is not as responsive to the systematic risk as it is believed be. The significant intercept coefficient of the empirical CAPM suggests that there might be other risks, in addition to the systematic risk, that investors in New Zealand are care about. Therefore, it is possible that the theoretical CAPM may underestimate the cost of equity required by the New Zealand market.

On the other hand, above results may be imprecise or even incorrect due to a flawed estimation method – with estimated stock betas used as the independent variable in the FM cross sectional regression, the error-in-variable problem may refrain from the precise estimation of the empirical CAPM. In order to minimize the error-in-variable problem (hopefully), estimated portfolio betas take over of the role of stock betas since Fama and MacBeth (1973) argued that the former can be more precisely estimated than the latter under certain conditions. Therefore, results estimated using portfolio betas as the independent variable in the FM cross sectional regressions are presented and analyzed in following contents.

Figure 4.2: The plot of the empirical and theoretical CAPM
– FM SB: OLS and SW

(a) FM SB: OLS

(b) FM SB: SW

4.2 Estimation results: the FM method with the portfolio betas (PB) as the independent variable (FM PB)

4.2.1 The baseline case result: monthly data with the OLS beta-estimation method

Baseline results are built upon the monthly data, and most importantly, portfolio betas, estimated using the OLS method, are used as the independent variable in the FM cross-sectional regressions.

Stock betas (estimated using the OLS method) have already been estimated and presented in the previous section (Table 4.1). Portfolios are formed and returns are calculated in the way described in Chapter 2, which yield 10 portfolios with returns from Jan-93 to Dec-06. Then, their betas are estimated using the OLS method.

The summary statistics of 10 portfolios betas are presented in Table 4.4 (As $\hat{\alpha}$ are so insignificant, they are scaled up by multiplying 100 to be visible in 3 decimal places). Notably, $\hat{\alpha}$ are positive for all portfolios, which suggests that the CAPM may understate expected returns for all securities from Jan-90 to Dec-06. Although, only two $\hat{\alpha}$ are significant at 5% level, Black et al. (1972) points out that interpreting t-statistics should be undertaken with caution because of the violation of normality assumptions, and suggested that low t-statistics for $\hat{\alpha}$ can still indicate an alternative form of CAPM or missing independent variables.

Table 4.4: The summary of portfolio betas estimated using the OLS method

Portfolio betas are estimated by regressing portfolio returns on market portfolio returns for the entire period – 1993-2006. $\hat{\alpha}$ is the estimated intercept coefficient, which is scaled up by 100 times in order to be visible in 3 decimal places; $\hat{\beta}$ is the estimated slope coefficient. The bracketed numbers are t-statistics of null hypothesis if $\alpha = 0$ or $\beta = 0$.

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
$100\hat{\alpha}$	0.36	0.59	0.85	0.02	0.71	0.43	0.18	0.25	0.98	0.03
	(0.85)	(1.77)	(2.76)	(0.05)	(1.91)	(1.46)	(0.61)	(0.74)	(2.83)	(0.07)
$\hat{\beta}$	0.69	0.59	0.55	0.91	0.85	0.95	0.94	1	0.98	1.3
	(6.79)	(7.4)	(7.59)	(10.8)	(9.6)	(13.68)	(13.51)	(12.33)	(11.91)	(12.38)

The next step is to estimate $\gamma_{i,t}$ following the method described in Chapter 2. Their histograms are presented in Figure (4.3), where compared with Figure (4.1), there are more extreme values in $\hat{\gamma}_{i,t}$ estimated using portfolio betas than using stock betas.

Figure 4.3: The Histogram of $\hat{\gamma}_{i,t}$ – FM PB: OLS+OLS (a) $\gamma_{0,t}$ (b) $\gamma_{1,t}$

γ_i^* and $\hat{\sigma}(\gamma_i^*)$ are calculated applying (2.2) and (2.3) respectively. Table 4.5 summarizes key estimates. There is a feature worth noting from the table, which is that γ_i^* estimated using portfolio betas have larger standard errors than ones estimated using the stock betas. This feature may signal that using stock betas as the independent variable is superior to using the portfolio betas. The sign and the statistical significance of γ_i^* are exactly the same in both cases.

Table 4.5: The summary of γ_i^* – FM PB: OLS+OLS

This table summarizes key estimates of γ_i^* . “FM PB: OLS+OLS” means that 1.) portfolios betas are used as the independent variable in the FM cross sectional regressions and 2.) stock betas are estimated using the OLS method, and ranked to form 10 portfolios, whose betas are then estimated using the OLS method. (2.2) and (2.3) are applied to calculate γ_i^* and their standard errors respectively. The “P-value” are calculated under the alternative hypothesis of $\gamma_i^* \neq 0$. The covariance of γ_i^* is given in this table as well because it will be useful in calculating the standard error of the cost of capital in the later chapter. The covariance is calculated by

$$\text{cov}(\gamma_0^*, \gamma_1^*) = \frac{\text{cov}(\hat{\gamma}_{0,t}, \hat{\gamma}_{1,t})}{168}$$

	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$\text{cov}(\gamma_0^*, \gamma_1^*)$
γ_0	2.0%	0.72%	0.005	-5.26×10^{-5}
γ_1	-1.14%	0.82%	0.16	-5.26×10^{-5}

Using the information provided in Table 4.5, the empirical CAPM is then presented as

$$\tilde{R}_i = 2.0\% - 1.14\%\beta_i \quad (4.13)$$

The standard error of $\tilde{R}_i$ is calculated by

$$\hat{\sigma}(\tilde{R}_i) = \sqrt{(0.72\%)^2 + (0.82\%)^2\beta_i^2 + 2(-5.26 \times 10^{-5})\beta_i} \quad (4.14)$$

Once again, the newly estimated empirical CAPM is used to predict returns for two special securities – the zero systematic risk security and the unit systematic risk one. $\beta_i = 0$ and 1 are substituted into (4.13) and (4.14) to yield monthly estimates

$$\begin{aligned} \tilde{R}_i|(\beta_i = 0) &= 2.0\% \\ \hat{\sigma}(\tilde{R}_i)|(\beta_i = 0) &= 0.72\% \\ \tilde{R}_i|(\beta_i = 1) &= 0.86\% \\ \hat{\sigma}(\tilde{R}_i)|(\beta_i = 1) &= \sqrt{(0.72\%)^2 + (0.82\%)^2 + 2(-5.26 \times 10^{-5})} \\ &= 0.37\% \end{aligned} \quad (4.15)$$

Then, these monthly estimates are transferred to annual ones applying (4.4) and (4.5)

$$\begin{aligned} \tilde{R}_i^y|(\beta_i = 0) &= (1 + 2.0\%)^{12} - 1 = 26.8\% \\ \hat{\sigma}(\tilde{R}_i^y)|(\beta_i = 0) &\approx 12 \times 0.72\% = 8.66\% \\ \tilde{R}_i^y|(\beta_i = 1) &= (1 + 0.67\%)^{12} - 1 = 10.8\% \\ \hat{\sigma}(\tilde{R}_i^y)|(\beta_i = 1) &\approx 12 \times 0.37\% = 4.53\% \end{aligned} \quad (4.16)$$

Finally, the 95% confidence interval is constructed by applying (4.6)

$$\begin{aligned} \tilde{R}_i^{y,\text{lower}}|(\beta_i = 0) &= 26.8\% - 1.96 \times 8.66\% = 9.86\% \\ \tilde{R}_i^{y,\text{upper}}|(\beta_i = 0) &= 26.8\% + 1.96 \times 8.66\% = 43.8\% \\ \tilde{R}_i^{y,\text{lower}}|(\beta_i = 1) &= 8.3\% - 1.96 \times 4.7\% = 1.97\% \\ \tilde{R}_i^{y,\text{upper}}|(\beta_i = 1) &= 8.3\% + 1.96 \times 4.7\% = 19.7\% \end{aligned} \quad (4.17)$$

Table 4.6 summarizes annualized tax-adjusted excess returns and the 95% confidence interval for the zero and unit systematic risk securities. Compared with previous estimates (Table 4.3), returns for both securities are predicted with even wider confidence interval when portfolio betas are used as the independent variable in the FM cross sectional regressions. Moreover, the security with $\beta_i = 0$ is predicted of much greater return than the one with $\beta_i = 1$ (26.8% and 10.8%).

Table 4.6: The 95% confidence interval of the estimated annual tax-adjusted excess returns for two special securities: $\beta = 0$ and 1 – FM PB: OLS+OLS

This table shows the 95% confidence interval constructed for the estimated annual tax-adjusted excess returns of 2 special securities with $\beta = 0$ and 1 respectively.

	Lower bound	Mean	Upper bound	Std Err
$\beta=0$	9.86%	26.8%	43.8%	8.66%
$\beta=1$	1.97%	10.8%	19.7%	4.53%

The empirical and theoretical forms of the CAPM again are plotted. The empirical CAPM is constructed following the way described in the previous section. For each systematic risk ranging from 0 to 1, the corresponding annualized tax-adjusted excess return is calculated by

$$\tilde{R}_i^y = (1 + 2.0\% - 1.14\%\beta_i)^{12} - 1 \quad (4.18)$$

The 95% confidence interval is calculated by

$$\begin{aligned}\tilde{R}_i^{y, \text{lower}} &= ((1 + 2.0\% - 1.14\%\beta_i)^{12} - 1) \\ &\quad - 1.96 \times 12 \sqrt{(0.72\%)^2 + (0.82\%)^2 \beta_i^2 + 2(-5.26 \times 10^{-5})\beta_i} \\ \tilde{R}_i^{y, \text{upper}} &= ((1 + 2.0\% - 1.14\%\beta_i)^{12} - 1) \\ &\quad + 1.96 \times 12 \sqrt{(0.72\%)^2 + (0.82\%)^2 \beta_i^2 + 2(-5.26 \times 10^{-5})\beta_i}\end{aligned}\tag{4.19}$$

The theoretical form of the CAPM remains the same, where $\widehat{MRP}$ is again assumed to be 7%.

**Figure 4.4: The plot of the empirical and theoretical CAPM
– FM PB: OLS+OLS**

Results shown in Figure 4.4 and 4.2 share great similarities. The empirical

CAPM is always downward sloping with the wide-open confidence interval, which may suggest the ambiguous empirical relationship between the expected return and the systematic risk in New Zealand. The significantly positive return on the zero-beta security may indicate the possibility of the existence of risks that investors may require extra returns on, but being omitted by the theoretical CAPM.

The greater width of the confidence intervals shown in Figure 4.4 (v.s Figure 4.2) may cast doubts on the selected estimation methodology. The reason for using portfolio betas is to improve the precision of the independent variable, which may lead to less standard error in γ_i^* . However, it appears that using the stock betas as the independent variable may be a better choice, which may be due to two reasons.

The first reason is that the precision of estimated portfolio betas may not be better than that of estimated stock betas. As stated in Fama and MacBeth (1973), one condition that portfolio betas can be more precisely estimated is that errors in stock beta estimates should be significantly less than perfectly positively correlated, or in other words, the portfolio is well diversified. However, given the small size of the New Zealand Stock Market, this condition may not be achieved. If that is the case, then using stock beta estimates as the independent variable in the cross-sectional regressions is indeed a better choice.

The second reason is that methods for forming portfolios and estimating betas may worsen, instead of improving, the precision of portfolio beta estimates. As discussed in Chapter 2, the OLS estimator of a security's beta tends to bias downward when that security is infrequently traded. Given

the thin trading problem inhabited in the stock return data (Table 3.2), the OLS may not be the ideal method to estimate stock and portfolio betas. If that is the case, the precision of γ_i^* may be improved if using a better beta estimation method – the SW model.

Therefore, in the following section, I will adopt a different method to estimate stock and portfolio betas – the SW method (Scholes and Williams (1977)) – to take care of the thin trading problem. In addition, all analyses are carried on the weekly data. Reasons for doing so are two-folds. Firstly, it may be a chance to improve the precision of γ_i^* (less standard errors). Second, it offers an opportunity to perform robustness check on existing results.

4.2.2 Robust check I: monthly data with the SW beta-estimation method

The OLS method may not be sufficient for consistently estimating stock and portfolio betas because of the thin trading problem inhabited in the stock return data in New Zealand (Scholes and Williams (1977), Cohen et al. (1983) and Bartholdy and Riding (1994)). The SW model is therefore adopted to overcome this problem.

More specifically, the detailed methodology described in Chapter 2 remains except for switching the OLS to SW method. The SW method will be applied to estimate stock betas. Then, the SW method is applied again to estimate portfolio betas. As thin trading problem is largely alleviated in portfolio returns, the OLS method is applied to estimate portfolio betas as

well.

Stock and portfolio beta estimates are presented in Table 7.1 and 7.2 in the Appendix. These tables show that the SW and OLS method produce very similar beta estimates, which may be due to the use of the monthly data (the alleviated thin trading problem).

In order to facilitate the comparison, Table 4.7 summarizes all γ_i^* estimated using FM SB: OLS and SW, and FM PB: OLS+OLS, SW+SW and SW+OLS, where OLS+OLS, SW+SW and SW+OLS represent combinations of method. The first method is the method used to estimate stock betas while the second is for estimating portfolio betas.

As noted, the change in the beta-estimation method from the OLS to the SW model does not alter γ_i^* significantly. Results from OLS+OLS and SW+OLS are nearly identical. The only “outlier” seems to be the SW+SW method, where the γ_1^* is positive but still insignificant, and the γ_0^* is insignificant. The table also illustrate that the standard error of γ_i^* is much smaller when stock betas are used as the independent variable, which may confirm our guess that using portfolio beta as the independent variable in the FM cross sectional regressions may not be suitable for studies using the New Zealand stock data because of the limited number of firms. (Table 7.5 shows the same results with the capital gain returns as the proxy for the risk free rate. It appears that the change of the proxy for the risk free rate does not alter the final results critically.)

Table 4.7: The summary of γ_i^* – FM SB: OLS and SW; FM PB: OLS+OLS, SW+SW and SW+OLS

The table presents all γ_i^* . Results for FM SB: OLS and SW, and FM PB: OLS+OLS are obtained from Table 4.2 and 4.5 respectively. OLS+OLS, SW+SW and SW+OLS represent combinations of method. The first method is the method used to estimate stock betas while the second is for estimating portfolio betas. Results from the OLS+OLS combination are the baseline results while ones from the SW+SW and SW+OLS combinations are for the robustness check.

FM SB: OLS				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	1.16%	0.37%	0.002	-3.98×10^{-6}
$\gamma_{1,t}$	-0.23%	0.26%	0.38	-3.98×10^{-6}
FM SB: SW				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	1.20%	0.36%	0.001	-1.88×10^{-6}
$\gamma_{1,t}$	-0.20%	0.16%	0.21	-1.88×10^{-6}
FM PB: OLS+OLS				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
γ_0	2.0%	0.72%	0.005	-5.26×10^{-5}
γ_1	-1.14%	0.82%	0.16	-5.26×10^{-5}
FM PB: SW+SW				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	0.72%	0.73%	0.32	-4.1×10^{-5}
$\gamma_{1,t}$	0.30%	0.64%	0.64	-4.1×10^{-5}
FM PB: SW+OLS				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	1.8%	0.81%	0.026	-6.34×10^{-5}
$\gamma_{1,t}$	-0.84%	0.86%	0.33	-6.34×10^{-5}

To avoid repetition, only one set of estimation results of γ_i^* (FM PB: SW+SW) is used to show how to derive following results (results for (FM SB: SW+OLS) are derived following the same processes). The newly estimated empirical CAPM is expressed by

$$\tilde{R}_i = 0.72\% + 0.3\%\beta_i \quad (4.20)$$

The standard error of $\tilde{R}_i$ is given by

$$\hat{\sigma}(\tilde{R}_i) = \sqrt{(0.73\%)^2 + (0.64\%)^2\beta_i^2 + 2(-4.1 \times 10^{-5})\beta_i} \quad (4.21)$$

$\beta_i = 0$ and 1 are substituted into (4.20) and (4.21) to yield monthly estimates

$$\begin{aligned} \tilde{R}_i|(\beta_i = 0) &= 0.72\% \\ \hat{\sigma}(\tilde{R}_i)|(\beta_i = 0) &= 0.73\% \\ \tilde{R}_i|(\beta_i = 1) &= 1.02\% \\ \hat{\sigma}(\tilde{R}_i)|(\beta_i = 1) &= \sqrt{(0.73\%)^2 + (0.64\%)^2 + 2(-4.1 \times 10^{-5})} \\ &= 0.35\% \end{aligned} \quad (4.22)$$

Then, these monthly estimates are transferred to annual estimates applying (4.4) and (4.5)

$$\begin{aligned} \tilde{R}_i^y|(\beta_i = 0) &= (1 + 0.73\%)^{12} - 1 = 8.92\% \\ \hat{\sigma}(\tilde{R}_i^y)|(\beta_i = 0) &\approx 12 \times 0.73\% = 8.73\% \\ \tilde{R}_i^y|(\beta_i = 1) &= (1 + 1.02\%)^{12} - 1 = 12.9\% \\ \hat{\sigma}(\tilde{R}_i^y)|(\beta_i = 1) &\approx 12 \times 0.35\% = 4.08\% \end{aligned} \quad (4.23)$$

Finally, the 95% confidence interval is calculated applying (4.6)

$$\tilde{R}_i^{y,\text{lower}}|(\beta_i = 0) = 8.92\% - 1.96 \times 8.73\% = -8.19\% \quad (4.24)$$

$$\tilde{R}_i^{y,\text{upper}}|(\beta_i = 0) = 8.92\% + 1.96 \times 8.73\% = 26.0\%$$

$$\tilde{R}_i^{y,\text{lower}}|(\beta_i = 1) = 12.9\% - 1.96 \times 3.5\% = 4.93\%$$

$$\tilde{R}_i^{y,\text{upper}}|(\beta_i = 1) = 12.9\% + 1.96 \times 3.5\% = 20.9\%$$

Table 4.8: The 95% confidence interval of the estimated annual tax-adjusted excess returns for two special securities: $\beta = 0$ and 1 – a summary

Results for FM SB: OLS and SW, and FM PB: OLS+OLS are obtained from Table 4.2 and 4.5 respectively. This table shows the 95% confidence interval constructed for the estimated annual tax-adjusted excess returns of 2 special securities with $\beta = 0$ and 1 respectively. All results presented in this table are either drawn from previous results or newly estimated

$\beta=0$				
	Lower bound	Mean	Upper bound	Std Err
FM SB: OLS	6.23%	14.9%	23.5%	4.41%
FM SB: SW	7.02%	15.4%	23.7%	4.27%
FM PB: OLS+OLS	9.86%	26.8%	43.8%	8.66%
FM PB: SW+SW	-8.19%	8.92%	26.0%	8.73%
FM PB: SW+OLS	4.15%	23.2%	42.3%	9.73%
$\beta=1$				
	Lower bound	Mean	Upper bound	Std Err
FM SB: OLS	3.5%	11.8%	20.2%	4.25%
FM SB: SW	4.71%	12.7%	20.7%	4.10%
FM PB: OLS+OLS	1.97%	10.8%	19.7%	4.53%
FM PB: SW+SW	4.93%	12.9%	20.9%	4.08%
FM PB: SW+OLS	3.08%	11.6%	20.0%	4.34%

Table 4.8 summarizes all predicted returns and their 95% confidence intervals for the special case (beta=0 and 1) using all the empirical form of the CAPMs. It appears that there are large systematic errors around the empirical CAPMs. For example, the range of the return of the zero beta security can be from -9.6% (FM PB: SW+SW) to 39.8% (FM PB: OLS+OLS) whereas the range of the return of the unit beta security are from -0.86% (FM PB: OLS+OLS) to 20.9% (FM PB: SW+SW). Moreover, the mean value of predicted returns of zero beta security are, in most cases, greater than those of unit beta security. This table casts doubts on whether a security's beta is the risk, and the only risk, that the market is priced. (Table 7.6 shows the same results with the capital gain returns as the proxy for the risk free rate.)

The empirical and theoretical forms of CAPM are plotted in Figure 4.5. For a given beta, the return estimated using the empirical form is calculated by

$$\tilde{R}_i^y = (1 + 0.72\% + 0.3\%\beta_i)^{12} - 1 \quad (4.25)$$

The 95% confidence interval is calculated by

$$\begin{aligned} \tilde{R}_i^{y, \text{lower}} &= ((1 + 0.72\% + 0.3\%\beta_i)^{12} - 1) \\ &\quad - 1.96 \times 12 \sqrt{(0.73\%)^2 + (0.64\%)^2 \beta_i^2 + 2(-4.1 \times 10^{-5})\beta_i} \\ \tilde{R}_i^{y, \text{upper}} &= ((1 + 0.72\% + 0.3\%\beta_i)^{12} - 1) \\ &\quad + 1.96 \times 12 \sqrt{(0.73\%)^2 + (0.64\%)^2 \beta_i^2 + 2(-4.1 \times 10^{-5})\beta_i} \end{aligned} \quad (4.26)$$

The theoretical CAPM remains the same, where $\widehat{MRP}$ is again assumed to be 7%.

Compared with previous results (Figure 4.2 and 4.4), this figure also shows

that the theoretical CAPM consistently understates the expected return of any securities with beta ranging from 0 to 1. By using the monthly data, features illustrated from the empirical CAPM estimated using different methods are remarkably consistent. For example, they all have poor predictability because of the ambiguous empirical risk-return relationship, and they all require extra amount of returns for a given beta (ranging from 0 to 1) compared with the theoretical CAPM. In the next section, I will check if these features still hold when the weekly data are used. (Figure 7.2 shows the same results with the capital gain returns as the proxy for the risk free rate. It appears that the change of the proxy for the risk free rate does not alter final results critically.)

Figure 4.5: The plot of the empirical and theoretical forms of the CAPM – FM PB: SW+SW and SW+OLS

(a) FM PB: SW+SW

(b) FM PB: SW+OLS

4.2.3 Robust check II: the FM method with portfolio betas as the independent variable with the weekly data

In this section, whether results estimated using the monthly data will still hold on the weekly data is examined. As the CAPM theory does not specify any length of data period (annual, month, week or day), the difference in data frequency should not alter estimation results critically. Although the study by Handa et al. (1993) reported that the choice of return measurement interval has an effect on the risk-return relationship, their conclusions were based on the monthly and annual data from New York Stock Exchange-American Stock Exchange, which cannot be generalised to our study – weekly data from New Zealand Stock Exchange.

Methodology for the weekly data

Any technical issues discussed in Chapter 2 for monthly data (the FM method with portfolio betas used as the independent variable) are applied to weekly data except for some changes in the choice of stock beta estimation period.

The preliminary step is to estimate stock betas using one year stock returns. More specifically, for the first year period (1-week-90 to 52-week-90), we select stocks with returns available at least in the latest 30 weeks (23-week-90 to 52-week-90) and all returns available in the next year (1-week-91 to 52-week-91). After selecting stocks, all available returns from 1-week-90 to 52-week-90 are used to estimate stock betas.

The next step is to form 10 portfolios in the way was that described Chap-

ter 2. Portfolio returns in the next period (1-week-91 to 52-week-91) are computed by value weighting stock returns at the same period.

By repeating the above process except for moving one year forward, portfolio returns from 1-week-92 to 52-week-92 are calculated. That is: for the period of 1-week-91 to 52-week-91, stocks – with returns at least available from 23-week-91 to 52-week-91 and all returns available for 1-week-92 to 52-week-92 – are selected; then, using all available stock returns from 1-week-91 to 52-week-91 to estimate stock betas; 10 portfolios are formed and returns from 1-week-92 to 52-week-92 are calculated using value-weighting stock returns from the same period. By repeating the process, portfolio returns from 1-week-91 to 52-week-06 are computed. Finally, 10 portfolio betas are estimated by regressing all portfolio returns from 1-week-91 to 52-week-06 on returns of the market portfolio from the same period.

Again, it is assumed that stocks in each portfolio have their betas equal to the portfolio beta which they belong to. For each week, stock returns are regressed on stock betas. There are in total 832 regressions (16 years $\times$ 52 weeks) performed from 1-week-91 to 52-week-06, which yield time series estimates of $\gamma_{i,t}$. Finally, γ_i^* are calculated by

$$\gamma_i^* = \frac{1}{832} \sum_{t=1\text{-week-91}}^{52\text{-week-06}} \hat{\gamma}_{i,t}, \quad (4.27)$$

and their variances are calculated by

$$\hat{\sigma}^2(\gamma_i^*) = \frac{1}{832(832-1)} \sum_{t=1\text{-week-91}}^{52\text{-week-06}} (\hat{\gamma}_{i,t} - \gamma_i^*)^2 \quad (4.28)$$

As γ_i^* are estimated using the weekly data, the tax-adjusted excess return and the standard error predicted from the empirical CAPM are also in weekly frequency. They have to be transferred to the annual outputs for facilitating the comparison with results using the monthly data.

For the tax-adjusted excess return, it is compounded up by 52 (assuming there are always 52 weeks in a year) to yield annual outputs. That is

$$\tilde{R}_i^y = (1 + \tilde{R}_i)^{52} - 1, \quad (4.29)$$

where $\tilde{R}_i^y$ is the estimated annual tax-adjusted excess return for security i . The “Delta method” is applied to transfer the weekly standard error to the annual one, which implies

$$\hat{\sigma}(\tilde{R}_i^y) \approx 52\hat{\sigma}(\tilde{R}_i), \quad (4.30)$$

where $\hat{\sigma}(\tilde{R}_i^y)$ is the annualized standard error.

Estimation results from the weekly data

Stock and portfolio beta estimates are shown in Table 7.3 and 7.4, which are consistent with their counterparts – ones estimated using monthly estimates.

Table 4.9 summarizes γ_i^* estimated from different combinations of the method with the weekly data. One important feature of the this table is that results from “OLS+OLS” and “SW+OLS” combinations are almost identical, which is also shown in Table (4.7). By using weekly data, all γ_1^* are negative which is different from the monthly data case, where the “SW+SW” produces a positive value. However, no γ_1^* is statistically significant at the

5% level. Although results from this table and Table (4.7) cannot be compared directly because of different data frequencies used, both tables give the impression that γ_i^* are robust to different data frequencies.

Table 4.9: The summary of γ_i^* – FM PB: OLS+OLS, SW+SW and SW+OLS (weekly data)

The table presents all γ_i^* estimated from the different combinations of the method with the weekly data. In this table, “OLS+OLS”, “SW+SW” and “SW+OLS” represent the method combinations. The first method is the method used to estimate stock betas while the second is used to estimate portfolio betas.

FM PB: OLS+OLS				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	0.20%	0.097%	0.04	-1.44×10^{-6}
$\gamma_{1,t}$	-0.26%	0.17%	0.13	-1.44×10^{-6}
FM PB: SW+SW				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	0.06%	0.125%	0.63	-1.2×10^{-6}
$\gamma_{1,t}$	-0.008%	0.114%	0.94	-1.2×10^{-6}
FM PB: SW+OLS				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	0.22%	0.121%	0.07	-2.0×10^{-6}
$\gamma_{1,t}$	-0.28%	0.187%	0.13	-2.0×10^{-6}

Results from “SW+OLS” are omitted for the great similarity to ones from “OLS+OLS”. To avoid repeated analyses, I will focus on exploring results

from “OLS+OLS” and any coming exploration and explanation on the results from “SW+SW” are executed following the same process.

The estimated empirical CAPM is expressed by

$$\tilde{R}_i = 0.2\% - 0.26\%\beta_i \quad (4.31)$$

The standard error of $\tilde{R}_i$ is given by

$$\hat{\sigma}(\tilde{R}_i) = \sqrt{(0.097\%)^2 + (0.17\%)^2\beta_i^2 + 2(-1.44 \times 10^{-6})\beta_i} \quad (4.32)$$

$\beta_i = 0$ and 1 are substituted into (4.31) and (4.32) to yield weekly estimates

$$\tilde{R}_i|(\beta_i = 0) = 0.2\% \quad (4.33)$$

$$\hat{\sigma}(\tilde{R}_i)|(\beta_i = 0) = 0.097\%$$

$$\tilde{R}_i|(\beta_i = 1) = -0.06\%$$

$$\begin{aligned} \hat{\sigma}(\tilde{R}_i)|(\beta_i = 1) &= \sqrt{(0.097\%)^2 + (0.17\%)^2 + 2(-1.44 \times 10^{-6})} \\ &= 0.098\% \end{aligned}$$

Then, these weekly estimates are transferred to the annual estimates applying (4.29) and (4.30)

$$\tilde{R}_i^y|(\beta_i = 0) = (1 + 0.2\%)^{52} - 1 = 10.9\% \quad (4.34)$$

$$\hat{\sigma}(\tilde{R}_i^y)|(\beta_i = 0) \approx 52 \times 0.097\% = 3.1\%$$

$$\tilde{R}_i^y|(\beta_i = 1) = (1 - 0.06\%)^{52} - 1 = -3.3\%$$

$$\hat{\sigma}(\tilde{R}_i^y)|(\beta_i = 1) \approx 52 \times 0.0098\% = 5.2\%$$

Finally, the 95% confidence interval is calculated applying (4.6)

$$\begin{aligned}
 \tilde{R}_i^{y, \text{lower}} | (\beta_i = 0) &= 10.9\% - 1.96 \times 3.1\% = 4.7\% \\
 \tilde{R}_i^{y, \text{upper}} | (\beta_i = 0) &= 10.9\% + 1.96 \times 3.1\% = 17\% \\
 \tilde{R}_i^{y, \text{lower}} | (\beta_i = 1) &= -3.3\% - 1.96 \times 5.2\% = -13.5\% \\
 \tilde{R}_i^{y, \text{upper}} | (\beta_i = 1) &= -3.3\% + 1.96 \times 5.2\% = 6.9\%
 \end{aligned} \tag{4.35}$$

Table 4.10: The 95% confidence interval of estimated annual tax-adjusted excess returns for two special securities: $\beta = 0$ and 1 – a summary (weekly data)

This table shows the 95% confidence interval constructed for estimated annual tax-adjusted excess returns of 2 special securities with $\beta = 0$ and 1.

$\beta=0$				
	Lower bound	Mean	Upper bound	Std Err
FM PB: OLS+OLS	4.7%	10.9%	17.0%	4.7%
FM PB: SW+SW	-3.9%	3.0%	9.9%	3.5%
$\beta=1$				
	Lower bound	Mean	Upper bound	Std Err
FM PB: OLS+OLS	-13.5%	-3.3%	6.9%	5.2%
FM PB: SW+SW	-7.0%	2.5%	12.1%	4.9%

The most noticeable feature is that annualized standard errors are much less than results shown in Table 4.8 (from monthly data), which may not suggest that the empirical CAPM estimated from the weekly data gener-

ate more accurate predictions. That is because 1.) assuming $(1 + \tilde{r}_i)^{51} = 0$ (where $\tilde{r}_i$ is the true weekly tax-adjusted excess return for security i) in the implementation of the “Delta method” may severely underestimate the annualized the standard error; 2.) the weekly data have large manipulation errors as all raw data come in either monthly or daily frequencies.

The empirical and theoretical CAPM are plotted in Figure 4.6. For a given beta, the estimated return predicted using the empirical CAPM is calculated by

$$\tilde{R}_i^y = (1 + 0.2\% - 0.26\%\beta_i)^{52} - 1 \quad (4.36)$$

The 95% confidence interval is calculated by

$$\begin{aligned} \tilde{R}_i^{y, \text{lower}} &= ((1 + 0.2\% - 0.26\%\beta_i)^{52} - 1) \\ &\quad - 1.96 \times 52 \sqrt{(0.097\%)^2 + (0.17\%)^2 \beta_i^2 + 2(-1.44 \times 10^{-6})\beta_i} \\ \tilde{R}_i^{y, \text{upper}} &= ((1 + 0.2\% - 0.26\%\beta_i)^{52} - 1) \\ &\quad + 1.96 \times 52 \sqrt{(0.097\%)^2 + (0.17\%)^2 \beta_i^2 + 2(-1.44 \times 10^{-5})\beta_i} \end{aligned} \quad (4.37)$$

The theoretical form remains the same, where $\widehat{MRP}$ is again assumed to be 7%.

Compared this figure with previous ones from the monthly data, the empirical CAPMs estimated from weekly data also show the same feature – an almost flat line with the wide confidence intervals. There may be some subtle differences in some return estimates, however, this is nothing significant. Therefore, results from the robustness check demonstrate that the estimation of the empirical form of the CAPM is robust to different beta-estimation methods and data frequencies (weekly and monthly).

**Figure 4.6: The plot of the empirical and theoretical CAPM
– a summary (weekly data)**

(a) FM PB: OLS+OLS

(b) FM PB: SW+SW

4.3 A discussion on all estimation results

This chapter provides the estimation results of the empirical CAPM using different beta-estimation methods with both monthly and weekly data. Table 4.11 summarizes all methods and data used for the estimation of the empirical CAPM. The empirical CAPM, using the FM method with stock betas as the independent variable and the monthly data, is first estimated. Results show that the empirical CAPM is virtually a flat line with wide confidence intervals, which predicts returns (with beta ranging 0 to 1) being consistently greater than ones predicted from the theoretical CAPM. Under the belief that using portfolio betas can improve the precision of the independent variable and hence γ_i^* , the baseline method (FM PB: OLS+OLS) was executed. The newly estimated empirical CAPM illustrates same features except for an increase in the standard error of γ_i^* . Discussed previously, this situation may be resulted from using portfolio betas at the independent variable as they may have not been more precisely estimated than individual stock betas due to the small number of firms.

Table 4.11: The summary of all methodologies and data used for estimating the empirical CAPM

Monthly Data	
<i>Method</i>	<i>Descriptions</i>
FM SB: OLS	Using stock betas as the independent variable Stock betas are estimated using the OLS method
FM SB: SW	Using stock betas as the independent variable Stock betas are estimated using the SW method
FM PB: OLS+OLS	Using portfolio betas as the independent variable OLS+OLS – the OLS method for estimating stock and portfolio betas
FM PB: SW+SW	Using portfolio betas as the independent variable SW+SW – the SW method for estimating stock and portfolio betas
FM PB: SW+OLS	Using portfolio betas as the independent variable SW+OLS – the SW method for estimating stock and the OLS method for estimating portfolio betas
Weekly Data	
<i>Method</i>	<i>Descriptions</i>
FM PB: OLS+OLS	Using portfolio betas as the independent variable OLS+OLS – the OLS method for estimating stock and portfolio betas
FM PB: SW+SW	Using portfolio betas as the independent variable SW+SW – the SW method for estimating stock and portfolio betas
FM PB: SW+OLS	Using portfolio betas as the independent variable SW+OLS – the SW method for estimating stock and the OLS method for estimating portfolio betas

In the robustness check, I kept working on the monthly data with the SW method used for estimating betas. Once again, estimation results are very similar. After switching to the weekly data, there are still no critical changes in the empirical CAPM. However, there is a difference worth noting: the empirical CAPM seems to predict lower returns with lower standard errors for any given beta from 0 to 1 than ones estimated using the monthly data. This may be due to the use some approximations (in the “Delta method”) that might artificially underestimate results, and the “bad” nature of the weekly data. As all dataset come in either monthly or daily frequencies, converting those to weekly and then back to annual frequency may generate great manipulation errors. All in all, the robustness check shows that it is highly likely that the empirical CAPM estimates are reliable. In addition, results (Table 7.5 and 7.6; Figure 7.2) presented in the Appendix demonstrates that the empirical CAPM estimates are also robust to the different choices of proxies of the risk free rate.

Estimation results from all dimensions exhibit similar features of the empirical CAPM – an virtually flat line with wide confidence intervals. This feature may be due to great ambiguities in the empirical relationship between the expected return and the systematic risk in New Zealand. Moreover, most results illustrate that predicted returns are significantly greater than zero given the systematic risk equal to 0, which may suggest that there might be other risks (in addition to the systematic risk) that are not included in the CAPM, but are required extra returns by the market and investors in New Zealand.

All these features illustrated from the empirical CAPM will naturally cast

doubts on results from the Commission's reports (Commerce Commission (2005b) and Commerce Commission (2007)), in which they are calculated using the theoretical CAPM (1.3). In order to better understand what the cost of capital are required by the market might actually be, in the next chapter the Commission's analyses are replicated using the empirical CAPM.

The implication of the empirical CAPM

5.1 Estimating the WACC using the empirical CAPM

In contrast to the application of the theoretical CAPM to calculate the WACC, using the empirical CAPM will provide estimates of the cost of equity, and hence the WACC that reflect the expected return on capitals required by the market. In addition, it also gives better estimates of the size of estimation errors than ones reported from the Commission's studies. Following contents explains how to calculate the WACC and its standard error using the empirical CAPM.

As the empirical CAPM estimates are robust to different data frequencies shown in Chapter 4, results estimated using the monthly data are adopted to calculate the WACC because they tend to contain less data-manipulation errors. For example, it is more accurate to transfer daily

stock returns to monthly returns, than to weekly returns, as the number of days in a month is relatively more stable than one in a week¹. Hence, I believe that results from the monthly data are more appropriate in the calculation the WACC.

The WACC is formulated as

$$\text{WACC} = (1 - L)R_e + L(1 - T_c)k_d, \quad (5.1)$$

where L is the financial leverage ratio; k_d is the cost of debt; R_e is the cost of equity capital and T_c is the corporate tax rate.

By assuming every component in (5.1) are estimated without errors except for the cost of equity – R_e , the estimated WACC can be expressed as

$$\widehat{\text{WACC}} = (1 - L)\hat{R}_e + L(1 - T_c)k_d \quad (5.2)$$

and the estimated standard error is presented by

$$\hat{\sigma}(\widehat{\text{WACC}}) = (1 - L)\hat{\sigma}(\hat{R}_e), \quad (5.3)$$

where $\hat{\sigma}(\hat{R}_e)$ is the standard error of the estimated cost of equity. The 95% confidence interval for the estimated WACC is calculated by

$$\begin{aligned} \widehat{\text{WACC}}^{\text{lower}} &= \widehat{\text{WACC}} - 1.96\hat{\sigma}(\widehat{\text{WACC}}) \\ \widehat{\text{WACC}}^{\text{upper}} &= \widehat{\text{WACC}} + 1.96\hat{\sigma}(\widehat{\text{WACC}}) \end{aligned} \quad (5.4)$$

¹There are 5 and 20 trading days in each week and month respectively. Public holidays will have relatively greater marginal effects on the weekly data than on the monthly data

5.2 Two case studies

Since the Part 4 of the *Commerce Act 1986* (the Act) came to effect on 8 August 2001, the Commission has been required to implement a targeted control regime for the regulated businesses such as electricity line and gas pipeline businesses. The intention of the regime is to promote better quality products with lower prices to benefit consumers. Under the Act, the Commission must set thresholds for the declaration of control of goods and services provided by those businesses, one of which thresholds is the cost of capital that imposes control on the maximum return that can be earned by investors in these businesses. Calculating such a vital threshold has been carefully documented by the Commission in its draft guideline (Commerce Commission (2005a)), which advocates using the capital asset pricing model to estimate the cost of equity that is then incorporated in the calculation of the weighted average cost of capital to determine the maximum return earned by investors.

Two reports, *Regulation of Electricity Lines Businesses Target Control Regime Intention to Declare Control Unison Networks Limited* (Commerce Commission (2005b)) and *Authorisation for the Control of Supply of Natural Gas Distribution Services by Powerco Ltd and Vector Ltd* (Commerce Commission (2007)), have been published by the Commission to present how its obligation is fulfilled under the Act, a part of which is to calculate the WACC. Two case studies are then derived from these reports by switching the method for the calculation of the cost of equity from the theoretical CAPM (applied by the Commission) to the empirical CAPM (estimated in this paper). With other parameter values for calculating the WACC remaining

the same as provided in the Commission's reports, each case study will generate comparable WACC estimate.

Table 5.1 summarizes all information estimated from previous chapter to calculate the cost of equity, where in order to transfer tax-adjusted excess returns to returns, the tax adjusted risk free rate is added back. That is

$$\hat{R}_i^y = \tilde{R}_i^y + R_f(1 - T_I) ,$$

where $\hat{R}_i^y$ is the estimated annual return for security i . $\hat{\sigma}(\hat{R}_i^y)$ is the estimated standard error for the return, which is equal to $\hat{\sigma}(\tilde{R}_i^y)$ since R_f and T_I are assumed to be estimated without errors.

Table 5.1: The summary of the annual cost of equity and its standard errors esitimated using the empirical CAPM

This table shows the “annualized” empirical CAPM estimated using different methods with the monthly data. Results shown are all taken from Chapter 4. These empirical CAPMs will predict annual returns (instead of tax-adjusted excess returns), which is achieved by adding back the tax adjusted risk free rate. That is

$$\hat{R}_i^y = R_f(1 - T_I) + \tilde{R}_i^y ,$$

where $\hat{R}_i^y$ is the estimated annual return for a security i . $\hat{\sigma}(\hat{R}_i^y)$ is the estimated standard error for the return, which is equal to $\hat{\sigma}(\tilde{R}_i^y)$ since R_f and T_I are assumed to be estimated without errors.

Method	Estimates
FM SB: OLS	$\hat{R}_i^y = R_f(1 - T_I) + (1 + 1.16\% - 0.23\%\beta_i)^{12} - 1$ $\hat{\sigma}(\hat{R}_i^y) = 12\sqrt{(0.37\%)^2 + (0.26\%)^2\beta_i^2 + 2(-3.98 \times 10^{-6})\beta_i}$
FM SB: SW	$\hat{R}_i^y = R_f(1 - T_I) + (1 + 1.20\% - 0.20\%\beta_i)^{12} - 1$ $\hat{\sigma}(\hat{R}_i^y) = 12\sqrt{(0.36\%)^2 + (0.16\%)^2\beta_i^2 + 2(-1.88 \times 10^{-6})\beta_i}$
FM PB: OLS+OLS	$\hat{R}_i^y = R_f(1 - T_I) + (1 + 2.0\% - 1.14\%\beta_i)^{12} - 1$ $\hat{\sigma}(\hat{R}_i^y) = 12\sqrt{(0.72\%)^2 + (0.82\%)^2\beta_i^2 + 2(-5.26 \times 10^{-5})\beta_i}$
FM PB: SW+SW	$\hat{R}_i^y = R_f(1 - T_I) + (1 + 0.72\% + 0.3\%\beta_i)^{12} - 1$ $\hat{\sigma}(\hat{R}_i^y) = 12\sqrt{(0.73\%)^2 + (0.64\%)^2\beta_i^2 + 2(-4.1 \times 10^{-5})\beta_i}$
FM PB: SW+OLS	$\hat{R}_i^y = R_f(1 - T_I) + (1 + 1.8\% - 0.84\%\beta_i)^{12} - 1$ $\hat{\sigma}(\hat{R}_i^y) = 12\sqrt{(0.81\%)^2 + (0.86\%)^2\beta_i^2 + 2(-6.34 \times 10^{-5})\beta_i}$

The WACC plays two important roles in the Commission's study on regulating electricity line businesses (Commerce Commission (2005b)). According to the study, the WACC is used to impose the electricity price cap, and for measuring excess profits. The report employs the "Brennan-Lally" version CAPM (1.3) to estimate the cost of equity and the WACC formula (1.1) to calculate the cost of capital for electricity line businesses, where relevant parameter values are listed below

$$\begin{aligned} L &= 0.4 & \beta_e &= 0.67 & T_c &= T_I = 0.33 \\ R_f &= 0.063 & MPR &= 0.07 & k_d &= 0.075 \end{aligned}$$

where L is the financial leverage ratio; β_e is the equity beta; MPR is the market risk premium; T_c is the corporate tax rate; R_f is the risk-free rate and k_d is the interest rate on the debt capital. The WACC is reported to be 7.35% (on page 39 of Commerce Commission (2005b))².

On the other hand, the WACC calculated using the empirical CAPM is reported in following contents. The cost of equity and its standard error are estimated by substituting R_f , T_I and β_e into each equation in Table 5.1. Taking the result from the "FM SB: OLS" as an example, the cost of equity is calculated by

$$\begin{aligned} \hat{R}_e &= 6.3\%(1 - 0.33) + (1 + 1.16\% - 0.23\% \times 0.67)^{12} - 1 & (5.5) \\ &= 17.0\% \end{aligned}$$

²The figure is calculated by applying the CAPM and WACC formula. That is

$$\begin{aligned} R_e &= 0.063(1 - 0.33) + 0.07 \times 0.67 = 0.089 \\ \text{WACC} &= 0.089(1 - 0.4) + 0.073(1 - 0.33)0.4 = 0.0735 \end{aligned}$$

The standard error is then calculated by

$$\begin{aligned}\hat{\sigma}(\hat{R}_e) &= 12\sqrt{(0.37\%)^2 + (0.26\%)^2 \times 0.67^2 + 2 \times 0.67(-3.98 \times 10^{-6})} \quad (5.6) \\ &= 4.1\%\end{aligned}$$

By substituting above estimates into (5.2) and (5.3), the estimated WACC and its standard error are obtained. That is

$$\begin{aligned}\widehat{\text{WACC}} &= (1 - L)\hat{R}_e + L(1 - T_c)k_d \quad (5.7) \\ &= (1 - 0.4)17\% + 0.4(1 - 0.33)7.5\% \\ &= 12.2\%\end{aligned}$$

and

$$\begin{aligned}\hat{\sigma}(\widehat{\text{WACC}}) &= (1 - L)\hat{\sigma}(\hat{R}_e) \quad (5.8) \\ &= (1 - 0.4)4.1\% \\ &= 2.46\%\end{aligned}$$

The 95% confidence interval for the WACC estimate is constructed using (5.4). That is

$$\begin{aligned}\widehat{\text{WACC}}^{\text{lower}} &= \widehat{\text{WACC}} - 1.96\hat{\sigma}(\widehat{\text{WACC}}) \quad (5.9) \\ &= 12.2\% - 1.96 \times 2.46\% = 7.38\% \\ \widehat{\text{WACC}}^{\text{upper}} &= \widehat{\text{WACC}} + 1.96\hat{\sigma}(\widehat{\text{WACC}}) \\ &= 12.2\% + 1.96 \times 2.46\% = 17.0\%\end{aligned}$$

Table 5.2 summarizes estimated WACCs, the 95% confidence interval and the standard error. The WACC estimated using the empirical CAPM is approximately 5% greater than the Commission's estimate (7.35%) that

Table 5.2: The WACC and its 95% confidence interval for the electricity line business estimated using the empirical CAPM

This table shows the WACC and its 95% confidence interval estimated using the empirical CAPM.

Estimation methods	Lower bound	Mean	Upper bound	Std Err
FM SB: OLS	7.38%	12.2%	17.0%	2.46%
FM SB: SW	7.94%	12.7%	17.4%	2.42%
FM PB: OLS+OLS	9.23%	14.1%	18.9%	2.48%
FM PB: SW+SW	5.81%	11.5%	17.2%	2.90%
FM PB: SW+OLS	8.43%	13.7%	19.0%	2.70%
Commission's estimates		7.35%		

roughly equal the lower bound of the confidence interval. It may be a sign that the New Zealand market requires more returns to compensate the systematic risk of investing in electricity line businesses than what the Commission has reported.

Table 5.3: The probability of the underestimation of the Commission's WACC: case study 1

This table shows the probability of the underestimation of the Commission's WACC. The mean and standard error of the "true" WACC are assumed to be the estimate from this paper. Test statistics, Z , is calculated using (5.10). "P-value" is calculated under the alternative hypothesis of $H_a : WACC^{cc} < WACC$. The probability of underestimation is given by $(1 - \text{P-value})$

Method	Mean	Std Err	Z	P-value	Probability of underestimation
FM SB: OLS	12.2%	2.46%	-1.97	0.02	0.98
FM SB: SW	12.7%	2.42%	-2.21	0.01	0.99
FM PB: OLS+OLS	14.1%	2.48%	-2.72	0.01	0.99
FM PB: SW+SW	11.5%	2.90%	-1.43	0.08	0.92
FM PB: SW+OLS	13.7%	2.70%	-2.35	0.01	0.99

There is an interesting question that how certain (or what the probability will be that) the WACC is understated by the Commission await to be answered. By assuming that the WACC distributions estimated from this paper are the true distributions, the probability can be calculated by testing $H_0 : WACC^{cc} = WACC$ and $H_a : WACC^{cc} < WACC$, where $WACC^{cc}$ is the Commission's value whereas $WACC$ is the true value. The test statistics is

$$Z = \frac{WACC^{cc} - WACC}{\sigma(WACC)}, \quad (5.10)$$

where $WACC$ and $\sigma(WACC)$ is assumed to be equal to estimates from this

paper. Then, $(1 - \text{P-value})$ from the above test can be interpreted as the probability of the underestimation of the Commission's WACC. As shown in Table 5.3, the null hypothesis of $H_0 : \text{WACC}^{cc} = \text{WACC}$ is rejected at the 5% level, or in other words there is, on average, 95% chance that the Commission's WACC for electricity line businesses is underestimated.

The second case study is derived from a more recent Commission report (Commerce Commission (2007)) which is on regulating the supply control of the natural gas distribution services by Powerco Ltd and Vector Ltd. The WACC in the study has similar roles compared to the one in electricity line businesses, which is for the price control and the profits assessment. The relevant parameters are given by

$$\begin{aligned} L &= 0.4 & \beta_e &= 0.93 & T_c = T_I &= 0.33 \\ R_f &= 0.0588 & MRP &= 0.07 & k_d &= 0.0688 \end{aligned}$$

The Commission used the same methodology to calculate the WACC which is reported equal to 8.13%³ and the standard deviation is equal to 1.5%. As noted, the equity beta in this case study is higher than the one in the previous study. Consequently, with other parameter values remaining the same or being similar, applying the theoretical CAPM will yield a higher WACC (8.13% v.s. 7.3%).

On the other hand, will the same situation occur if the empirical CAPM is applied? Table 5.4 summarizes the WACC and its 95% confidence interval (the methods used are the same as in the first case study with some pa-

³In the report, it uses the different corporate tax rate and investor tax rate after the 31th Mar 2008. Therefore, it produces two WACC estimates before and after this date. I use the one calculated before the date as our data period is not beyond 2008.

parameter values changed, e.g, R_f and k_d), and the Commission's estimates.

Table 5.4: The WACC and its 95% confidence interval for gas pipeline businesses estimated using the empirical CAPM

This table shows the WACC and its 95% confidence interval estimated using the empirical CAPM and the Commission's method. Reported from the commission's study, the WACC is equal to 8.13% and the standard deviation equals 1.5%, which gives a 95% confidence interval of 5.19% ($8.13\% - 1.96 \times 1.5\%$) and 11.07% ($8.13\% + 1.96 \times 1.5\%$).

Estimation methods	Lower bound	Mean	Upper bound	Std Err
FM SB: OLS	6.52%	11.4%	16.36%	2.51%
FM SB: SW	7.16%	11.9%	16.71%	2.43%
FM PB: OLS+OLS	6.36%	11.3%	16.34%	2.55%
FM PB: SW+SW	6.94%	11.8%	16.63%	2.47%
FM PB: SW+OLS	6.75%	11.6%	16.50%	2.49%
Commission's estimates	5.19%	8.13%	11.07%	1.5%

Again, Table 5.4 illustrates that the Commission may underestimate the WACC compared with ones estimated using the empirical CAPM. Overall, the mean value of the WACC is underestimated by approximately 3%.

In addition, it seems that the Commission calculated a relatively small standard error (1.5%) for the WACC estimate, which is, however, not produced from any statistically-sound way. For example, with the standard

errors being provided for the relevant parameters, the Commission simply aggregates them in line with the CAPM formula to yield the standard error of the cost of equity, which only takes into account of possible errors in the input variables and ignores the potential for specification error in the model.

In contrast, this paper provides a solution for the problem arising from the Commission's method to calculate the standard error. The possible errors arising from the specification in the model are captured in γ_i^* . Furthermore, as the empirical CAPM is estimated from the market data, it automatically incorporates estimation errors from proxies of the risk free rate and the return on the market portfolio respectively. With two sources of error retaining in the WACC estimates, the standard error reported in this paper is consequently larger.

Figure 5.1 shows distributions for all WACC estimates. Knowing the mean and standard deviation, each curve in the graph is plotted assuming the WACC is normally distributed. The distribution of the Commission's WACC is to the left of mine with less spread, which is because the less mean value and the smaller standard error. On the other hand, distributions of WACC estimated using empirical CAPMs are very similar to each other, which signals the robustness of estimation results⁴.

Table (5.5) is constructed in the same way as for the first case study. It

⁴All results presented in this chapter are replicated and presented in the Appendix (Table 7.7 and Figure 7.3), where the alternative proxy for the risk free rate, the capital gain return on the bond, is used. It appears that results are robust to the different choice of proxies for the risk free rate as well.

Figure 5.1: The distribution of the WACC calculated using the theoretical and empirical CAPM

also illustrates that the Commission' WACC for gas pipeline businesses is, highly likely (90% on average), underestimated if assuming the true WACC distribution is equal to what have been estimated in this paper.

These case studies demonstrate that the Commission's WACC estimates may understate the cost of capital required by the New Zealand market (with high probabilities). The WACCs are understated by approximately 5.5% for electricity line business and 3.5% for the gas pipeline businesses. Although our estimates have larger standard errors, they are estimated in a statistically-sound way compared to the Commission's estimates.

Table 5.5: The probability of the underestimation of the Commission's WACC: case study 2

This table shows the probability of the underestimation of the Commission's WACC. The mean and standard error of the "true" WACC are assumed to be the estimate from this paper. Test statistics, Z , is calculated using (5.10). "P-value" is calculated under the alternative hypothesis of $H_a : WACC^{cc} < WACC$. The probability of underestimation is given by $(1 - \text{P-value})$

Method	Mean	Std Err	Z	P-value	Probability of underestimation
FM SB: OLS	11.4%	2.51%	-1.30	0.1	0.90
FM SB: SW	11.9%	2.43%	-1.55	0.06	0.94
FM PB: OLS+OLS	11.3%	2.55%	-1.24	0.11	0.89
FM PB: SW+SW	11.8%	2.47%	-1.48	0.07	0.93
FM PB: SW+OLS	11.6%	2.49%	-1.39	0.08	0.92

Moreover, the estimated empirical CAPMs show that the market is not as responsive to the systematic risk as the CAPM theory states, therefore, using the theoretical model may misinterpret how systematic risks are truly valued by the market. Consequently, the Commission's WACC estimates may not be the true value or an even a good estimate of the true value.

Last but not least, applying either the theoretical or empirical CAPM to calculate the WACC should be viewed with great caution, especially for businesses in regulatory settings. Both forms of CAPM only value sys-

tematic risks without considering unsystematic risks that are often taken into account by regulated-business-investors. Therefore, applying either the theoretical or empirical CAPM as a part for calculating the WACC may undervalue the cost of capital required by the market and investors in New Zealand.

Conclusion

Outlined in the Commission’s draft guideline, the cost of equity, the key component in the calculation of the cost of capital, is estimated using the “strict” version of the capital asset pricing model, whose empirical reliability has been consistently questioned for the past 3 decades. In contrast, I propose to use the empirical CAMP to calculate the cost of equity, which is then used in the calculation of the cost of capital for regulated businesses. I believe there are two advantages for using the empirical CAPM. It provides a point estimate that reflects the expected return required by the market, in addition, it also offers a better estimate of the size of the estimation error of the cost of equity and the cost of capital.

The empirical CAPM is estimated using the framework from Fama and MacBeth (1973) with the latest New Zealand data. In the process of estimating the empirical CAPM, the ordinary least square method was applied to estimate stock and portfolio betas. Scholes and Williams (1977) model was adopted to overcome the thin trading problem. The monthly

data and the OLS method are used for the baseline case while weekly data and Scholes-William model are applied for a check of robustness.

Results from the baseline case and the robustness check show that the empirical CAPM is virtually a flat line with wide confidence intervals. The cost of equity calculated using the empirical CAPM contains large standard errors, which reflect the great uncertainty in the empirical relationship between the expected return and the systematic risk in New Zealand.

The estimated empirical CAPM is applied to calculate the costs of capital for two case studies derived from the Commission's reports on electricity line businesses (Commerce Commission (2005b)) and gas pipeline businesses (Commerce Commission (2007)). It is found that the empirical-CAPM-based-WACC has relatively larger standard errors compared with ones calculated using the theoretical CAPM. This may be because that the Commission does not use any statistically-sound way to estimate its standard errors. The Commission's WACC estimates also tend to underestimate the cost of capital required by the investor compared with estimates from this paper – 7.35% compared to 12.5% (on average) for electricity line businesses, and 8.13% compared to 11.5% (on average) for gas pipeline businesses.

Caution should be used when applying either model in a regulatory setting. The cost of capital for regulated businesses can directly affect investors' incentives for investment, which can have profound effects on capital investments in New Zealand and national economic growth. Using either the theoretical or empirical CAPM to determine the cost of capital should just be the starting point, not the conclusion for decision makers.

Chapter 7

Appendix

Table 7.1: Stock betas estimated using the SW method with monthly data

This table shows stock beta estimates. Stocks are selected based on 2 criteria: 1.) a selected stock must have at least two latest years of returns available (out of 3 years of beta estimation period) in order to estimate stock betas. 2.) a selected stock must also have all returns available in the following year in order to perform the cross-sectional regression. The second column shows the number of stocks meeting both conditions. Following columns present the summary statistics of estimated stock betas. The last column shows the value-weight stock betas for each estimation period, which is calculated by taking the average of market capitalization of each stock at each period (90-92, 91-93 etc), dividing it by the sum of all stocks' average market capitalization at the same period to yield w_i , and finally applying Equation (4.1).

	No. of stocks	Min	Median	Max	Mean ^V
1990-1992	86	-1.31	0.85	3.85	1.05
1991-1993	90	-10.69	0.11	12.13	0.85
1992-1994	99	-2.25	0.74	7.13	1.00
1993-1995	104	-1.58	1.12	6.89	1.07
1994-1996	112	-2.59	0.98	4.75	1.05
1995-1997	110	-2.94	1.00	9.18	1.19
1996-1998	107	-1.13	1.07	5.13	1.08
1997-1999	113	-1.47	0.93	4.86	0.90
1998-2000	104	-2.80	1.04	3.36	0.91
1999-2001	105	-5.94	0.73	6.48	1.05
2000-2002	116	-3.70	0.85	4.86	0.99
2001-2003	119	-6.20	0.79	6.87	1.05
2002-2004	114	-4.44	1.08	9.04	0.88
2003-2005	113	-2.80	1.11	5.37	1.00

Figure 7.1: The Histogram of $\hat{\gamma}_{i,t}$ estimated where the SW method is used to estimate stock betas

(a) $\gamma_{0,t}$

(b) $\gamma_{1,t}$

Table 7.2: The summary of portfolio betas estimated using the combination of the SW and OLS method

Portfolio betas are estimated by regressing portfolio returns on market portfolio returns for the entire period – 1993-2006. $\hat{\alpha}$ is the estimated intercept coefficient, which is scaled up by 100 times in order to be visible in 3 decimal places; $\hat{\beta}$ is the estimated slope coefficient. The bracketed numbers are t-statistics of null hypothesis if $\alpha = 0$ or $\beta = 0$. “SW+SW” means the SW method is used for estimating stock betas while the same method is used for estimating portfolio betas. “SW+OLS” means the SW method is used for estimating stock betas while the OLS method is used for estimating portfolio betas.

SW+SW										
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
100α	0.12	0.44	0.15	0.12	0.42	0.94	0.43	0.49	0.18	0.65
	(0.25)	(1.05)	(0.39)	(0.33)	(1.31)	(1.79)	(1.7)	(1.33)	(0.43)	(0.92)
β	0.98	1.01	0.60	0.99	0.90	1.26	0.99	1.15	1.04	0.84
	(8.43)	(10.53)	(6.51)	(12)	(12.57)	(10.28)	(17.27)	(12.67)	(10.33)	(4.71)
SW+OLS										
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
100α	0.42	0.58	0.02	0.19	0.48	1.07	0.48	0.39	0.28	0.48
	(0.96)	(1.7)	(0.05)	(0.62)	(1.83)	(2.44)	(2.13)	(1.03)	(0.71)	(0.66)
β	0.63	0.74	0.82	0.85	0.83	0.98	0.90	1.22	0.90	1.04
	(6.04)	(9.16)	(9.69)	(11.78)	(13.44)	(9.4)	(16.69)	(13.58)	(9.61)	(6.04)

Table 7.3: Stock betas estimated using the OLS and SW methods with weekly data

This table shows the information on the estimation of stock betas. Stocks are selected based on the 2 criteria: 1.) a selected stock must have at least the 30 latest weeks of returns available (out of 1 year of beta estimation period) in order to estimate the stock beta. 2.) a selected stock must also have all returns available in the following year in order to form portfolio returns. The second column shows the number of stocks meeting both conditions. The following columns present the summary statistics of the estimated stock betas. The last column shows the value-weight stock beta for each estimation period, which is calculated by taking the average of market capitalization from each stock in each period (1-week90 to 52-week-90 etc), dividing it by the sum of all stocks' average market capitalization at the same period to yield w_i and finally applying Equation (4.1).

		OLS	SW
No. of stocks		Mean ^V	Mean ^V
1990	82	0.98	0.91
1991	86	0.94	1.04
1992	92	1.00	0.98
1993	102	0.95	0.96
1994	123	1.00	1.00
1995	119	0.90	0.95
1996	120	0.97	1.03
1997	122	1.02	1.03
1998	120	0.90	0.86
1999	122	0.91	0.98
2000	114	0.99	0.97
2001	124	0.87	0.94
2002	126	0.99	1.01
2003	128	0.98	0.96
2004	128	0.97	1.02
2005	131	0.93	1.03

Table 7.4: The summary of portfolio betas estimated using combinations of the SW and OLS methods with weekly data

Portfolio betas are estimated by regressing portfolio returns on market portfolio returns for the entire period – 1993-2006. $\hat{\alpha}$ is the estimated intercept coefficient, which is scaled up by 100 times in order to be visible in 3 decimal places; $\hat{\beta}$ is the estimated slope coefficient. The bracketed numbers are t-statistics of null hypothesis if $\alpha = 0$ or $\beta = 0$. “SW+SW” means the SW method is used for estimating stock betas while the same method is used for estimating portfolio betas. “SW+OLS” means the SW method is used for estimating stock betas while the OLS method is used for estimating portfolio betas and so forth.

OLS+OLS										
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
100α	0.48	0.25	0.22	0.17	0.1	0.28	0.14	-0.09	-0.11	0.03
	(2.88)	(2.9)	(2.96)	(2.4)	(1.43)	(3.87)	(1.76)	(-1.25)	(-1.36)	(0.36)
β	0.45	0.4	0.33	0.48	0.47	0.6	0.59	0.62	0.84	0.95
	(5.64)	(9.97)	(9.38)	(14.63)	(14.26)	(17.7)	(16.17)	(18.12)	(22.59)	(21.83)
SW+SW										
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
100α	0.12	0.17	0.17	0.16	0.04	0.07	-0.07	-0.09	0.05	0.12
	(0.89)	(2.17)	(1.69)	(2.23)	(0.52)	(1.26)	(-0.99)	(-1.25)	(0.63)	(1.18)
β	0.52	1.06	0.92	0.62	1.05	0.9	1.19	1.11	1.45	0.84
	(7.64)	(26.27)	(19.17)	(17.99)	(30.29)	(30.18)	(33.2)	(30.23)	(41.24)	(15.63)
SW+OLS										
	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
100α	0.14	0.25	0.21	0.17	0.08	0.11	-0.02	-0.04	0.12	0.11
	(1.02)	(2.84)	(2.27)	(2.37)	(1.05)	(1.7)	(-0.26)	(-0.54)	(1.57)	(0.91)
β	0.39	0.43	0.5	0.48	0.69	0.54	0.73	0.64	0.81	0.98
	(6.13)	(10.47)	(11.22)	(14.09)	(19.83)	(17.21)	(21.26)	(18.12)	(23.45)	(17.61)

Table 7.5: The summary of γ_i^* – FM SB: OLS and SW; FM PB: OLS+OLS, SW+SW and SW+OLS (with the alternative risk free rate)

The table presents all γ_i^* . Results for FM SB: OLS and SW, and FM PB: OLS+OLS are obtained from Table 4.2 and 4.5 respectively. OLS+OLS, SW+SW and SW+OLS represent combinations of method. The first method is the method used to estimate stock betas while the second is for estimating portfolio betas. Results from the OLS+OLS combination are the baseline results while ones from the SW+SW and SW+OLS combinations are for the robustness check.

FM SB: OLS				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	1.54%	0.37%	0	-4.96×10^{-6}
$\gamma_{1,t}$	-0.42%	0.30%	0.16	-4.96×10^{-6}
FM SB: SW				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	1.46%	0.34%	0	-2.05×10^{-6}
$\gamma_{1,t}$	-0.20%	0.20%	0.32	-2.05×10^{-6}
FM PB: OLS+OLS				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
γ_0	2.66%	0.71%	0	-5.15×10^{-5}
γ_1	-1.71%	0.82%	0.037	-5.14×10^{-5}
FM PB: SW+SW				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	1.14%	0.74%	0.12	-5.41×10^{-5}
$\gamma_{1,t}$	0.065	0.81%	0.94	-5.41×10^{-5}
FM PB: SW+OLS				
	γ_i^*	$\hat{\sigma}(\gamma_i^*)$	P-value	$c\hat{o}v(\gamma_0^*, \gamma_1^*)$
$\gamma_{0,t}$	1.63%	1.01%	0.11	-1.13×10^{-4}
$\gamma_{1,t}$	-0.49%	1.17%	0.68	-1.13×10^{-4}

Table 7.6: The 95% confidence interval of the estimated annual tax-adjusted excess returns for two special securities: $\beta = 0$ and 1 – a summary (with the alternative risk free rate)

This table shows the 95% confidence interval constructed for the estimated annual tax-adjusted excess returns of 2 special securities with $\beta = 0$ and 1 respectively. All results presented in this table are either drawn from previous results or newly estimated

$\beta=0$				
	Lower bound	Mean	Upper bound	Std Err
FM SB OLS	11.4%	20.2%	28.9%	4.45%
FM SB SW	11.0%	19.1%	27.1%	4.09%
FM PB OLS+OLS	20.4%	37.0%	53.7%	8.5%
FM PB SW+SW	-2.75%	14.6%	32.0%	8.9%
FM PB SW+OLS	-2.47%	21.5%	45.5%	12.2%
$\beta=1$				
	Lower bound	Mean	Upper bound	Std Err
FM SB OLS	5.96%	14.3%	22.7%	4.27%
FM SB SW	8.25%	16.2%	24.2%	4.01%
FM PB: OLS+OLS	2.95%	12.0%	21.1%	4.63%
FM PB: SW+SW	7.33%	15.5%	23.7%	4.18%
FM PB: SW+OLS	5.48%	14.7%	23.9%	4.71%

Figure 7.2: The plot of the empirical and theoretical forms of the CAPM – a summary (with an alternative risk free rate)

(a) FM SB: OLS

(b) FM SB: SW

(c) FM PB: OLS+OLS

(d) FM PB: SW+SW

(e) FM PB: SW+OLS

Table 7.7: The WACC of electricity line businesses and gas pipeline businesses estimated using the empirical CAPM – a summary (with an alternative risk free rate)

This table shows the estimated WACC and its 95% confidence interval calculated from using the empirical CAPM and the Commission's method. From the commission's study, the WACC is equal to 8.13% and the standard deviation equals 1.5%, which gives a 95% confidence interval of 5.19% ($8.13\% - 1.96 \times 1.5\%$) and 11.07% ($8.13\% + 1.96 \times 1.5\%$).

Case study I: electricity line businesses				
Estimation methods	Lower Bound	Mean	Upper bound	Std Err
FM SB OLS	9.58%	14.3%	19.0%	2.40%
FM SB SW	10.22%	14.8%	19.4%	2.35%
FM PB (OLS+OLS)	11.16%	16.4%	21.2%	2.45%
FM PB (SW+SW)	8.86%	13.7%	18.5%	2.46%
FM PB (SW+OLS)	9.35%	14.7%	20.0%	2.73%
Commission's estimates		7.35%		
Case study II: pipeline business				
Estimation methods	Lower Bound	Mean	Upper bound	Std Err
FM SB OLS	8.12%	13.1%	17.98%	2.51%
FM SB SW	9.33%	14.1%	18.79%	2.41%
FM PB (OLS+OLS)	7.31%	12.4%	17.48%	2.60%
FM PB (SW+SW)	8.84%	13.5%	18.13%	2.37%
FM PB (SW+OLS)	8.29%	13.3%	18.34%	2.56%
Commission's estimates	5.19%	8.13%	11.07%	1.5%

Table 7.8: The probability of the underestimation of the Commission's WACC for both studies – a summary (with alternative risk free rate)

This table shows the probabilities of the underestimation of the Commission's WACC. The mean and standard error of the "true" WACC are assumed to be estimates from this paper. Test statistics, Z , is calculated from (5.10). "P-value" is calculated under the alternative hypothesis of $H_a : WACC^{cc} < WACC$. The probability of underestimation is given by $(1 - \text{P-value})$

Case study I: electricity line businesses					
Method	Mean	Std Err	Z	P-value	Probability of underestimation
FM SB: OLS	14.3%	2.40%	-2.90	0.002	0.998
FM SB: SW	14.8%	2.35%	-3.17	0.001	0.999
FM PB: OLS+OLS	16.4%	2.45%	-3.69	0.001	0.999
FM PB: SW+SW	13.7%	2.46%	-2.58	0.005	0.995
FM PB: SW+OLS	14.7%	2.73%	-2.96	0.0041	0.996
Case study II: pipeline business					
Method	Mean	Std Err	Z	P-value	Probability of underestimation
FM SB: OLS	13.1%	2.51%	-1.98	0.024	0.976
FM SB: SW	14.1%	2.41%	-2.48	0.007	0.993
FM PB: OLS+OLS	12.4%	2.60%	-1.64	0.05	0.95
FM PB: SW+SW	13.5%	2.37%	-2.27	0.01	0.99
FM PB: SW+OLS	13.3%	2.56%	-2.02	0.02	0.98

Figure 7.3: The distribution of the WACC calculated using the theoretical and empirical CAPM (with an alternative risk free rate)

Bibliography

- Banz, R. W. (1981). The relationship between return and market value of common stocks. *Journal of Financial Economics*, 9(1):3–18.
- Bartholdy, J., Fox, K., Gilbert, C., Hibbard, R., McNoe, W., Potter, M., Shi, J., and Watt, K. (1996). Estimating beta for new zealand companies. *New Zealand Investment Analyst*, pages 14–22.
- Bartholdy, J. and Riding, A. (1994). Thin trading and the estimation of betas: The efficacy of alternative techniques. *Journal of Financial Research*, 17/2:241–254.
- BASU, S. (1983). The relationship between earnings' yield, market value and return for nyse common stocks: Further evidence. *Journal of Financial Economics*, 12:129–156.
- Black, F., Jensen, M. C., and Scholes, M. (1972). *The Capital Asset Pricing Model: Some Empirical Tests*. Studies In The Theory Of Capital Markets (Praeger).

- Brennan, M. (1970). Taxes, market valuation and corporate financial policy. *National Tax Journal*, 23:417–427.
- Bryant, P. S. and Eleswarapu, V. R. (1997). Cross-sectional determinants of New Zealand share market returns. *Accounting and Finance*, 37:181–205.
- Casella, G. and Berger, R. L. (2002). *Statistical Inference*. 2nd edition.
- Cliffe, C. and Marsden, A. (1992). The effect of dividend imputation on company financing decisions and the cost of capital in New Zealand. *Pacific Accounting Review*, 4:1–30.
- Cohen, K. J., Hawawini, G. A., Maier, S. F., Schwartz, R. A., and Whitcomb, D. K. (1983). Friction in the trading process and the estimation of systematic risk. *Journal of Financial Economics*, 12(2):263–278.
- Commerce Commission (2005a). Draft guidelines: The Commerce Commission's approach to estimating the cost of capital.
- Commerce Commission (2005b). Regulation of electricity lines businesses, targeted control regime, intention to declare control. Technical report, New Zealand Commerce Commission.
- Commerce Commission (2007). Authorisation for the control of supply of natural gas distribution services by Powerco Ltd and Vector Ltd. Technical report, New Zealand Commerce Commission.
- Dimson, E. (1979). Risk measurement when shares are subject to infrequent trading. *Journal of Financial Economics*, 7:197–226.

- Fama, E. F. and French, K. R. (1992). The cross-section of expected stock returns. *Journal of Finance*, 47(2):427–465.
- Fama, E. F. and MacBeth, J. D. (1973). Risk, return and equilibrium: Empirical tests. *Journal of Political Economy*, 81(3):607–636.
- Fowler, D. J. and Rorke, C. H. (1983). Risk measurement when shares are subject to infrequent trading: Comment. *Journal of Financial Economics*, 12:279–283.
- Handa, P., Kothari, S. P., and Wasley, C. (1993). Sensitivity of multivariate tests of the capital asset-pricing model to the return measurement interval. *Journal of Finance*, 48(4):1543–1551.
- Jagannathan, R. and Wang, Z. (1996). The conditional capm and the cross-section of expected returns. *Journal of Finance*, 51(1):3–53.
- Kan, R. and Zhang, C. (1999). Two-pass tests of asset pricing models with useless factors. *Journal of Finance*, LIV(1):203–235.
- Lally, M. (1992). The CAPM Under Dividend Imputation. *Pacific Accounting Review*, 4:31–44.
- Lintner, J. (1965). The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics*, 47(1):13–37.
- Roll, R. (1977). A critique of the asset pricing theory's tests: On past and potential testability of the theory. *Journal of Financial Economics*, 4(2):129–176.

- Roll, R. and Ross, S. A. (1994). On the cross-sectional relation between expected returns and betas. *Journal of Finance*, 49(1):101–121.
- Scholes, M. and Williams, J. (1977). Estimating betas from nonsynchronous data. *Journal of Financial Economics*, 5:309–327.
- Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3):425–442.