Posteriori Reconstruction and Denoising for Path Tracing

by

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Abstract

Path tracing is a well-established technique for photo-realistic rendering to simulate light path transport. This method has been widely adopted in visual effects industries to generate high quality synthetic images requiring a large number of samples and a long computation time. Due to the high cost to produce the final output, intermediate previsualization of path tracing is in high demand from production artists to detect errors in the early stage of rendering. But visualizing intermediate results of path tracing is also challenging since the synthesized image with limited samples or improper sampling usually suffers from distracting noise. The ideal solution would be to provide a highly plausible intermediate result in the early stages of rendering, using a small fraction of samples, and apply a posteriori manner to approximate the ground truth.

In this thesis, this issue is addressed by providing several efficient posteriori reconstructions and denoising technique for previsualization of path tracing. Firstly, we address the problem for the recovery of the missing values to construct low rank matrices for incomplete images including missing pixel, missing sub-pixel, and multi-frame scenarios. A novel approach utilizing a convolutional neural network which provides fast precompletion for initializing missing values, and subsequent weighted nuclear norm minimization with a parameter adjustment strategy efficiently recovers missing values even in high frequency details. The result shows better visual quality compared to the recent methods including compressed sensing based reconstruction.

Furthermore, to mitigate the computation budgets of our new approach, we extend our method by applying a block Toeplitz structure forming

a low-rank matrix for pixel recovery, and tensor structure for multi-frame recovery. In this manner, the reconstruction time can be significantly reduced. Besides that, by exploiting temporal coherence of multi-frame with a tensor structure, we demonstrate an improvement in the overall recovery quality compared to our previous approach.

Our recovery methods provide satisfying solution but still require plenty of rendering time at prior stage compared with denoising solutions. Finally, we introduce a novel filter for denoising based on convolutional neural network, to address the problem as conventional denoising approach for rendered images. Unlike a plain CNN that applies fixed kernel size in each layer, we propose a multi-scale residual network with various auxiliary scene features to leverage a new efficient denoising filter for path tracing. Our experimental results show on par or better denoising quality compare to state-of-the-art path tracing denoiser.

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Chapter 1

Introduction

1.1 Motivations and Problem Statement

In the visual effects industries, rendering is one of the most crucial and time-consuming tasks. As a general estimate, rendering one frame in production quality could require hours or days in today's computer animated movies, and the time of rendering in production keeps increasing for realism and fidelity.

The core process of rendering is to simulate all possible light paths in a virtual scene, which has been well addressed by path tracing in computer graphics. Path tracing is usually evaluated by Monte Carlo integration technique, which is a general approach for computing the integral over all light paths. Monte Carlo integration technique can robustly estimate the high-dimensional integrals in the rendering equation [5], making it a powerful tool for computing the light transport in complex scenes. However, Monte Carlo integration is also known to be inefficient due to its relatively slow converging rate of $\frac{1}{\sqrt{N}}$, here N is the number of samples. Using insufficient samples causes undesirable artifacts in the final image (e.g., noise, blur, and banding). A naive path tracing algorithm often requires a large number of samples to be drawn and evaluated in order to reconstruct a high quality image, but a long rendering time is required accordingly. Due

to the high cost to produce the final output, intermediate previsualization of path tracing is in high demand from production artists to detect errors in the early stage of rendering.

Addressing this problem is challenging, various approaches have been presented to address the noise or variance reduction issue. Such as efficient sampling patterns (importance sampling, Metropolis sampling), statistical analysis models (density estimation, variance control), and image processing techniques (frequency domain analysis, non-linear filtering).

In this thesis, rather than pursuing analysis of light transport equation and sampling pattern, we explore posteriori approaches that provide a highly plausible intermediate result in the early stages of rendering, using a small fraction of samples, and apply a post process on the rendered data to efficiently approximate the ground truth rendering. We address the problem as reconstruction and denoising.

Reconstruction: It exploits the coherence of the visual signal to reconstruct missing samples based on partially and randomly rendered ones. Several previous techniques have been proposed.

Recent inpainting methods in image processing show promising results to recover missing pixels from limited known pixel samples [6, 7]. However, these methods have limitations when used for previsualization of path tracing. Due to the random nature of rendered pixels, the methods cannot provide the localized scene structure required to prevent boundary edge blurring. Recent example based inpainting methods address the issue more efficiently, but they often suffer from long recovery time [6, 4, 8]; one of the important requirements for previsualization is to show believable intermediate results within an acceptable time.

Recent compressive rendering (CR) has been proposed as a previsualization solution by Sen et al. [9]. CR is based on compressed sensing (CS), which can reconstruct an unknown signal from a few samples provided the signal has a sparse representation in some basis. CR shows believable reconstruction of missing pixels in an incompletely rendered image within

an acceptable time. However, it has limitations, when the sampling rate is low (like 10%), and the final output quality relies on the choice of basis in the transform domain.

Denoising: A large amount of works have been proposed to render a noisy result in a quick manner with low sample budget and then apply a denoising filter to output the noise-free result [10, 11, 12, 13, 14, 15]. These methods are scene feature based filters, like the non-local mean filter and bilateral filter, which can utilize additional scene feature buffers (e.g., world position, depth information) to guide the denoising process. Since the additional scene information is closely related to scene details, these features based denoising filters could achieve a good quality result. However, the primary challenge is how to set the filter weights for different features properly. Many approaches use different error estimation metrics to generate the optimum filter parameters. Since the error metrics are noisy under a low sample budget, the accuracy of the selected parameters are not the optimum. Therefore, the denoised results from these methods may suffer from the over-blurred area, especially in regions with high-frequency details.

Based on the analyses of this section, the issues/questions that deserve investigation are described as follows.

- Reconstruction based methods have demonstrated the ability to augment various rendering algorithms and scenes as a postprocessing step. The remaining technical issue is how to generate visually believable result under the consideration of both low sampling rate and short recovery time.
- As denoising based filters can utilize additional scene features, the main issue is how to provide suitable filter parameters with regard to various scene features. More importantly, a denoising filter work robustly and efficiently even with low sampling budget.

Applying these challenges to the path tracing previsualization prob-

lem, we have the following objectives to investigate:

1. Develop reconstruction based method for previsualization of path tracing in feature film production. Existing reconstruction based methods fall into two categories, inpainting and compressed sensing based methods. Specifically, inpainting methods often suffer from long recovery time that fails to show believable intermediate results within an acceptable time. In contrast, compressed sensing based methods mainly work well under a high sampling rate. Therefore, the new reconstruction based framework will aim for both efficiency and robustness.
2. Design and develop new denoising based previsualization method, which can robustly handle suitable parameters for multiple scene features. Although a wide range of denoising techniques of path tracing have been proposed, how to perform consistently well and easy of adoption is still an issue worth exploiting.

1.2 Thesis Contributions

In this thesis, our goal is to provide efficient and optimal postprocessing methods for previsualization of path tracing. The problem is addressed in two manners: reconstruction and denoising. The main difference of the two categories is that reconstruction based methods restore the missing or unrendered samples based on rendered ones, while denoising based methods remove the noise through adjusting the rendered pixel values.

To develop reconstruction based methods, we exploit the coherence of the visual signal to reconstruct missing samples, using a low-rank matrix completion and tensor completion framework. Matrix completion and tensor completion have been recently introduced [16, 17, 18] and successfully applied to many problems in image processing [19, 20, 21], but have not been adopted in the computer graphics community. Applied to our problem, the basic strategy is to construct a matrix from a group of similar

pixels or patches, and then use matrix completion to estimate the missing samples. Additionally, applying the tensor structure to represent multi-frames and recover missing samples by tensor completion.

To explore more complex and general denoising based method, we utilize a deep learning based denoising filter that leverages convolutional neural network (CNN) architecture. The CNN has proven the ability to learn complex non-linear models. Once the neural network is trained, it can be easily and efficiently applied without complex parameter selection. Such a framework can more efficiently handle the task of previsualization of path tracing in real feature film production.

The contributions of this thesis include new algorithms and new analysis, which can be summarized as follows:

- Chapter 3 formalizes the recovery of missing data in incompletely rendered images as a matrix completion problem. Specifically, we adopt the low-rank matrix completion scheme for previsualization of path tracing. Although matrix completion and separation have been used for many lights rendering [22] and inpainting [4] recently, they have not yet been applied to recover sub-pixel samples of rendering or pixel samples in multiple frames. To form a low-rank matrix, we show how to formulate pixel, sub-pixel, and multi-frame sample problems as matrix completion. Our method contributes to reducing spatial and temporal noise in reconstructed images.

Additionally, we have developed novel nuclear norm minimization methods using a multi-weighted approach (WNNM) with a progressive adjustment strategy (PAWNNM). Our method has been evaluated against state-of-the-art methods and shows better results regarding visual quality including CS based reconstruction. In particular, our method outperforms with limited samples (e.g., $\leq 40\%$).

The work in this chapter was published in the following paper:

P. Liu, J. Lewis and T. Rhee, "Low-Rank Matrix Completion to Re-

construct Incomplete Rendering Images,” in *IEEE Transactions on Visualization and Computer Graphics*, vol. 24, no. 8, pp. 2353-2365, 1 Aug. 2018.

- Based on Chapter 3, we present novel methods in Chapter 4 to construct enhanced low-rank matrices and low-rank tensor for dealing with missing pixel and multi-frame scenarios respectively. Specifically, we adopted the latest enhanced matrix completion in pixel recovery and tensor completion for the multi-frame case. Then we apply weighted nuclear norm minimization with a parameter adjustment strategy, which can more efficiently recover missing values even in high-frequency details.
- In Chapter 5, we present a novel denoising approach using convolutional neural network. Motivated by recent deep learning-based works, we apply a denoising network based on multi-scale residual learning. Unlike the previous CNN architectures, which employ a single scale filter in a convolutional layer or perform multi-scale concept through down/up scaling the feature maps, we use different filters during the convolution step to preserve both long and short contextual information. Additionally, we develop a separated residual learning strategy to predict the noise. Also we generate a virtual flash image as one of the input scene feature buffers to enhance the denoising performance in high-frequency area. Our method shows on par or better denoising quality compare to the state-of-the-art methods.

1.3 Thesis Outline

The rest of this thesis is organized as follows: Chapter 2 presents the background information and related works. Chapter 3 to Chapter 5 present major contributions of this thesis aiming to achieve efficient previzualiza-

tion for path tracing. Additionally, Chapter 6 presents conclusions and future directions.

- Chapter 2 provides a brief introduction of path tracing. Existing reconstruction based works and denoising based works are reviewed, which form the motivation of the work in this thesis.
- Chapter 3 introduces a low-rank matrix completion based reconstruction method for previsualization of path tracing. This method is described to handle sub-pixel, pixel and multi-frame recovery.
- Chapter 4 is built upon Chapter 3, which introduces enhanced matrix completion to reduce the computation time for pixel recovery in the previous chapter. In addition, a tensor completion based reconstruction method is proposed to achieve a more efficient and accurate recovery for the multi-frame case.
- Chapter 5 describes a new convolutional neural network based denoising filter. Specifically, we introduce a multi-scale filter based residual learning network for path tracing previsualization. This denoising method brings the power of neural work to remove noise from path tracing
- Chapter 6 provides major conclusions to the objectives achieved in this work along with limitations and several avenues for future research enabled by this thesis.

Chapter 2

Background and Related work

In this chapter, we start with a review of the background of path tracing (Section 2.1). The work described in this thesis takes a two-pronged post-process to accelerate previsualization of path tracing: (1) partially rendering a set of samples and recovering the missing ones based on the captured samples; (2) a denoising filter based on deep learning which filters the noise image rendered with a few samples per pixel. We review relevant works in two areas: reconstruction based techniques for recovering missing samples (Section 2.2), and denoising filter based techniques for removing the path tracing noise (Section 2.3). Given the topic of this thesis, the emphasis in this chapter will be previous works relating to reconstruction and denoising techniques.

2.1 Path Tracing

Rendering an image which simulates global illumination is a basic issue concerning the computation of pixel values. The pixel value refers to the radiance shooting from the visible surface point to the viewing camera. In addition, the radiance represents a function $L(x \rightarrow \omega)$ which satisfies the

following rendering equation:

$$L(x \rightarrow \omega) = L_e(x \rightarrow \omega) + \int_{\Omega} f_r(x, \omega', \omega) L(x \leftarrow \omega') \cos(n_x, \omega') d\sigma(\omega'). \quad (2.1)$$

According the equation, the amount of energy $L(x \rightarrow \omega)$ reflected from the object surface x to the direction ω that can be equivalent to the emitted radiance $L_e(x, \omega)$, with addition of the hemisphere integral on the received radiance $L(x \leftarrow \omega)$ that is weighted by the cosine of the angle between the normal vector n_x and ω' , and the BRDF term f_r . Finally, it measures the integration of solid angle. Figure 2.1 illustrates the core concept of the rendering equation. Sometimes Equation (2.1) could also be called the solid angle formulation of the rendering equation and use of the equivalent surface-area formulation is better, wherein incoming radiance is parameterized at surfaces instead of the directions.

$$L_o(x \rightarrow \omega) = L_e(x \rightarrow \omega) + \int_M f_r(x, \omega', \omega) L(x' \rightarrow x) G(x \leftrightarrow x') dA(x'). \quad (2.2)$$

Here, x' denotes the surface point, while $A(x')$ denotes the measure of the surface area. The geometry term $G(x \leftrightarrow x')$ is defined as follows:

$$G(x \leftrightarrow x') = \frac{\cos\theta_x \cos\theta_{x'}}{\|x - x'\|^2} V(x, x'). \quad (2.3)$$

where $V(x; x')$ is visibility term, and it equals to 1 when two surface points x and x' are visible between each other. Otherwise it will be 0. According to the rendering equation, reasons for difficulty of the problem can be found. Firstly, it is global since the energy of the outgoing radiance from a surface point is decided by the incoming radiance from various surface points. Secondly, the integral is continuous and should be described into sum. In the end, the light may recursively arrive at the camera by different times of bounces.

We can also express the rendering equation (2.1) in a path integration form. This form can derive algorithms which can be hardly established from Equation (2.1).

Figure 2.1: The rendering equation describes the total amount of radiance from a surface point x along a viewing direction and can be computed by adding the emission of and the integration of the incoming radiance over the hemisphere of the point, weighted by the BRDF term.

Through a given pixel j , the illumination I_j is denoted by

$$I_j = \int_{\Omega} f_j(\bar{x}) d_{\mu}(\bar{x}) \quad (2.4)$$

Where $x = x_0 x_1 \dots x_n$ denotes a complete light path. For example, x_0 is the point from a light source and x_n denotes the viewing camera. The measurement of surface area on each point x_i is defined as $\mu(\bar{x})$. The contribution $f_j(\bar{x})$ can be generated from the appropriate light emission, geometry terms, BRDFs and sensor response:

$$f_j(\bar{x}) = E(x \rightarrow \omega) G(x_0 \leftrightarrow x_1) f_r(x_0 \rightarrow x_1 \rightarrow x_2) G(x_1 \leftrightarrow x_2) \dots \dots f_r(x_{n-2} \rightarrow x_{n-1} \rightarrow x_n) G(x_{n-1} \leftrightarrow x_n) \quad (2.5)$$

Theoretically, each surface from the scene is emissive and reflective, but this rarely happens. As for a more frequent scenario, there are distinct light sources and only the illumination of them is reflected by the scene. Based

on this fact, global illumination is often divided into direct illumination L_d , emission L_e , and indirect illumination L_i :

$$L(x \rightarrow \omega) = L_e(x \rightarrow \omega) + L_d(x \rightarrow \omega) + L_i(x \rightarrow \omega), \quad (2.6)$$

here, the direct illumination refers to the total radiance over the surface from n light sources:

$$L_d(x \rightarrow \omega) = \sum_{i=1}^n \int_{A_i} f_r(x, \omega', \omega) L(y \rightarrow x) G(x \leftrightarrow y) dA(y), \quad (2.7)$$

where A_i denotes the area involved by the i -th light source. In general, the direct illumination can be deemed as a much easier problem in comparison with the indirect illumination. This is correct in most cases when the set of emissive surfaces under the situation comprise a few numbers of area lights or point lights. But many small light sources, large area light and scenes with environment maps often appear. In many-light formulation, all these situations are represented by many point lights. Based on certain proper probability distribution $p(x)$, Monte Carlo integration can generally compute the integral I concerning the function $f(x)$ towards the domain D with N random samples:

$$I = \int_D f(x) dx \approx \frac{1}{N} \sum_{i=1}^N \frac{f(x_i)}{p(x_i)}. \quad (2.8)$$

This is an unbiased method, since the expected value of the result equals to the computed integral. Ideally, a large number of samples could approximate the value of the integral.

Samples Issue. The main disadvantage of path tracing is that it is inherently a noisy process, which converges rather slowly to the right result. The noise in Monte Carlo integration corresponding to the estimation error due to considering only a finite number of samples, which leads to integral values that can be too high or too low (or pixels that are too dark or too bright in the rendering image). The noise can be reduced by increasing the sample budget per pixel and vanishes within the limit, as the

amount of samples increases to infinity, but the noise reduction is not linear with respect to the sampling rate. Actually, to reduce the magnitude of noise by a factor of two, we need to increase the sampling rate by a factor of four. This leads to rapidly increasing sampling budgets for non trivial scenes. Yet, the computation cost of path tracing is still problematic for offline rendering, and prohibitive for interactive or real-time rendering.

2.2 Related Work

In this section, the existing methods of reconstruction and denoising based previsualization for path tracing will be discussed.

2.2.1 Reconstruction Techniques for Path Tracing

Reconstruction techniques like compressive rendering [9] and compressive image reconstruction [23] have been proposed to address efficient previsualization of path tracing. As described in previous section, artifacts shown in intermediate previsualiations of path tracing are mainly due to improper or limited number of evaluated samples. Therefore, it is reasonable to formulate the issue as a data reconstruction problem to estimate missing samples (or pixels) using evaluated samples. The existing reconstruction based works can be divided into three branches: 1) inpainting based reconstruction; 2) compressed sensing based reconstruction; 3) matrix completion and separation based reconstruction.

Inpainting based Reconstruction

Image inpainting is widely discussed in various fields such as computer graphics, computer vision and image processing. The basic task of inpainting targets prediction of incomplete image pixels. The missing pixels include scratches, damaged photo stains and small holes. Inpainting is a

positive research field, wherein the methods can be separated into semi-automatic inpainting methods, partial derivative equations (PDE) based algorithms, algorithms that are based on templates/models, texture synthesis methods, and hybrid technologies based on specific features. According to the PDE model, the first approach was proposed by Bertalmio et al. [24], wherein the information is spread in wide areas, via isophote lines across the edges. The algorithm has high efficiency for images which suffer narrow damage, because it applies anisotropic diffusion leading to reduction in blurring effects. The major disadvantage of the method is embodied in the fact that it cannot set up textures [25]. The methods developed by Taschler [2] belong to the same type. The authors have come up with an algorithm according to partial differential equations of the second order with use of diffusion and the previous improved one [26]. The significant problem was not solved, namely, the algorithm could not set up the textures. Tschumperle and Deriche [27] presented a method with application of partial differential equations of higher order. It is not an image restoration technique, but it generates good results for images which have occluded regions of small area and narrow results.

As for types of the semiautomatic inpainting methods, Sun et al. [28] came up with a technique that needs two steps for restoration. First, the user must describe contours of the object located in the occluded region, which begins from the outside to the inside, and then use texture synthesis that applies blocks or pixel images as a texture source. The method developed by Oliveira et al. [29] applies isotropic diffusion, which could maintain the contours and separate the edges from the mask. Hence, they do not suffer influences of smoothing. Telea [30] tried to make the improvement based on the estimated pixel value, which is restored by pixel neighborhoods only once for each pixel. In comparison with the iterative methods, a seam carving method was shown in [31], which addressed the efficiency issue.

With regard to the texture synthesis methods, the technique developed

by Efros and Leung [32] takes a pixel as a starting point, while it is set on the edge of the occluded area and selects a window around it, so similar blocks in the region can be found. This method restores the texture for each pixel; therefore, the proposed algorithm overcomes the limitations of Bertalmio's algorithm [24].

Heeger and Bergen [33] presented a texture reconstruction method with application of intermediate images collections that establish an image pyramid. Their method comprises the interaction, where the image pyramid is created through division of the damaged image and the one which embodies the source. Their method can obtain satisfactory texture results through repeating the iterative steps, but is only valid with regard to the stochastic types. Bonet [34] proposed an improvement to reproduce the regular textures by considering dependencies among different levels of texture granularity. Igehy and Pereira [35] discussed another version of the algorithm that was proposed by Heeger and Bergen. The method involved a new step with a mask containing subunit values and the information from the original image involved in texture synthesis. The similar techniques could involve an algorithm based on templates, which was proposed by Criminisi et al. [36]. The authors are describing a technique that emphasizes the order of the restored pixels. The algorithm starts from the edge of the occluded area, while each pixel from the edge is assigned a priority. Texture synthesis is completed with blocks by reusing the information from a source area, which is based on the priority value determined for each pixel.

The algorithm proposed by Drori et al. [37] involves specific granularity levels, which are used for estimating the best levels. It then sets a filling order with means of a confidence value, followed by the step of search similar to Efros and Leung. Their algorithm applies several different orientations of the block. The inpainting algorithm of Guillemot et al. [38] tries to find k -nearest neighbors concerning the damage to be filled. Then the k -nearest neighbor search is improved with linear regres-

sion. Hays and Efros [39] came up with an approach which adopts a big image collection and uses it as a restoration database. As pointed out, the possibility in region restoring in a natural manner increases in view of the information quantity contained in the large images set. The restoration process includes checking of each item in the database aiming at possible matching of the damaged region with application of an image descriptor. The same approach was embodied by Le Meur and Guillemot [40], which provides an exemplar based inpainting framework. A coarse version is inpainted at first, so the computational complexity and noise sensitivity can be reduced, while the dominant orientations of image structures can be extracted. A novel sparsity concept aiming at the level of patch is introduced by Sun and Xu [41], so that the priority and representation of a patch can be modeled into two core processes for propagation in exemplar based method. Furthermore, Ladjal et al. [42] proposed experimental proof with regard to the exemplar based methods could establish the local geometric information. After implementation of minimization to variational models, geometry and especially smooth edges can be reconstructed.

One of the methods for hybrid inpainting was proposed by Bertalmio et al. [43], who developed an algorithm by dividing the original image into two layers. One of them involves the characteristics of structure while the second one includes the texture. The first image can be processed with a structural inpainting algorithm [24] and the second one can be processed with the texture synthesis algorithm that was put forward by Efros and Leung in [44]. The results generated from both operations lead to the final image. Atzori and de Natale [45] put forward another hybrid method. The restoration process starts from matching of the contours that runs through the edge of the occluded area. This operation will generate smaller regions which can be filled through copying of blocks from the outside. As proposed by Rares et al. [46], the global and local information should be treated as integration of edge contours which crosses the damaged area. Thus, more accurate pairs of lines can be achieved, but the matching pro-

cess will be more complex. Recovered pixel values are then assigned based on the pixels near the new contours in the occluded area.

State of the art inpainting methods can reasonably recover missing pixels but often suffer from blurry artifacts in high-frequency areas. Recently, Li et al. [4] proposed a method to combine numerical and example based methods to address the issue. Although it requires a long recovery time, it shows promising results for recovering missing pixels in detailed regions.

Compressed Sensing based Reconstruction

Compressed sensing (CS) has been applied in the signal processing community to recover a signal when it satisfies sparse property under a transform domain. Specifically, it indicates that any sparse signal (like visual data transformed with a suitable basis) can be established with high quality based on a few measurements. That means existing rendering techniques can be accelerated through measuring or computing only a small number of samples, and the data can be accurately estimated by the CS method.

The performance exerted by a CS method is generally decided by the selection of a suitable basis, which can represent a signal. Since an image shows various sparsity patterns on different basis, to minimize the reconstruction error, the signal's representation must be as sparse as possible [47]. Most previous CS based image synthesis algorithms depend on the analytical basis, such as wavelets bases [9]. For this purpose, Sen et al. [9] only rendered a subset of pixels in image domain and applied a greedy, simple CS based algorithm for estimating the wavelet transformation of the image after rendering. Since the image's energy in the wavelet domain is concentrated more compactly, only a small set of samples is needed to obtain a satisfied result in comparison with the conventional spatial-domain rendering. However, the natural images deliver diverse features and information, a fixed analytical basis cannot get a sparse enough rep-

resentation with the purpose of an accurate signal recovery. Hence, the reconstruction quality is poor and artifacts like speckle noise, ringing or blurring can be noticed.

Miandji et al. [23] presented a CS based algorithm with a learning based dictionary, which can accurately reconstruct incomplete visual data, including light fields, animations and images. The learning based approach allows the dictionary exploiting the sparsity patterns involving the visual data, and obtains a more sparse representation than the fixed analytical dictionaries. This is in comparison with the previous work [48], in which a single 1D-learned dictionary was used to apply CS for light fields. Miandji conducted training of 2D dictionary to represent the locally sparse data, which allows sparse representation in each cluster. Their results proved that signals can be embodied with the reduced union-of-subspaces model of signals [49], and the sparse representation can be admitted with a minimal size of dictionary. The key point of their method is that correlations in 2D can be explored by the dictionary. This allows the optimal recovery and requires a very small footprint of memory. The ensemble can be trained only once with regard to different signal types (light fields, images and videos).

Compressed sensing based methods generate the believable reconstruction of missing pixels in a partially rendered image. But it is limited in the case of low sampling rate, and the final output quality depends on selecting a suitable basis in transform domain.

Matrix Completion and Separation based Reconstruction

Matrix completion methods have been recently introduced in image processing to recover missing data in a low-rank matrix [16, 17]. Although solving the rank minimization problem is NP-hard, it can be approximated, and exactly solved in some useful cases, as a nuclear norm minimization (NNM) problem. Cai et al. provided a singular value threshold-

ing algorithm (SVT) for solving NNM [50]. They demonstrated that the low-rank matrix completion problem could be solved by a soft thresholding operation on the singular values of the observed matrix.

Matrix completion and separation schemes have not been adopted in computer graphics until recently and we could find only two previous works. Recently, Hou et al. [22] took advantage of the low-rank property of a many lights matrix for solving many lights rendering problems using a matrix separation scheme. Li et al. [4] adapted matrix completion for inpainting while exploring a low rank matrix by grouping similar patches within an image. In the framework, they begin by searching and grouping similar patches with regard to the input image, and then estimating the missing pixels of the grouped patches by using matrix completion technique. The results are finally obtained by synthesizing all reconstructed patches. The main limitations of their method involve the accuracy of the patch matching and low-rank matrix completion. For the first issue, a method concerning robust patch matching was proposed. With regard to the second task, they applied the multipliers with alternating directions technique. Although they tested their inpainting scheme to recover missing pixels of a rendered image, their main focus was inpainting to recover missing pixels in photos, so they did not formulate sub-pixel and multi-frame samples as in this thesis. Therefore, the proposed reconstruction based work in this thesis is unique in addressing the screen space recovery of incomplete path-tracing images using matrix completion across low rank matrices of sub-pixel, pixel, and multi-frame samples.

2.2.2 Denoising Techniques for Path Tracing

Despite the above-discussed reconstruction based method, another category of the solutions is the denoising based method. Extensive works have been proposed to address path tracing noise. Denoising techniques render an image with a few samples per pixel, and then remove the noise

with a post-filtering operation. Some approaches also adaptively sample to further improve the results. Previous denoising based techniques can be divided into two general branches: 1) color based denoising filters that utilize sample color value for filtering, and 2) scene feature based denoising filters that utilize additional scene features.

Color Based Denoising Filters

Color based denoising filters for path tracing are often enlightened with traditional image denoising methods and solely utilize the rendered pixel color to remove path tracing noise. Redner and Lee [51] proposed a seminal work which defines alpha-trimmed filter based on the median/mean filter. Christensen and Jensen [52] applied a median filter or Gaussian filter within a local area, which the light has been reflected at least two times. The trilateral filter proposed by Choudhury [53] concerns the image gradient and an adaptive neighborhood function. Another classic bilateral filter was developed by Pattanaik and Xu [54] which is different from the work proposed by Redner and Lee [51], as classical bilateral filters will not remove the outliers. To address this limitation, the bilateral filter weights proposed by Pattanaik and Xu are calculated via the denoised image. Bala et al. [55] presented an efficient filter with an edge image. In addition, Egan et al. [56] applied frequency analysis for shearing of filter and distributed effects like motion blur and occlusion or shadowing. Mehta et al. [57] applied correlated analysis to generate simple formulas that use variance of Gaussian filter of the screen space to remove noise from specific rendering effects.

More complex filters apply sample information based on the renderer for filtering on the pixel level. Ward and Rushmeier [58] set an influence region of noisy pixel based on comparing whether the variance of noisy pixel is smaller than a certain threshold after several iterations. Each noisy sample can make a contribution to other denoised pixels, and no energy

will be leaked since the filter is normalized. McCool [59] introduced another classical filter by applying geometric information of pixels. It is an anisotropic diffusion that removes the path tracing noise. Normal and depth information collected in rendering (included in G-buffer) and the color coherence map are used, while the conductivity function of the diffusion scheme in the pixel is estimated from the coherence map. More recently Tschumperle et al. [10] proposed a scheme with an efficient wavelet filter for global illumination of path tracing. Hence, the filter adopts information of the buffer concerning indirect or direct illumination, as well as the scene feature buffer based on point position or vector normals. Additionally, Xu et al. [60] invoked the bilateral filter for denoising images generated with light paths, which are complicated in fog or smoke. According to the path gradient direction, in their paper, additional bilateral weights are used for better guidance of the denoising scheme. Most of these last approaches also adaptively position samples with the analysis.

In order to remove noise of the general rendering effects, a shrinkage of wavelets is adapted to path tracing noise reduction by Overbeck et al. [61]. Rousselle et al. [62] chose an appropriate scale for the Gaussian filter on each pixel to reduce the recovery error. In addition, they made improvements of it by using the non-local means filter [63]. Since the path tracing noise at every pixel can be estimated with the absolute median deviation. Sen and Kalantari [64] can use the arbitrary image denoising method for path tracing. Lastly, Delbracio et al. [2] put forward a method subject to non-local means filtering, which predicted the distance between the selected patches together with the color histograms. The color based methods can be easily adapted to handle different distributed effects, but they usually request many samples to obtain satisfy results. For challenging scenes, they may fail to generated reasonable results at low sampling budget.

Scene Feature Based Denoising Filters

With this type of methods, additional feature buffers (such as texture values, shading normals and world positions) calculated with the render can be leveraged. Thus, they can generate higher-quality results in comparison with the color based methods in previous section. For efficiently rendering global illumination of a scene, Laine et al. [65] and Segovia et al. [66] applied the geometry buffer. For removing the global illumination noise of path tracing, Dammertz et al. [10] integrated the wavelet information and bilateral filter, while Bauszat et al. [67] applied a filter with the guided image. In addition, a G-buffer was applied by Shirley et al. [68] to deal with depth-of-field and motion blur effects, and Huo et al. [69] made combination of a G-buffer and variance for filtering the noise from depth-of-field. These scene feature based approaches only concern several rendering effects which are not general. For removing different path tracing noise with auxiliary features from render, Sen and Darabi [15] demonstrated the benefit of adaptively setting the filters feature weights through the image. They made computation with these weights and mutual information, so they can approach the functional dependencies existing in random parameters and scene features. Li et al. [1] applied a cross-bilateral filter with Steins unbiased risk estimator (SURE) [70] for estimating the proper spatial filter parameter, and the cross terms weights is conducted by hard coding. Rousselle et al. [71] made improvement on this with the SURE metric for selection among three candidates across the non-local mean filters, and weight features and colors differently.

Darabi and Sen [72] applies all information concerning rendering and each numerical photon for denoising with use of bilateral filtering. In addition, the bilateral filter also considers the spatial neighborhood and the sample position, the scene features (texture values, world space position and normal), the random synthesis parameters, and the sample color finally. The computation of mutual information concerns dependence of statistics involving randomly generate filter parameters at the position

that share identical colors or features. They achieved outstanding results based on a low sample budget per pixel, but the high computation of the method means it is not suitable for previsualization. In addition, Lehtinen et al. [73] introduced a denoising method with the anisotropy property of temporal light field, which can handle various effects like depth-of-field, motion blur and soft shadows. Specifically, the effective sampling rate increases by a big factor subject to efficient reuse of pixel samples. Recently Lehtinen et al. [74] conducted generalization of these ideas for the indirect illumination. Through contemplating of the diffuse surface properties, their algorithm can interpolate the light field for the denoised results, which is as similar as the image rendered with many samples. For instance, for the input path-tracing image rendered with 8 samples per pixel, the denoised result of this input have similar visual quality with the reference image that rendered with 512 samples per pixel. The main problem of scene feature based methods is that the weight of every filter term is estimated according to error metric and/or heuristic rules which will contain noise at low sampling rates. Therefore, they cannot predict proper filter weights in challenging cases.

Deep Learning Based Denoising Filters

Recently, convolutional neural network (CNN) has appeared as a ubiquitous model in machine learning, which achieved state-of-the-art performance in various tasks such as speech processing [75] and image classification [76]. CNNs have also been widely applied with regard to various image-processing, low-level tasks. Particularly, several works have taken into account the natural image denoising [77, 78] and image super-resolution [79] which is highly correlated.

Unlike the aforementioned techniques, deep learning based path tracing denoising methods learn the complex relationship between noisy rendering and reference data by leveraging both the color values and the aux-

iliary scene features. Our proposed denoising based method in Chapter 5 is also inspired by the recent deep learning based denoising techniques. Kalantari et al.[80] firstly applied a plain neural network to predict parameters for a cross-bilateral or non-local filter. In their method, a complicated relation between the noisy scene data and the ideal filter parameters is observed, while this relationship is learned with a nonlinear regression model. For this purpose, they used a perceptron neural network with multiple layers and made combination with a matching filter during testing and training. The network is first trained offline with noisy images including various distributed effects. Then, during running, the trained network can estimate the filter parameters on different rendering to generate the denoised results that close to the reference rendering.

Later, Bako et al.[81] improved this approach by using a kernel prediction network to compute the filtered pixel value from its local neighbors. They put forward a supervised learning method which enables the filtering kernel become more complicated and general by applying a deep convolutional neural network (CNN) architecture. As one embodiment of their framework, the CNN makes direct prediction of the denoised pixel value through a non-linear representation of the rendered feature buffers. As for the second approach, they applied a kernel-prediction network that adopts the CNN for estimation of the local weighting kernels for computing each denoised pixel from the surrounding neighbors. Recently, Vogels et al.[82] extended the concept over a temporal window predicting kernels for an entire sequence of subsequent animation frames. Chaitanya et al.[83] propose a convolutional architecture which enforces temporal stability via recurrent connections that provide a mechanism to incorporate information from past frames in a sequence. But under a low sampling budget, these methods may generate over blurred results and fail to provide accurate previsualization.

Chapter 3

Matrix Completion for Path Tracing

As discussed in Chapter 1, path tracing is well established for photo-realistic rendering to simulate light path transport. The method has been adapted in many applications to produce high quality synthetic images requiring a huge number of samples and long computation time. Due to the high cost to produce the final output, intermediate previsualization of path tracing is in high demand from production artists to detect errors in the early stage of rendering.

Visualizing intermediate results of path tracing is challenging since the synthesized image with limited samples or improper sampling usually suffers from distracting noise. The conventional practice is to render the frames at a lower resolution and upsample them for previsualization. However, this has clear drawbacks due to the limitation in visual quality as well as a waste of resources: we cannot reuse the low-resolution outputs for final rendering. The ideal solution would be to provide a highly plausible intermediate result in the early stages of rendering, using a small fraction of samples, and also adapt additional samples in a progressive manner to minimize any unnecessary rendering. Furthermore, if the previsuali-

alization relies only on samples, and not on scene information, it can augment various rendering algorithms as an additional postprocessing step. In this chapter, we present a screen space reconstruction that addresses these challenges.

In this chapter, we present a novel reconstruction method using *matrix completion* that efficiently recovers missing data in incompletely rendered images, which is randomly and sparsely rendered under fixed sampling rate (e.g. 40%). The fundamental task of matrix completion is to estimate missing entries in a matrix. Recently this problem has been successfully solved by making use of the assumption that the matrix has low-rank property. This assumption requires that the reconstructed signal be as simple as possible in a certain sense, and plays a similar role to the generic smoothness assumption that is often employed in inpainting and other problems. However, unlike the smoothness assumption, the low-rank assumption allows edges in certain cases. For example, an $n \times n$ matrix $M_{i,j} := \text{sign}(j - n/2)$ has exactly one non-zero singular value.

Applied to our problem, the basic strategy is to construct a matrix from a group of similar pixels or patches, and then use matrix completion to estimate the missing samples. The availability of surrounding, similar pixels or patches helps guide the recovery.

We approach the low-rank matrix construction step differently for pixel samples and for sub-pixel samples (a fraction of samples within a pixel) of images, and for multi-frame samples in animation sequences.

For pixel samples, we form a pixel patch using neighboring pixels, and then group similar pixel patches from elsewhere in the image to form a low-rank matrix [4]. However, identifying similar pixel patches across an image has extra challenges due to missing pixel values. A sophisticated pre-completion step is required to estimate missing pixel values in each patch [4]. In this chapter, we use a pre-trained convolutional neural network (CNN) to precompute the missing values efficiently in a short time.

For multi-frame samples, the intuitive assumption to form a low-rank

matrix is to use redundancy across nearby frames. We therefore adapt a similar approach, gathering similar patches from nearby frames rather than from within a single image. The scheme has been applied to missing pixel recovery in multiple adjacent frames and efficiently reduces temporal noise in a video.

For sub-pixel samples, we apply the same general idea, but rather than using similar patches in the image or adjacent frames; we draw samples from adjacent pixels (Figure 3.5). Considering the approximate self-similarity of image statistics, it would seem that the same strategy could be used at both pixel and sub-pixel levels. For example, an edge with missing pixels is easier to identify when more of the edge is present, even if the ratio of known to missing pixels remains constant (Figure 3.6). Our choice of differing strategies can be motivated in terms of a speed/quality trade-off. The expected variance within a region grows as the region size increases, so simply choosing adjacent pixels is less likely to be successful at larger scales. On the other hand, using local information is cheaper than searching elsewhere in the image, and exact recovery is less likely to be crucial at sub-pixel scales, so we make use of only local information in this case.

Finally, the missing values of a low-rank matrix formed from either missing pixel, sub-pixel, or multi-frame samples are recovered by our novel matrix completion step. For efficient recovery of high-frequency images, we adapt a multi-weighted nuclear norm minimisation (WNNM) scheme that optimizes both high and low singular values differently. Then, we added a novel progressive parameter (PA) adjustment strategy to WNNM (PAWNNM). Our PAWNNM reduces the WNNM weights during each iteration of the optimization, resulting in a progressive quality increase.

This chapter is started by introducing the background of matrix completion (Section 3.1). To utilizing matrix completion, we need to show how we generate the related low matrix in different scenarios (Section 3.2). Our optimized algorithm is described in Section 3.3. Furthermore, we compare

our results with other reference works and demonstrate that our technique is more efficient and optimal.

3.1 Overview

Matrix completion has been recently introduced [17, 16] and successfully applied to many problems in image processing [20, 21], but has not been adopted in the computer graphics community until recently; we found only one recent paper, which addresses many lights rendering [22]. We first make a fundamental review of the standard matrix completion method.

The values of a sparsely and randomly sampled matrix M can be approximated by matrix completion if M has low-rank [17]. Then, the problem can be formulated as a low-rank matrix minimization:

$$\begin{aligned} \min_X \quad & \text{rank}(X) \\ \text{s.t.} \quad & X_{ij} = M_{ij}, \quad (i, j) \in \Omega \end{aligned} \tag{3.1}$$

where X is an estimate to be solved by rank minimization constraint by observed samples, and Ω contains the positions of the observed samples.

Although solving the rank minimization problem is NP-hard, Candes and Recht showed this problem has a convex relaxation as a nuclear norm minimization (NNM) problem [17]:

$$\begin{aligned} \min_X \quad & \|X\|_* \\ \text{s.t.} \quad & X_{ij} = M_{ij}, \quad (i, j) \in \Omega \end{aligned} \tag{3.2}$$

where the nuclear norm $\|X\|_*$ is defined as $\|X\|_* = \sum_{s=1}^n \Sigma_s(X)$, and $\Sigma_s(X)$ are singular values of matrix X .

In our method, we use singular value thresholding (SVT) algorithm for solving the NNM. SVT iteratively applies a soft thresholding operation to the singular values of X [50]. The solution to the SVT formulation in Equation (3.3) converges to Equation (3.2) for large threshold λ .

$$\begin{aligned}
\min_X \quad & \frac{1}{2} \|X - M\|_F^2 + \lambda \|X\|_* \\
\text{s.t.} \quad & P_\Omega(X) = P_\Omega(M)
\end{aligned} \tag{3.3}$$

where $\|\cdot\|_F^2$ is the Frobenius norm, and P_Ω is the orthogonal projector onto the span of matrices vanishing outside of Ω , so that the (i, j) component of $P_\Omega(X)$ is equal to X_{ij} for $(i, j) \in \Omega$ and zero otherwise.

It can be solved by iterating Equation (3.4) from an initial matrix $Y_0 = 0$ with a given $\lambda > 0$ until a stopping criterion is reached. It has been shown that the sequence X^k obtained via Equation (3.4) exactly converges to the unique solution of Equation (3.3) [17].

In Equation (3.4), D_λ is the singular value shrinkage operator on each singular value Σ_i of Y : $D_\lambda(Y) = US_\lambda(\Sigma)V^T$, where $S_\lambda(\Sigma) = \text{diag}(\max(\Sigma_i - \lambda, 0))$, and σ_k is a positive step size sequence.

$$\begin{cases} X^k = D_\lambda(Y^{k-1}) \\ Y^k = Y^{k-1} + \sigma_k P_\Omega(M - X^k) \end{cases} \tag{3.4}$$

The Lagrangian for problem (3.3) is given by

$$L(X, Y) = \lambda \|X\|_* + \frac{1}{2} \|X\|_F^2 + \langle Y, P_\Omega(M - X) \rangle \tag{3.5}$$

where Y is the Lagrangian dual variable. Cai et al. show that SVT indeed optimizes the dual function $f(Y) = \inf_X L(X, Y)$ via the gradient descent method[50]. The SVT algorithm shows great efficiency for solving the matrix completion task, especially for a large low-rank matrix. Besides matrix completion, it has been widely applied to solving other nuclear norm minimization based problems, such as robust principle component analysis [16, 84], texture rectification and denoising [85] and low-rank representation for subspace clustering[86].

Figure 3.1: Low-rank matrix construction for pixel patch: from the left, the input image (20% pixel samples), pre-completion and grouping, and low-rank matrix M_p . From the initialized image (middle), similar pixel patches (red squares) for a current pixel patch (green square) are selected, and stored into columns of the matrix M_p .

3.2 Low-Rank Matrix Construction

In order to estimate missing values in an incomplete path tracing image using matrix completion, we need to provide a proper low-rank matrix that contains sparse random samples evaluated by path tracing rendering. We present three different schemes for constructing low-rank matrices in different scenarios such as an image containing a fraction of missing pixels, missing pixels in multi-frames, and a fraction of missing sub-pixels within a pixel. Accordingly, the constructed low-rank matrix in the three scenarios is as follows.

Pixel Patch: We recover missing pixels in an incompletely rendered image (Figure 3.1) using matrix completion. We pre-complete the missing pixels using a convolutional neural network (CNN). A set of similar pixel patches (Figure 3.1, middle) are computed to form a low-rank representation across the whole image space. The similar pixel patches are fed into column vectors of a matrix M_p (Figure 3.1). The precompleted missing values of M_p are recovered by matrix completion (see Section 3.3). The details

of the construction process of M_p are discussed in Section 3.2.1.

Multi-frame Patch: Pixels in adjacent frames have a high chance to have similar values, and therefore similar values across frames provide a low-rank form. Similar pixel patches of adjacent frames are computed and fed into a matrix M_m as in Figure 3.4. The missing pixels in adjacent frames are simultaneously recovered by matrix completion.

Sub-pixel Patch: A group of randomly selected samples within a tiny area such as within a pixel have a high chance to be similar. The proposed idea is to form a low-rank matrix that contains a fraction of sub-pixels within a pixel and estimates the missing sub-pixels using low-rank matrix recovery. A fraction of a sub-pixel grid is evaluated by path tracing. The surrounding sub-pixel grids are grouped to form a sub-pixel patch and fed into a low-rank matrix M_s . The details are discussed in Section 3.3.3.

3.2.1 Pixel Patch

In CR [9], missing pixels in an incomplete image I_m are estimated by compressed sensing: I_m consists of a portion of randomly selected and rendered pixel samples as well as missing pixel samples. We address the problem using a matrix completion scheme while constructing multiple low-rank matrices, M_p s consists of similar pixel patches, $\tilde{P}_p$ s within an image. The idea is inspired by a non-local image processing scheme [87], [63] such that a group of mutually similar patches, $\tilde{P}_p$ s provides low-rank property [4].

Pre-completion using CNN: Computing similar pixel patches is technically challenging due to many missing pixels. Li et al. [4] proposed a pre-completion step that initializes missing pixels [4]. They address pre-completion for low frequency and high-frequency images differently. For a low frequency image, matrix completion is directly applied for pre-completion, and a total variation (TV) regularized reconstruction [88] is used for high-frequency images that do not meet the low-rank condition.

Although it provides promising initialization, the process requires long computation time that is a clear limitation to adapting in the method for previsualization.

We present a novel pre-completion scheme using a convolutional neural network (CNN). A CNN is trained by pairs of patches from a reference image I_r and an incomplete image I_y that contains randomly removed pixel values in I_r . Then, the pre-trained CNN is used to initialise missing values for pre-completion. For our neural network, a loss function $L(\Theta)$ is defined to measure error between the estimated data $F(y; \Theta)$ that contains patches of I_y and the reference data R that contains patches of I_r :

$$L(\Theta) = \frac{1}{n} \sum_{i=1}^n \|F(I_y^i; \Theta) - R_i\|^2 \quad (3.6)$$

where n is the number of training samples and Θ is a network parameter to learn. $F(y_i)$ is the pre-completion result defined by:

$$F_l(y) = \hat{f}(W_l \otimes F_{l-1}(I_y) + B_l) \quad (3.7)$$

where l is the number of layers, W_l are convolutional filters having different sizes ($s_l \times s_l$) and numbers (n_l) in each layer l , and B_l are the biases, $\otimes$ is the convolution operator, and $\hat{f}$ is the sigmoid function: $\hat{f}(x) = 1/(1+e^{-x})$. The network parameter Θ in Equation (3.6) is then defined as $\Theta = \{W_i, B_i\}$ [89], and $L(\Theta)$ is minimized by standard back propagation:

$$\Delta_{i+1} = 0.9 \times \Delta_i + \eta \times \frac{\partial L}{\partial W_i^l} \quad (3.8)$$

$$W_{i+1}^l = W_i^l + \Delta_{i+1} \quad (3.9)$$

where Δ is momentum, η denotes the learning rate, and i is the iteration. The basic structure of our CNN is illustrated in Figure 3.2. The CNN is a three-layer network. The filter size s_l and number of filter n_l of W_l in three layers are initialized as: $s_1 = 9, s_2 = 3, s_3 = 5, n_1 = 128$, and $n_2 = 64$. The weights of convolutional filter W_l in Equation (3.7) are set by a Gaussian

Figure 3.2: Three layered CNN architecture for pre-completion: the red square is a convolutional filter in layer 1 (W_1), the green square is a convolutional filter in layer 2 (W_2), and the blue square is a convolutional filter in layer 3 (W_3).

distribution with zero mean with 0.001 standard deviation, and basis B_l is initialized by zero. We use 130k patches from 500 images from Flickr (<https://www.flickr.com/>) for the training set with the learning rate $\eta = 0.00001$ in Equation (3.8). The network is trained with different missing rate of input data.

Table 3.1: PSNR compared with CNN and TV initialization under 10% input pixels. The recovery time are measured in seconds.

Input 10% pixels	CNN	TV
Toasters	39.74	30.59
Bottles	30.17	19.73
Bedroom	31.82	23.91
Bamboo	26.03	23.01
Recovery Time	4s	90s

In Table 3.1, we compare the PSNR (peak signal-to-noise ratio) of our CNN initialisation results with TV based initialisation, used in [4]. Even under a low sampling rate, our CNN based initialisation shows better re-

sults than TV in shorter time. An image comparison of the toasters scene is in Figure 3.3. More comparison results can be found in Appendix A.

Patch matching and grouping: After pre-completion, we create a pixel patch $P_p(i, j)$ in pixel (i, j) having $n_p \times m_p$ pixels (e.g., green square in Figure 3.1), and the similar pixel patches $\tilde{P}_p$ s within the whole image space (e.g., red squares in Figure 3.1) are computed by weighted L_2 distance. In the weighted scheme, we assign more weights ($w=0.75$) for rendered pixels than initialized pixels from the pre-completion. The selected similar patches are stored in each column of a low-rank matrix M_m as in Figure 3.1, and the missing values in the matrix are estimated by our low-rank matrix recovery step (see Section 3.3). After completing missing values in patch $P_p(i, j)$, the next patch $(P_p(i + \delta_p, j), 1 \leq \delta_p < \min(n_p, m_p)/2)$ will be iteratively processed.

Figure 3.3: A visual quality comparison of initialisation between Total Variation and our CNN for a toasters scene, and the corresponding recoveries. Our recovery also achieve better recovery in terms of SSIM (the structural similarity) and PSNR.

3.2.2 Multi-frame Patch

Path tracing is often used to produce multi-frame images for a video. Intuitively, a pixel (i, j) in a frame k , $p_k(i, j)$ has a high chance to have similar values in adjacent pixels of nearby frames such as $p_{k-1}(i-1, j-1)$, ..., $p_{k+1}(i+1, j+1)$. The similar values across frames provide low-rank property to form a matrix M_m for matrix completion to estimate missing values in multiple frames simultaneously. Images of each frame are sparsely and randomly sampled as in the previous section. A pixel patch $P_p(i, j)$ of a pixel (i, j) in frame k , $P_p^k(i, j)$ having $n_p \times m_p$ elements is created. Then, pixel patches $\tilde{P}_{p,i}^k$, $(0 \leq i \leq n_f)$ that are similar to P_p^k are selected within the frame k and grouped as $F_k = \{\tilde{P}_{p,0}^k, \dots, \tilde{P}_{p,n_f}^k\}$. Similar pixel patches in nearby frames are also selected and grouped (i.e., $F_{k-1}, \dots, F_{k+1}$). Since each F_k consists of a mixture of rendered and missing samples, the missing samples are initialized using a pre-trained CNN, which is same CNN in pixel recovery. Then, a weighted L_2 norm is applied to measure the similarity across the initialized patches. Regions for searching similar patches are limited to the adjacent areas of the reference pixel (i, j) across nearby frames. Similar pixel patches in each frame F_k are assigned to columns of a low-rank matrix M_m . For example, similar pixel patches in 3 nearby frames $[F_{k-1}, F_k, F_{k+1}]$ can be assigned to M_m as in Figure 3.4. Missing elements of pixel patches in multiple frames are estimated simultaneously in the subsequent matrix recovery step.

3.2.3 Sub-pixel Patch

Distracting artifacts in a path tracing image can be caused by the limited number of sub-pixel samples available for evaluating each pixel value. Therefore, we propose to address the problem as matrix completion that estimates missing sub-pixel samples required to correct each pixel value. The basic idea is to form a low-rank matrix that contains a fraction of the sub-pixel samples within a pixel and estimates the missing samples using

Figure 3.4: An example of multi-frame patches F_k and the constructed low-rank matrix M_m : similar patches across multiple frames (squares) are stored in columns of a low-rank matrix.

our low-rank matrix recovery method. We form sub-pixel patches P_s as in Figure 3.5 to form a low-rank matrix M_s for matrix completion. A uniform sub-pixel grid $G_s(i, j)$ having $n_s \times m_s$ elements is associated with a pixel (i, j) as in Figure 3.5 (left). A fraction of elements in G_s are sparsely and randomly selected, and the values are evaluated by path tracing. In most cases of rendering, G_s has the low-rank properties required for M_s , based on the fact that a group of randomly selected samples within a tiny area such as a pixel will have similar values within the local area. In some cases, such as an edge boundary, sub-pixel samples within a pixel might have high-frequency. In order to handle this case, we construct a sub-pixel patch $P_s(i, j)$ that consists of a group of sub-pixel grids surrounding the pixel (i, j) . One example of a P_s having 3×3 elements is $P_s(i, j) = \{G_s(i-1, j-1), G_s(i-1, j), \dots, G_s(i+1, j+1)\}$ as in Figure 3.5. The assumption behind our sub-pixel patch is that although a pixel value may be evaluated by high-frequency sub-pixel samples (e.g., edges), the surrounding pixels will have high probability to have low frequency sub-pixel samples (e.g., non-edge), and therefore the associated matrix can be a low-rank (Figure 3.6). Elements of each sub-pixel patch in P_s are stored in a matrix M_s and the missing elements of M_s (the missing sub-pixel samples of P_s) are reconstructed in the following matrix recovery step (see Section 3.3). Our idea has been evaluated with various scenes including distribution noise (motion blur, soft shadow) as well as path tracing noise (see Section 3.4.4).

3.3 Low-rank Matrix Recovery

NNM: The low-rank matrix created in the three different scenarios is constructed using matrix completion [17]. In particular, missing values in the low-rank matrix are recovered using the SVT algorithm [50]. Here, M denotes the incomplete low-rank matrix and X is the original matrix we want to approximate. The nuclear norm minimization (NNM) problem is

Figure 3.5: A sub-pixel grid G_s (green) and 3×3 sub-pixel patch P_s (red) fed into a low-rank matrix M_s : black dots are rendered samples.

formulated in Equation (3.3) and the energy function can be defined as:

$$\hat{X} = \arg \min_X \frac{1}{2} \|M - X\|_F^2 + \lambda \|X\|_* \quad (3.10)$$

where λ is a positive weight, and $\hat{X}$ can be obtained by

$$\hat{X} = US_\lambda(\Sigma)V^T \quad (3.11)$$

where $M = U\Sigma V^T$ is the SVD of M , and $S_\lambda(\Sigma)$ is the soft-thresholding function on the diagonal matrix Σ with constant weight λ . For each diagonal element Σ_{ii} in Σ , we set

$$S_\lambda(\Sigma)_{ii} = \max(\Sigma_{ii} - \lambda, 0) \quad (3.12)$$

WNNM: In a standard nuclear norm minimization, each singular value is treated equally and the soft thresholding operator shrinks each singular value with a constant λ . In order to optimize high-frequency details efficiently, we adapt the weighted nuclear norm minimization (WNNM) algorithm [90] to utilize the prior knowledge of singular values in the matrix. The main idea is that the larger singular values should be shrunk less to preserve the major data components. To adapt the WNNM, we modified the constant weight λ in Equation (3.12) as multiple weights λ_i :

$$\lambda_i = \omega_i / (\Sigma_{ii}(X) + \epsilon) \quad (3.13)$$

Figure 3.6: In a sub-pixel area, an edge may be difficult to localize correctly (left); the orange dots and the blue square(s) are rendered samples around an edge. But the orientation of the edge (purple line) becomes more clear when a sub-pixel patch is formed (right), even when the ratio of known-to-unknown samples remain constant.

where, $\Sigma_{ii}(X)$ is the i^{th} singular value of X , ω_i is the weight for the i^{th} singular value, and ϵ is a small regularizer to avoid dividing by zero.

PAWNNM: For efficient optimization, we add a parameter adjustment (PA) strategy, reducing ω with a constant $c = 0.9$ in each image recovery iteration, which can progressively increase recovery quality during the optimization loop.

$$\omega_{i+1} = c\omega_i \quad (3.14)$$

The PA strategy is based on the observation that recovered image quality keeps increasing within fixed iterations. Compared with a hard threshold, using a gradually reduced soft threshold helps to achieve a better result, as demonstrated by Mancera and Portilla [91]. Their idea is to solve a single gradient descent step while slowly reducing the threshold iter-

Table 3.2: PSNR comparison for NNM, WNNM and PAWNNM under 20% input pixels.

Input 20% pixels	NNM	WNNM	PAWNNM
Bamboo	30.74	31.01	32.29
Bedroom	37.85	39.35	39.96
Bottles	35.37	36.96	37.15

atively. We compared our PAWNNM with NNM and WNNM in pixel recovery. Table 3.2 shows PSNR results of the three methods and our PAWNNM demonstrates better recovery.

Algorithm 1 shows the whole recovery process, based on our PAWNNM scheme. The inputs are the rendered data, M , and the corresponding sampling mask, Φ , that the renderer has used to determine which samples to take. The first step in each iteration is to construct a low-rank matrix, X , using one of the schemes proposed in the previous section. We then apply our PAWNNM to recover the missing data. We iterate on this process to produce a good reconstructed result.

3.4 Results

3.4.1 Experimental Setup

Our method has been tested on various scenes including detailed scenes with high-frequency content for which it is challenging to recover missing information without introducing blurry artifacts. The test scenes include the Bamboo, Bedroom, and Bottles scenes in Figure 3.9, the San Miguel scene in Figure 3.8 and motion blur and soft shadow effect scenes in Figure 3.13. In order to create partially sampled rendering images as well

Algorithm 1 Recovery via low-rank matrix completion**Require:** partially rendered data M , sampling mask Φ

- 1: Initialization: $\hat{x}^0 = CNN(M)$ and set threshold $\omega = \omega_0$;
- 2: **for** $i = 1 : k$ **do**
- 3: Form a low-rank matrix X_i based on $\hat{x}^i$;
- 4: Singular value decomposition for each matrix X_i : $[U_i, \Sigma_i, V_i] = \text{SVD}(X_i)$;
- 5: Apply weighted singular value thresholding: $\hat{\Sigma}_i = S_{\lambda_i}(\Sigma_i)$;
- 6: Generate the updated result: aggregate $\hat{X}_{i+1} = U_i \hat{\Sigma}_i V_i^T$ to get $\hat{x}^{i+1}$;
- 7: Iterative regularization: $\hat{x}^{i+1} = \Phi M + \hat{x}^i - \Phi \hat{x}^i$;
- 8: Parameter shrinkage: $\omega_{i+1} = c\omega_i$;
- 9: **end for**

Ensure: reconstructed result $\hat{x}^k$.

as the reference images, we use two open source renderers, LuxRender and PBRT. The rendered static images are 1280×960 pixels, and animation sequences are 311×500 pixels. All of our experiments were performed on a workstation with a quad-core 3.0 GHz Intel Xeon CPU with 8G RAM. We set initial values by experiment based on accuracy and efficiency considerations. The initial values in Equation (3.14) are $\omega = 80.5$, and $c = 0.9$. The number of iterations in Algorithm 1 is set at $k = 45$.

3.4.2 Pixel Recovery

Path Tracing rendering: A proportion of random pixels samples (20%, 40%, 60%) is selected and rendered using LuxRender and PBRT. Randomly selected pixels in each pixel patch P_p (7×7) are rendered and the positions are stored in a corresponding binary matrix. The patch size (7×7) has been selected experimentally while testing recovery quality of three different scenes under different patch sizes (Figure 3.7). The missing values of the patches are initialized by CNN and similar pixel patches within the image

Figure 3.7: The corresponding PSNR of three test scenes with different patch size choices.

are grouped and create a low-rank matrix M_p as explained in Section 3.2.1. Then, our reconstruction method estimates the values of missing pixels in M_p . After reconstructing values in a pixel patch, we process the next pixel patch while overlapping the patches by two pixels to avoid block artifacts. The size of the pixel patches are chosen by experiment to balance computation time and quality.

Figure 3.8 shows reconstructed images at different sampling rates (10%, 20%, 40%). Even with low sampling rates, our method reconstructs missing samples efficiently and shows believable results required for previsualization.

The reconstruction results are compared with CS based reconstruction: CR [9] and CIR [23] in Figure 3.9 (we implemented CR ourselves and use the author’s code for CIR). Our method with PAWNNM shows better results in terms of both visual quality and PSNR computed between reference and reconstructed images (see Table 3.3). CIR was compared with

Figure 3.8: Examples of missing pixel recovery (10%, 20%, 40% pixel samples).

Figure 3.9: A visual quality comparison of CR (Compressive Rendering), CIR (Compressive Image Reconstruction), NNM (nuclear norm minimization) and our PAWNNM. Input: 40% pixel samples.

Table 3.3: PSNR/SSIM results of three test scenes under different sampling rates.

Scene	Sampling Rate	CR	CIR	NNM	PAWNNM
<i>Bamboo</i>	60%	31.54/0.928	38.33/0.965	37.86/0.972	40.62/0.980
	40%	28.01/0.904	34.56/0.953	34.11/0.939	36.65/0.962
	20%	25.39/0.837	30.81/0.917	30.74/0.912	32.29/0.931
<i>Bedroom</i>	60%	40.43/0.961	47.15/0.984	46.05/0.982	48.68/0.990
	40%	35.57/0.953	43.54/0.974	42.63/0.971	45.32/0.985
	20%	30.92/0.872	38.37/0.956	37.85/0.960	39.96/0.967
<i>Bottles</i>	60%	34.25/0.906	42.46/0.964	41.69/0.957	43.51/0.970
	40%	32.19/0.865	39.85/0.922	39.02/0.920	41.77/0.951
	20%	29.56/0.801	36.03/0.873	35.37/0.911	37.15/0.913

various filtering algorithms [7], [6], [19] in their paper [23] and showed better visual quality. Since our method outperforms CIR, by transitivity our methods have been compared with these three filtering methods.

Our recovery time depends on the size of the low-rank matrix and the number of similar pixel patches in an image. Using our test setups, the reconstruction time is around 17 minutes to recover missing pixel samples in 1280×960 images regardless the total number of samples. We have im-

Table 3.4: Relative reconstruction cost comparison under different sampling rates.

Scene	Sampling Rate	CR	CIR	PAWNNM
<i>Bedroom</i>	60%	0.8%	6%	1.4%
	40%	0.8%	5%	1.4%
	20%	0.8%	4%	1.4%

Table 3.5: Given a fixed time budget (17 minutes), the PSNR of the recovery under CR, CIR and PAWNNM.

Scene	Sampling Rate	CR	CIR	PAWNNM
<i>Bedroom</i>	60%	40.43	47.15	48.68
	40%	35.57	43.54	45.32
	20%	30.92	38.37	39.96

plemented CR on Linux and used the authors' code for CIR working on Windows. Due to different implementations, run time evaluation is not a fair comparison. Therefore, we used an relative reconstruction cost (reconstruction time/partial rendering time) for evaluation in Table 3.5. The efficiency of our PAWNNM is between CR and CIR, but the PAWNNM can achieve best reconstruction.

Inpainting: Although our main focus is to provide solutions for pre-visualisation of path tracing rendering, our method can be applied to inpainting. We have tested our method and compared the results with recent inpainting methods such as group-based sparse representation (GSR) [92] and non-local inpainting (NLI) [4].

We selected three standard test images (House, Boat and Hill) and compared the recovery results with 20% and 30% input random pixels. For GSR, we reference the author's implementation and NLI uses our implementation based on their paper. As shown in Table 3.6 and Figure 3.10 our method with PAWNNM outperforms the other two recent inpainting methods. Both the NLI and GSR fail to recovery the missing pixels without blurring image details, while our PAWNNM preserved more high-frequency information compared with the original images. More comparison results can be found in Appendix B.

3.4. RESULTS

Figure 3.10: A visual quality comparison of NLI, GSR, NNM and our proposed PAWNNM. Input: 20% pixel samples.

Figure 3.11: Example of multi-frame recovery (with PAWNNM) under different sampling rate.

Table 3.6: PSNR/SSIM results of inpainting applications.

Image	Sampling Rate	NLI	GSR	NNM	PAWNNM
<i>House</i>	30%	33.83/0.893	36.58/0.947	36.06/0.941	37.23/0.949
	20%	32.52/0.882	34.36/0.903	34.17/0.921	35.47/0.927
<i>Boat</i>	30%	30.48/0.905	31.36/0.911	30.85/0.901	32.74/0.928
	20%	28.23/0.876	28.97/0.892	28.39/0.884	30.29/0.897
<i>Hill</i>	30%	31.31/0.889	33.01/0.913	32.02/0.906	34.17/0.925
	20%	29.45/0.807	31.44/0.891	30.15/0.882	32.56/0.908

3.4.3 Multi-frame Recovery

We have tested our method for recovering missing pixels in a video consisting of 147 frames, which is fully rendered in 24 hours. Portions of random pixel samples of each image are selected, rendered, and initialized as in Section 3.2.2. Pixel patches of size 7×7 are created and the similar patches in seven consecutive frames are selected and grouped. Ten similar patches are selected in each frame. Parameters in our test are selected by experiment to balance reconstruction time and visual quality. The selected similar patches construct a low-rank matrix M_m , and missing values of the matrix are estimated by our recovery method; boundary frames are overlapped and values are averaged.

The reconstructed result is shown in Figure 3.11. PSNR and SSIM measured between the reference and reconstructed images is shown in Table 3.7. Since we have reconstructed missing values in multiple frames concurrently while considering temporal coherence, potential temporal noise is minimized (multi-frame reconstruction shows lower standard deviation in Table 3.7). For frame-by-frame reconstruction, we used 70 similar patches per frame, but only 10 similar patches are used for multi-frame reconstruction. As shown in Table 3.7 our multi-frame reconstruction shows faster and better results than frame-by-frame reconstruction. The total re-

Table 3.7: Result of multi-frame recovery (with PAWNNM): Incomplete video frames sampled at 10%, 20% and 40% are reconstructed independently (single-frame) and 7-frame concurrently (multi-frames). Time is measured in minutes.

Input Samples	Recovery Method	Average PSNR/SSIM	Standard Deviation	Recovery Time	Full Render Time
10%	single-frame	27.42/0.867	1.62	1029	1440
	multi-frames	29.38/0.872	1.29	147	
20%	single-frame	31.74/0.905	1.25	1029	
	multi frame	34.23/0.931	0.97	147	
40%	single-frame	35.17/0.934	1.08	1029	
	multi frame	39.41/0.956	0.65	147	

covery time is 10% of total rendering time.

3.4.4 Sub-pixel Recovery

For testing recovery of missing samples within a pixel, we assigned a uniform sub-pixel grid $G_S(i, j)$ having 16×16 elements to each pixel (i, j) . A fraction of the sub-pixel grid is randomly selected (10% and 25%) and rendered using PBRT. Then, the grids of nearby pixels are grouped as a sub-pixel patch P_s ; 3×3 nearby pixels are selected in our test to balance reconstruction time and quality. The low-rank matrix M_s is constructed as in Section 3.2.3, and the missing sub-pixel samples are estimated by our matrix recovery step using PAWNNM. The reconstructed image of the San Miguel scene is shown in Figure 3.12. Our method efficiently recovers missing samples and shows promising results even in high-frequency regions. Our matrix completion contributes to provide efficient anti-aliasing in edge boundaries with fewer samples.

We compared our sub-pixel recovery results with three recent denoising filter techniques: ray histogram fusion (RHF) [2], learning based filter-

Figure 3.12: Examples of sub-pixel recovery under different sampling rates (10% and 25%), From the top, input image rendered by 25 sub-pixels per pixel (spp) and 64 spp, our PAWNNM reconstructions, and the reference image rendered by 256 spp. The unit of rendering time (top and bottom), and reconstruction time is seconds.

ing (LBF) [3], and Stein’s unbiased risk estimator (SURE) [1] using incomplete input images with missing sub-pixel samples. As shown in Figure 3.13, our method achieves better results compared with RHF and SURE in all test scenes, and outperforms against LBF in scenes that have high-frequency detail, such as Teapot and Sponza. In our tests, LBF could not estimate appropriate filtering weights under a low sampling rate in these two scenes. We performed an equal time comparison with RHF and LBF (Figure 3.14): we fixed the total time including both rendering and recovery (or filtering) time. Within a fixed total time, RHF and LBF can have more input samples, since their filtering time is shorter than our recovery time (Table 3.8). Even with this advantage of more samples, RHF still shows worse results than our method. For the same total time, our method and LBF produce similar results. Since our current implementation is not optimal, our recovery time and the quality of the equal time comparison could be improved, as discussed in the following section.

Table 3.8: The time cost in equal time comparison test: RHF, LBF (rendering time/ filtering time) and ours PAWNNM (rendering time/ recovery time).

	RHF	LBF	PAWNNM
Plants	93s/90s	175s/15s	85s/100s
Sibenik	168s/116s	260s/20s	107s/180s

3.5 Summary and Discussion

In this chapter, we present an efficient reconstruction method for recovering missing values in an incomplete path tracing image using matrix completion. Our novel construction schemes efficiently create low-rank matrices for images having missing pixels, missing pixels in multiple frames, and missing sub-pixel samples within a pixel. Our CNN based pre-compl-

Figure 3.13: Comparison with filtering methods (SURE [1], RHF [2] , LBF [3]), and our reconstruction schemes with PAWNNM under the same input samples (sub-pixels per pixel (spp)).

Figure 3.14: Equal time comparison with filtering methods (RHF, LBF), and our reconstruction schemes (PAWNNM).

etion provides the fast initialization required to find similar patches in the incomplete image(s). Our multi-weighted NNM with progressive parameter adjustment strategy (PAWNNM) recovers missing values in low-rank matrices while recovering high-frequency details efficiently. Our method shows better reconstruction than recent compressed sensing based techniques and filtering methods at the same sampling rates.

Since artifacts in an intermediate image of path tracing can be caused by insufficient samples, we claim that matrix completion is an efficient choice as a screen space reconstruction for previsualisation. This is the first time matrix completion has been adapted for screen space reconstruction to recover missing values in sub-pixels and multiple frames for previsualization of path tracing. Our method provides an efficient image space solution to previsualize early stage intermediate results (e.g., $\leq 40\%$ samples) with minimal blurry artifacts in high-frequency details, and to reduce temporal noise in a video. Furthermore, our PAWNNM shows promising results for inpainting providing better recovery compared with recent methods such as GSR [92] and NLI [4] in our experiments.

Although we show promising results, our method has some limitations. The performance of matrix completion relies on the size and number of low-rank matrices. Over several experiments, we determined appropriate patch sizes and an appropriate number of similar patches in an image or multi-frames, but careful analysis of scene information may provide better optimal choices. However, it is challenging to do this because, in general, detailed scene information cannot be known before a significant number of samples have been taken.

Our method is challenged when attempting to recover values under very low sampling rates ($< 10\%$). Previous recovery methods, such as the compressive sensing (CS) based approaches, suffer from similar issues. Our method shows better and more reliable results than CS based approaches at very low sampling rates.

Figure 3.15: A comparison between our pixel recovery and sub-pixel recovery under a same sampling rate (20%).

We tested our method in three different recovery scenarios; pixel, multi-frame, and sub-pixel. In pixel and multi-frame recovery, our reconstruction methods show reasonable *relative reconstruction cost* (i.e., reconstruction time $\leq 2\%$ rendering time of input image). This performance is at least as good as CR or CIR and our recovery quality outperforms their results.

On the other hand, the current implementation of the sub-pixel recovery is not efficient compared with previous filtering methods in terms of the total time. Equal time comparisons (Figure 3.14) show that LBF performs better than our method. However, because the input images for the filtering methods will have more samples in the equal time case (Table 3.8), our method shows better recovery result under the same sampling rates (Figure 3.13). In terms of efficiency, there are two ways in which we could feasibly improve performance: (1) our current implementation is based on Matlab without careful code optimization, and (2) our reconstruction is fully independent across patches, and therefore each matrix recovery step could be computed in parallel on the GPU.

In our current implementation we have fixed the sub-pixel grid size at 16×16 , since recovery time increases according to grid size, and we need to consider the *relative reconstruction cost* to balance the reconstruction

and rendering time. Therefore, we have limitations to evaluate sufficient samples in complex scenes such as San Miguel (Figure 3.12).

We have tested the recovery results of pixel and sub-pixel samples under the same sampling rate (Figure 3.15). In general, computed values for pixel samples are more reliable, since each evaluated pixel has been averaged with relatively more samples, and therefore each pixel has a better chance to have less noise than sub-pixel samples. Since matrix completion relies on the computed values, the following recovery can provide better estimation for missing pixels.

In some challenging cases, filtering techniques may not output better quality image even with input having many more samples (e.g. 8000 samples per pixel). We performed equal time comparison between our pixel recovery and RHF to recover (or filter) an incomplete rendering of a complex scene (the Villa scene, Figure 3.16); in our experiment, LBF could not handle many samples due to its high memory requirement. The light source in this scene is located outside and therefore the scene requires a large number of samples for proper rendering. Under a fixed time budget, our pixel recovery method outperformed RHF. Therefore, for previsualisation of complex scene such as the Villa scene requiring many samples per pixel, our pixel recovery can be a better choice than filtering techniques. To this end, recall that our key target application, the movie post production pipeline, often requires rendering of very complex scenes with a large number of samples.

Since denoising filters can relieve the variance issue, a hybrid approach, combining filtering and reconstruction approaches, could be a future practical solution.

Figure 3.16: Under a fixed time budget (1.8 hours), our pixel recovery (in the middle) can generate better visual quality results than filter based method, RHF (on the right).

Overall, our method shows an efficient reconstruction of incomplete path tracing images within a reasonable time, which can provide efficient screen space previsualization acceptable for many applications including visual effects rendering for films. The main limitation is that the extra time is spent on grouping similar patches for pixel recovery, we will offer another solution in the next chapter. Additionally, the proposed multi-frames reconstruction can only explore limited coherence information within the selected local area across multi-frames. To utilize global information of animation data, we also provide a new approach to this issue in Chapter 4.

Chapter 4

Enhanced Completion for Path Tracing

The method described in Chapter 3 delivered an accurate and efficient solution compared to other reference solutions. However, there are limitations in terms of pixel recovery and multi-frame recovery. First, the process of grouping similar patches to form a low-rank matrix in pixel recovery still requires extra time. Second, the coherence exploited in multi-frame recovery is mainly within a local area across frames.

In this chapter, we present novel reconstruction methods that efficiently recover missing data in incompletely rendered images and multi-frames using enhanced matrix completion and tensor completion respectively. Both of the two techniques have been applied to problems in different areas, such as image processing [93] and machine learning [94], but has not been exploited in computer graphics area. Enhanced Matrix Completion (EMaC) was proposed by Chen et al. [69]. The method is based on matrix completion and begins with converting the incomplete matrix into an enhanced form with Toeplitz or Hankel structure, then apply nuclear norm minimization (NNM) to compute the missing samples. Tensor completion explores the algebraic structure of high dimensional data. More recently, Kilmer et al. [18] proposed tensor singular value decomposition for ten-

tensor nuclear norm minimization, which demonstrates efficiency in recovery. Based on the two techniques, we apply different strategies to form a structure with low-rank property in pixel and multi-frame recovery.

For pixel recovery, we developed enhanced matrix reconstruction (EMR). Specifically, we form an enhanced pixel patch using a block Toeplitz structure [95]. Unlike our previous method in Chapter 3 that pre-completing the missing pixels and grouping similar patches together to obtain a low-rank matrix, the new enhanced pixel patch skips the computation of finding the similar patches. Therefore the whole recovery time could be reduced. We also adopt the multi-weighted nuclear norm minimization with progressive parameter adjustment strategy in Chapter 3, to efficiently recover high-frequency details.

For multi-frame recovery, we proposed to use tensor completion based reconstruction (TCR). The intuition is to treat the multi-frames as a tensor structure and use redundancy across nearby frames. We accordingly adopt tensor completion here rather than a frame by frame recovery scheme. Specifically, we utilize tensor singular value decomposition (t-SVD) for minimizing the tensor nuclear norm and apply our PAWNNM in the iterations. As a comparison with previous method in Chapter 3, the proposed method can speed up the recovery process and preserve better accurate details.

We begin this chapter by briefly revisiting the enhanced matrix completion and tensor completion (Section 4.1). We then introduce how to generate enhanced matrix structure and tensor structure for pixel and multi-frame recovery respectively (Section 4.2). In section 4.3, the recovery algorithms for enhanced matrix completion and tensor completion are described. In addition, Section 4.4 discusses results on selected test data and conduct the comparison between the proposed methods and exiting methods.

4.1 Overview

4.1.1 Basic Notation

This section introduces the essential notations used in this chapter. Specifically, vectors are represented as lowercase letters (like x, y, z); matrices are represented as uppercase letters (like X, Y, Z); and tensors are represented as calligraphy letters (like $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$).

A three-way real tensor is a three-dimensional structure in $R^{n_1 \times n_2 \times \dots \times n_N}$. A slice of a tensor is defined as a two-dimensional matrix by fixing two indices of the tensor. Specifically, $\mathcal{X}(i, :, :)$ stands for the i -th horizontal slice, where the first index is held constant for the tensor $\mathcal{X}$. Similarly, the i -th lateral and frontal slice are represented by $\mathcal{X}(:, i, :)$ and $\mathcal{X}(:, :, i)$ receptivity. Additionally, we use $\mathcal{X}^i$ as the notation of the i -th frontal slice, $\mathcal{X}(:, :, i)$ of $\mathcal{X}$. $\hat{\mathcal{X}} = \text{fft}(\mathcal{X}, [], 3)$ stands for the Fourier transform along the third dimension of $\mathcal{X}$. While $\mathcal{X} = \text{ifft}(\hat{\mathcal{X}}, [], 3)$ stands for the inverse Fourier transform.

Especially, we denote the block diagonal matrix of a given tensor $\mathcal{X} \in R^{n_1 \times n_2 \times n_3}$ as :

$$BD(\mathcal{X}) = \begin{bmatrix} \mathcal{X}^{(1)} & & \\ & \ddots & \\ & & \mathcal{X}^{(n_3)} \end{bmatrix} \quad (4.1)$$

4.1.2 Enhanced Matrix Completion

The matrix completion problem has been demonstrated that exact completion can be handled with nuclear norm minimization, which is a convex relaxation of the rank minimization problem [17]:

$$\begin{aligned} \min_{\mathbf{X}} \quad & \|\mathbf{X}\|_* \\ \text{s.t.} \quad & X_{ij} = M_{ij}, \quad (i, j) \in \Omega \end{aligned} \quad (4.2)$$

where M is the complete matrix, X is an estimation to be solved by rank minimization constraint on observed samples, and Ω contains the positions of the observed samples. The nuclear norm $\|X\|_*$ is defined as $\|X\|_* = \sum_{j=1}^n S_j(X)$ (n is the number of singular values of X), and $S_j(X)$ are singular values of matrix X .

In the work by Chen et al. [69], the authors proved that the effectiveness of the enhanced structure relies on the shift-invariance of harmonic structure, which promotes the application of Toeplitz structure to address compressed sensing problem in frequency domain. Different from their work, we explore the low rank property in image domain using the Toeplitz structure. Thus, given an enhanced matrix X_E that was generated by block Toeplitz matrix as described above, the EMaC problem can be described as:

$$\begin{aligned} \min_X \quad & \|X_E\|_* \\ \text{s.t.} \quad & X_{ij} = M_{ij}, \quad (i, j) \in \Omega \end{aligned} \tag{4.3}$$

4.2 Low Rank Matrix from an Image

In order to solve matrix completion using NNM, we need to build a low rank matrix for X . We present a numerical method to build the low rank matrix using a block Toeplitz matrix (BTM) from a patch of an image having missing or damaged pixels to recover.

4.2.1 Block Toeplitz Matrix (BTM)

A Toeplitz matrix has been introduced by [96] in mathematics. The block Toeplitz structure is comprised of multiple Toeplitz matrices, providing an efficient numerical solution to create a low rank matrix. Chen et al. [69] adapt this structure to solve compressed sensing problems. To the best of our knowledge, this is the first time the BTM has been used for solving the

Figure 4.1: Given a 5 by 5 matrix, a corresponding BTM can be constructed from it (see Equation (4.4) and (4.5)).

matrix completion problem. A Toeplitz matrix is defined as:

$$T = \begin{bmatrix} c_0 & c_{-1} & c_{-2} & \cdots & \cdots & c_{1-n} \\ c_1 & c_0 & c_{-1} & \ddots & \ddots & \vdots \\ c_2 & c_1 & c_0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & c_{-2} \\ \vdots & \ddots & \ddots & \ddots & c_0 & c_{-1} \\ c_{n-1} & \cdots & \cdots & c_2 & c_1 & c_0 \end{bmatrix} \quad (4.4)$$

and the BTM, X^B , is defined using the blocks of the Toeplitz matrices:

$$X^B = \begin{bmatrix} T_0 & T_{-1} & T_{-2} & \cdots & \cdots & T_{1-n} \\ T_1 & T_0 & T_{-1} & \ddots & \ddots & \vdots \\ T_2 & T_1 & T_0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & T_{-2} \\ \vdots & \ddots & \ddots & \ddots & T_0 & T_{-1} \\ T_{n-1} & \cdots & \cdots & T_2 & T_1 & T_0 \end{bmatrix} \quad (4.5)$$

Figure 4.2: Given a selected pixel patch P_i from an incomplete path tracing image, a corresponding BTM can be constructed (see Equation (4.4) and (4.5)).

where T_i of X^B is a Toeplitz matrix. Figure 5.5 shows how to generate a BTM from a single 5 by 5 matrix.

4.2.2 Create BTM from an Image

Given an image patch P_i with size $(2n - 1)$ by $(2n - 1)$ from the incomplete rendered image I , we form a block Toeplitz structure X^B based on Equation (4.5). Specifically, each block T_i is another Toeplitz matrix that is constructed from a column C_i in patch P , the first row and column of T_i are $(C_i^0, \dots, c_i^{1-n})$ and $(C_i^0, \dots, c_i^{n-1})$ respectively. The $2n - 1$ columns will generate $2n - 1$ Toeplitz matrices: $(T_{n-1}, \dots, T_{1-n})$. Finally, the block Toeplitz matrix X^B can be formed from the $2n - 1$ Toeplitz matrices, the first row and column of X^B are $(T_0, \dots, T_{1-n})$ and $(T_0, \dots, T_{n-1})$ respectively. As shown in Figure 4.2, a selected patch from the partially rendered image can be transformed into block Toeplitz matrix.

4.3 Tensor Completion

Given an incomplete tensor $\mathcal{X}$, the task of tensor completion is to reconstruct missing values using the known values, which can be described as the following convex optimization problem:

$$\begin{aligned} \min_{\mathcal{X}} \quad & \|\mathcal{X}\|_* \\ \text{s.t.} \quad & \mathcal{X}_{ijk} = \mathcal{M}_{ijk}, \quad (i, j, k) \in \Omega \end{aligned} \quad (4.6)$$

where $\mathcal{M}$ is the complete tensor data, and Ω contains the positions of the observed samples. $\|\mathcal{X}\|_*$ stands for the tensor nuclear norm of $\mathcal{X}$.

Previous methods, like the CP decomposition [97] and Tucker decomposition [98], which have been applied to solve the tensor completion problem, but both of the two techniques suffer from high computation or local minimal solution. More recently, Kilmer et al. [18] proposed tensor-SVD to address the optimization problem (4.4), which is demonstrated as a more efficient and accurate solution for the tensor completion problem. Given a three-way tensor $\mathcal{X} \in R^{n_1 \times n_2 \times n_3}$, the t-SVD of $\mathcal{X}$ is defined as:

$$\mathcal{X} = \mathcal{U} * \mathcal{S} * \mathcal{V}^T \quad (4.7)$$

where $\mathcal{U}$ and $\mathcal{V}$ represent orthogonal tensors with size $n_1 \times n_1 \times n_3$ and $n_2 \times n_2 \times n_3$ respectively. While $\mathcal{S}$ stands for a diagonal tensor of size $n_1 \times n_2 \times n_3$, and $*$ denotes the tensor multiplication.

It has been proved that the tensor decomposition can be computed efficiently under Fourier domain by Kilmer et al. [18] and they use the Algorithm 2 to achieve the t-SVD product. Based on t-SVD, the tensor multi rank and tensor nuclear norm are defined as below:

Definition 1 (Tensor Multi-rank): The multi-rank of $\mathcal{X} \in R^{n_1 \times n_2 \times n_3}$ is a vector $r \in R^{n_3 \times 1}$ with the i^{th} element equal to the rank of the i^{th} frontal slice of $\hat{\mathcal{X}}$.

Definition 2 (Tensor Nuclear Norm): The tensor nuclear norm of a tensor $\mathcal{X} \in R^{n_1 \times n_2 \times n_3}$ is defined as: $\|\mathcal{X}\|_* = \left\| BD(\hat{\mathcal{X}}) \right\|_*$.

Algorithm 2 tensor-SVD**Input:** tensor $\mathcal{X} \in R^{n_1 \times n_2 \times n_3}$ **Output:** U, S, V ;

```

1:  $\hat{\mathcal{X}} = \text{fft}(\mathcal{X}, [], 3)$ 
2: for  $i = 1, 2, 3, \dots, n_3$  do
3:    $[U_i, S_i, V_i] = \text{SVD}(\hat{\mathcal{X}}_i)$ 
4:    $\hat{\mathcal{U}}_i = U_i, \hat{\mathcal{S}}_i = S_i, \hat{\mathcal{V}}_i = V_i$ 
5: end for
6:  $\mathcal{U} = \text{ifft}(\hat{\mathcal{U}}, [], 3), \mathcal{S} = \text{ifft}(\hat{\mathcal{S}}, [], 3), \mathcal{V} = \text{ifft}(\hat{\mathcal{V}}, [], 3).$ 

```

Thus, $\|BD(\hat{\mathcal{X}})\|$ can be computed through the multi-rank of n_3 sub-problems: $\|BD(\hat{\mathcal{X}})\|_* = \sum_{i=1}^{n_3} \|\hat{\mathcal{X}}_i\|_*$

4.3.1 Multi-frame Tensor

Despite the single rendered image, multi-frame images in a video is another case that requires previewing by visual effects artists. Intuitively, a set of adjacent frames could be treated as a tensor product. By exploiting the redundant temporal information between frames, the missing samples could be efficiently recovered through the low-rank property of a tensor.

Given an incomplete rendered frame set $\{F_1, F_2, \dots, F_i\}$, and each frame F_i is partially rendered like our single image recovery case. Unlike the previous matrix completion method in Chapter 3, which find and group similar patches across adjacent frames to generate low-rank matrix, applying tensor completion can skip the grouping similar patches and directly exploit the low-rank property and coherence among continuous frames. As shown in Figure 4.3, a few adjacent partially rendered frames are grouped to form a tensor structure.

Figure 4.3: A tensor structure consists of a number of adjacent animation frames: $\{F_1, F_2, \dots, F_i\}$.

4.4 Enhanced Matrix and Tensor Recovery

4.4.1 Pixel Recovery via Enhanced Matrix Completion

As we described at the beginning of this chapter, reconstruction for missing pixel samples involves two steps. First, generating an EM of a selected image patch of the rendering (Section 4.2). Then applying the enhanced matrix completion to recover the missing pixels, which will be addressed in this section.

Recall from Section 4.1, the enhanced matrix completion can be described as follow:

$$\begin{aligned} \min_X \quad & \|X_E\|_* \\ \text{s.t.} \quad & X_{E_{ij}} = M_{ij}, \quad (i, j) \in \Omega \end{aligned} \tag{4.8}$$

To efficiently solve the problem (4.8), we adopt the Alternating Direction Method of Multipliers (ADMM) [99]. It has been demonstrated that ADMM is high efficient for solving convex or non-convex problems in different application [99], the algorithm performs optimization in a low com-

putational cost. Based on EMaC and ADMM, the augmented Lagrangian function of problem (4.8) could be defined as:

$$L(X_E, Y, Z, \beta) = \|X_E\|_* - \langle Z, X_E - Y \rangle + \frac{\beta}{2} \|X_E - Y\|^2 \quad (4.9)$$

where X_E is the estimate to be solved, Y is an intermediate variable, Z stands for the Lagrange multiplier, β and $\langle \cdot, \cdot \rangle$ represent the penalty parameter and the inner product respectively. The involved variables X and Y are computed separately in alternating order since the main concept of ADMM is to divide the minimization problem into two sub-problems that have closed-form solutions.

Specifically, the optimization problem (4.9) can be addressed as three sub-problems.

For computing X_E . We fix Z_k and Y_k , and solve the minimization problem $L(X_E, Y_k, Z_k, \beta)$ as follows:

$$\begin{aligned} X_{E_{k+1}} &= \arg \min_{X_E} L(X_E, Y_k, Z_k, \beta) \\ &= \arg \min_{X_E} \|X_E\|_* + \beta/2 \left\| X_E - (Y_k - 1/\beta Z_k) \right\|^2 \end{aligned} \quad (4.10)$$

We can obtain the closed form solution of (4.10) based on the weighted nuclear norm minimization (WNNM) [90]:

$$X_{E_{k+1}} = W_\lambda(Z_k - \frac{1}{\beta} Y_k) \quad (4.11)$$

where W_λ is the weighted singular value shrink operator which is defined as follows:

$$\begin{aligned} W_\lambda(X) &= U D_\lambda(S) V^T \\ D_\lambda(S) &= \text{diag}(\max\{S_i(X) - \lambda_i, 0\}) \\ \lambda_i &= \frac{\omega_i}{S_i(X) + \epsilon} \end{aligned} \quad (4.12)$$

where U, S, V are the SVD product of X , S_i is the i^{th} singular value of X , ω_i is the weight for i^{th} singular value, and ϵ is a small regularizer to avoid dividing by zero.

We followed the parameter adjustment (PA) strategy in Chapter 3, to efficiently solving the optimization problem. The PA strategy reduces the singular values with a constant parameter C , which is described as:

$$\omega_{i+1} = c\omega_i \quad (4.13)$$

As discussed and tested in Chapter 3, the PA strategy is based on the observation that recovered image quality keeps increasing within fixed iterations. Compared with a hard threshold, using a gradually reduced soft threshold helps to achieve an accurate result, as demonstrated by Mancera and Portilla [91].

For computing Y . We fix X_k, Z_k to calculate Y_{k+1} as follows:

$$\begin{aligned} Y_{k+1} &= \arg \min L(X_k, Y, Z_k, \beta) \\ &= \arg \min \left\| Y - (X_k - \frac{1}{\beta} Z_k) \right\|^2 \end{aligned} \quad (4.14)$$

which has a closed form solution, and we fix the values at the observed entries and obtain

$$Y^{k+1} = \begin{cases} M_{ij} & \text{if } (i, j) \in \Omega \\ X_k - \frac{1}{\beta} Z_k & \text{otherwise} \end{cases} \quad (4.15)$$

For computing Z . We fix X_{k+1} and Y_{k+1} , Z_{k+1} which can be obtained from:

$$Z_{k+1} = Z_k + \beta(X_{k+1} - Y_{k+1}) \quad (4.16)$$

The whole procedure of the recovery is summarized in Algorithm 3.

4.4.2 Multi-frame Recovery via Tensor Completion

For multi-frames recovery, we treat a set of incomplete rendered frames as a tensor product $\mathcal{X}$. Based on the tensor nuclear norm and tensor-svd that introduced in Section 4.3, we describe the multi-frame recovery problem as:

$$\begin{aligned} \min_{\mathcal{X}} \quad & \|\mathcal{X}\|_* \\ \text{s.t.} \quad & \mathcal{X}_{ijk} = \mathcal{M}_{ijk}, \quad (i, j, k) \in \Omega \end{aligned} \quad (4.17)$$

Algorithm 3 Pixel Recovery via EMR**Require:** partially rendered data M , sampling mask Ω **Ensure:** optimal X_{E_k}

- 1: initial $X_{E_0} = M, \beta_0, c, Y = Z = 0, k=0$;
- 2: **repeat**
- 3: $Y_{k+1} = X_{E_k} - \frac{1}{\beta_k} Z_k$;
- 4: $\tau = \frac{1}{\beta_k}, X_{E_{k+1}} = W_\tau(Z_k - \tau Y_k)$;
- 5: $Z_{k+1} = Z_k + \beta_k(X_{E_{k+1}} - Y_{k+1})$;
- 6: $\beta_{k+1} = c\beta_k, k = k + 1$;
- 7: **until** $\|X_{E_{k+1}} - X_{E_k}\| \leq tol$ and $k < iter$

where $\mathcal{M}$ is complete tensor data, and Ω contains the positions of the observed samples. $\|\mathcal{X}\|_*$ stands for the tensor nuclear norm of $\mathcal{X}$.

Similar with the pixel recovery in previous section, we also adopt AD-MM to solve Equation (4.17), and the objective function can be defined as:

$$L(\mathcal{X}, \mathcal{Y}, \mathcal{Z}, \beta) = \|\mathcal{X}\|_* - \langle \mathcal{Z}, \mathcal{X} - \mathcal{Y} \rangle + \frac{\beta}{2} \|\mathcal{X} - \mathcal{Y}\|^2 \quad (4.18)$$

where $\mathcal{Z}_k$ and β are the Lagrange multiplier and penalty parameter respectively, $\langle \cdot, \cdot \rangle$ stands for the standard inner product. $\mathcal{Y}$ is an intermediate variable for computing $\mathcal{X}$.

The main difference between (4.18) and (4.8) is that the matrix term X_E, Y, Z are replaced by tensor $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ accordingly. Thus, problem (4.18) can be iteratively solved by the three steps introduced in Section 4.3.1.

For computing $\mathcal{Y}$ and $\mathcal{Z}$, we can directly adapt the solution from (4.14), which is described as follows:

$$\mathcal{Y}^{k+1} = \begin{cases} \mathcal{M}_{ijk} & \text{if } (i, j, k) \in \Omega \\ \mathcal{X}_k - \frac{1}{\beta} \mathcal{Z}_k & \text{otherwise} \end{cases} \quad (4.19)$$

$$\mathcal{Z}_{k+1} = \mathcal{Z}_k + \beta(\mathcal{X}_{k+1} - \mathcal{Y}_{k+1}) \quad (4.20)$$

For computing $\mathcal{X}$, the minimization problem with tensor product is

described as follows:

$$\mathcal{X}_{k+1} = \arg \min_{\mathcal{X}} \|\mathcal{X}\|_* + \beta/2 \left\| \mathcal{X} - (\mathcal{Y}_k - 1/\beta \mathcal{Z}_k) \right\|^2 \quad (4.21)$$

Unlike like the pixel recovery case, here we utilize tensor-SVD with PAWNNM to solve $\mathcal{X}$.

$$\begin{aligned} \mathcal{X}_{k+1} &= \text{ifft}(\hat{\mathcal{X}}_{k+1}) \\ \hat{\mathcal{X}}_{k+1} &= \hat{W}_\lambda(\hat{Z}_k - \frac{1}{\beta} \hat{Y}_k) \end{aligned} \quad (4.22)$$

where $\text{ifft}()$ stand for the inverse Fourier transform, $\hat{W}_\lambda$ stands for the weighted singular value shrink operator in Fourier space as follows:

$$\hat{W}_\lambda(X) = W_\lambda(\hat{X}) \quad (4.23)$$

here, the definition of the weighted singular value shrink operator W_λ is same with (4.11). The whole optimization process for multi-frame recovery is described in Algorithm 4. The input data are the partially rendered animation sequences, $\mathcal{M}$, and the corresponding sampling mask that the renderer has used to determine which samples to take. We iterate on the process based on the description in this section to generate a satisfying reconstructed result.

Algorithm 4 Multi-frame Recovery via TCR**Require:** partially rendered animation data $\mathcal{M}$, sampling mask Ω **Ensure:** optimal $\mathcal{X}_k$

```

1: initial  $\mathcal{X}_0 = \mathcal{M}$ ,  $\beta_0, c$ ,  $\mathcal{Y} = \mathcal{Z} = 0$ ,  $k=0$ ;
2: repeat
3:    $\mathcal{Y}_{k+1} = \mathcal{X}_k - \frac{1}{\beta_k} \mathcal{Z}_k$ ;
4:    $\tau = \frac{1}{\beta_k}$ ,  $\mathcal{L} = \mathcal{Z}_k - \tau \mathcal{Y}_k$ ,  $\hat{\mathcal{L}} = \text{fft}(\mathcal{L}, [], 3)$ ;
5:   for  $j = 1 : n_3$  do
6:      $\text{SVD}(\hat{\mathcal{L}}_j) = \hat{\mathcal{U}}_j \hat{\mathcal{S}}_j \hat{\mathcal{V}}_j^T$ ;
7:      $\lambda_i = \frac{\tau}{\hat{S}_i(\hat{\mathcal{L}}_j) + \varepsilon}$ 
8:      $D_{\lambda_i}(\hat{\mathcal{S}}_i) = \text{diag}(\max\{\hat{\mathcal{S}}_i(\mathcal{L}_j) - \lambda_i, 0\})$ 
9:      $\hat{\mathcal{P}}_j = \hat{\mathcal{U}}_j D_{\lambda_i}(\hat{\mathcal{S}}_j) \hat{\mathcal{V}}_j^T$ 
10:  end for
11:   $\mathcal{Y}_{k+1} = \text{ifft}(\hat{\mathcal{P}}, [], 3)$ 
12:   $\mathcal{Z}_{k+1} = \mathcal{Z}_k + \beta_k(\mathcal{X}_{k+1} - \mathcal{Y}_{k+1})$ ;
13:   $\beta_{k+1} = c\beta_k$ ,  $k = k + 1$ ;
14: until  $\|\mathcal{X}_{k+1} - \mathcal{X}_k\| \leq \text{tol}$  and  $k < \text{iter}$ 

```

4.5 Results

We evaluated our method using various test scenes, including the Bedroom, Bamboo and Bottles scenes. Due to all these scenes contain high-frequency content, recovering missing samples without blurry artifacts would be challenging. Similar to the experiment setting in Chapter 3, the PBRT and LuxRender are applied to generate the partially rendered images or animated sequences. The tested images for pixel recovery are 1280×960 pixels, and animated sequences are 311×500 pixels. All experiments are conducted on a desktop with a quad-core 3.0 GHZ Intel Xeon CPU with 8G RAM.

Figure 4.4: Given a selected patch, the block Toeplitz matrix is able to obtain a low rank property, similar to the patch grouping approaches; tested using a different percentage of randomly distributed input pixels (20%, 40%, and 100% as a reference).

Figure 4.5: Recovery quality and time of an incomplete path tracing image (San Miguel scene with 1024×1024 resolution with 40% pixels) under different patch sizes using a BTM; we select 25×25 to balance quality and recovery time.

4.5.1 Pixel Recovery

We evaluate our method with missing pixels of an incomplete path tracing image and partially damaged photograph. In our tests, we use PBRT [100] to render images with missing pixels, and the quality is quantitatively measured using the PSNR and SSIM image similarity metrics. All experiments are conducted on a desktop with a quad-core 3.0 GHZ Intel Xeon CPU with 8G RAM. In our tests, we set the patch to 25×25 , which we chose experimentally while testing recovery quality and time from eight different scenes under different patch sizes (Figure 4.5). The parameters

Figure 4.6: The PSNR of progressive recovery using iterative WNNM, going up to 100 iterations. This uses the Sanmiguel scene with 40% rendered pixels.

Figure 4.7: Examples of missing pixel recovery for sparsely rendered images with different sampling rates (10%, 20%, 40%).

Figure 4.8: The recovery comparison with CR (Compressive Rendering), CIR (Compressive Image Reconstruction), PAWNNM, and our method.

Figure 4.9: The recovery comparison of CR (Compressive Rendering), CIR (Compressive Image Reconstruction), PAWNNM, and our method.

Figure 4.10: Examples of repairing damaged photographs. Compared with the previous method [4], our methods takes less time to recover the damaged pixels with slightly better quality.

in Algorithm (1) are also chosen experimentally, initialized as: $\beta = 0.39$, $\omega = 80.5$, and $c = 0.9$. The number of iterations is set to 100, where the recovery quality is progressively improved with more iterations. For example, using 40% sampling rate, we can observe convergence as it approaches 100 samples in Figure 4.6.

In Figure 4.4 we show that our BTM achieves a low rank matrix numerically without the need to use a complex initialization and patch finding algorithm as used in the previous methods [4, 101]. Figure 4.7 shows our recovery of images under 10%, 20% and 40% input pixels, where our method can generate plausible results for pre-visualization even with 10% input pixels.

We compare our method with state-of-the-arts approaches for path tracing pre-visualization, which include CR [9], CIR [23] using compressed sensing, and PAWNNM [101] based on matrix completion. We evaluated our method using test scenes that exhibit various properties, such as: a fine detailed structure, motion blur, soft shadow and depth of field. We show in Figure 4.8 that our method has better visual quality than CS based approaches, and similar visual quality as PAWNNM, but our method re-

Table 4.1: PSNR/SSIM results of three test scenes under different input pixel sampling rates.

Scene	Pixel portion	CR	CIR	PAWNNM	Ours
<i>Sanmiguel</i>	60%	29.43/0.912	34.17/0.968	35.95/0.975	35.93/0.974
	40%	27.31/0.873	31.93/0.947	33.44/0.958	33.46/0.961
	20%	25.66/0.852	28.64/0.902	30.58/0.916	30.55/0.917
<i>Killeroos</i>	60%	32.40/0.949	37.31/0.967	38.68/0.975	38.61/0.974
	40%	29.38/0.876	33.46/0.951	35.52/0.967	35.59/0.969
	20%	26.71/0.822	28.29/0.873	31.96/0.926	31.26/0.925
<i>Toasters</i>	60%	35.63/0.962	39.42/0.978	40.68/0.980	40.63/0.979
	40%	32.57/0.902	36.58/0.961	38.36/0.977	38.34/0.975
	20%	28.92/0.875	32.76/0.917	35.64/0.963	35.62/0.963
<i>Spotfog</i>	60%	35.17/0.919	43.23/0.975	44.81/0.985	44.79/0.984
	40%	33.21/0.876	40.74/0.952	42.71/0.972	42.69/0.971
	20%	30.74/0.849	36.83/0.927	37.33/0.930	37.35/0.931

quires less computation time. Rather than finding similar patches as done by PAWNNM [101], the BTM uses a numerical method using only the selected patch, which reduces the computation time. In our tests, the reconstruction time is reduced from 17 minutes (in [101]) to 8 minutes in a 1024×1024 image. We use the relative construction cost (reconstruction time/partial rendering time) for evaluation in Table 4.2. Another two complex scenes that contain fine details are shown in Figure 4.9.

We also tested our method with inpainting applications, which often involves tasks such as photograph repair by removing scratches or overlaid text (Figure 4.10). Our method can effectively reconstruct the photographs which look similar to the original photographs. The previous method [4] took approximately 13 minutes to generate the repaired result, our method took only 36 seconds.

Table 4.2: Recovery performance measured by relative recovery cost with respect to the total rendering time. The reference image is rendered in 32 hours.

Scene	Pixel Portion	CR	CIR	PAWNNM	EMR
<i>Samiguel</i>	60%	0.8%	6%	1.4%	0.7%
	40%	0.8%	5%	1.4%	0.7%
	20%	0.8%	4%	1.4%	0.7%

4.5.2 Multi-frame Recovery

To evaluate our method for recovering the missing pixels from animation sequences, a video consisting of 150 frames is rendered for comparing with our previous methods PAWNNM in Chapter 3. In our test, we select ten frames as a tensor, and missing pixels values of the frames are recovered by the method described in Section 4.3.2; boundary frames of each tensor are overlapped and values are averaged.

Figure 4.11 shows the comparison with the patch based approach in Chapter 3. Our tensor completion based reconstruction can achieve better recovery quality than the patch based approach since we can consider the temporal coherence across frames with more global scene information rather than within a local region, and the temporal noise can be greatly reduced. We also demonstrate this by comparing the standard deviation of the recovery in Table 4.3. Benefiting from the tensor structure, the proposed method can escape the demanded of grouping similar patched from adjacent frames, our tensor-based multi-frame recovery also shows faster performance than the patch based approach (in Table 4.3). The total recovery time is only 6% of total rendering time.

Reference

Input 10% pixels

Input 20% pixels

Input 40% pixels

PAWNNM

TCR

Figure 4.11: Example of our proposed TCR and PAWNNM under different sampling rate.

Table 4.3: Result of multi-frame recovery (with TCR): Incomplete video frames sampled at 10%, 20% and 40% are reconstructed independently (single-frame) and 10-frame concurrently (multi-frames). Time is measured in minutes.

Input Samples	Recovery Method	Average PSNR/SSIM	Standard Deviation	Recovery Time	Full Render Time
10%	single-frame	27.42/0.867	1.62	1029	1440
	multi-frames	29.38/0.872	1.03	86	
20%	single-frame	31.74/0.905	1.25	1029	
	multi frame	34.23/0.931	0.80	86	
40%	single-frame	35.17/0.934	1.08	1029	
	multi frame	39.41/0.956	0.42	86	

4.6 Summary and Discussion

In this chapter, we have presented two efficient solutions for pixel recovery and multi-frame recovery. We first applied an enhanced matrix completion based on Toeplitz structure to reconstruct the missing pixels in a single rendered image. To exploit the low-rank property for the recovery of the missing pixel samples in an enhance matrix structure, we employed the ADMM algorithm with a parameter adjustment strategy that alternatively estimate the factorization matrices. Since our algorithm is SVD free, the computational time is faster than the approach presented in Chapter 3.

We then introduced tensor structure and apply tensor completion for recovering missing values of partially rendered animation sequences. We exploited the tensor singular value decomposition, which can characterize information and structural complexity of multi-frame rendering. By applying the PAWNNM to solve the tensor completion problem, our multi-frame recovery shows significant performance gain compared to the proposed patch based reconstruction in the previous chapter. The primary

contribution of the work presented in this chapter is the introduced enhanced matrix completion with Toeplitz structure, which helps to significantly reduce the overall recovery time compared to the pixel recovery in Chapter 3. Additionally, by adapting the tensor structure to multi-frame restoration, global coherence information can be explored across both the image domain and time domain. The proposed multi-frame reconstruction solution can generate a more accurate result within less time than the proposed technique in Chapter 3. Finally, by incorporating the progressive adjustment strategy with weighted nuclear norm minimization into standard enhanced matrix completion and tensor completion, our proposed strategy can achieve better recovery quality.

The proposed reconstruction methods have similar limitations with the methods in Chapter 3. One limitation is that the proposed reconstruction methods are challenged when attempting to recover pixel values under very low sampling rates ($< 10\%$). Another limitation of the reconstruction methods is the requirement of rendering captured pixels with sufficient samples, which means an extra time is spent before the post process. Differently, a denoising based method only requires a few samples to generate the input. Thus, it seems more efficient than a reconstruction based manner. In next chapter, we will introduce a novel denoising filter approach to efficiently achieve noise-free path tracing.

Chapter 5

Denoising Network for Path Tracing

As introduced in Chapter 1, rendering photorealistic images of a virtual scene requires simulating all the possible light paths between the light sources and view camera. Path tracing [5] provides a general approach to approximating this problem. The method has been widely applied in feature film and visual effects production since it can meet the high-quality requirement of the industry. Unfortunately, this task involves generating a large number of samples and lengthy computation time for noise-free renderings.

To mitigate this problem, previous two chapters have investigated the reconstruction based approaches to provide efficient and accurate solutions for path tracing which provides the ability to recover the missing samples based on rendered ones, but the reconstruction methods require rendering the captured pixels or sub-pixels with sufficient samples, which means less efficiency under a low sample budget. We will now turn to the denoising based approach. A large number of works have been proposed to generate a noisy rendering with a low sample budget quickly and in the process the application of a denoising filter to output a noise-free result [10, 11, 12, 13, 14, 15]. These methods usually apply a scene feature

based filter, like non-local mean filter and bilateral filter, which can utilize additional scene feature buffers (e.g. world position, depth information) to guide the denoising process. Due to the additional scene information is closely related to scene details; these features based denoising filters achieve a good quality result. But the main challenge is how to set the filter weights for different features properly. Many approaches use error estimation metrics to select the best filter parameters. Since the error metrics are noisy under a low sample budget, the accuracy of the selected parameters are not the optimum ones. Therefore, the denoised results from these methods may suffer from over blurred areas, especially in the region with high-frequency details.

More recently, the convolutional neural network has been applied and achieved better performance than the feature based denoising filters. Kalantari et al. [80] introduced a multi-layer perceptron learning-based model. A recurrent denoising autoencoder is proposed to reconstruct rendered animation sequences [83]. Also, kernel predicting based convolutional network is proposed to train and evaluate with production data [81]. The CNN based denoising network can learn the complex relationship between noisy rendering and corresponding ground-truth by leveraging both the color values and the auxiliary scene features, while showing state-of-the-art performance. Once the neural network is trained, it can be easily and efficiently applied without complex parameter selection. Such a framework can more efficiently handle the task of previsualization of path tracing.

Motivated by these deep learning based works, in this chapter, we propose to apply a multi-scale residual learning based denoising network. The basic task is to render a noisy result in a quick manner with only a few samples per pixel, and then apply the denoising network to output a noise-free result within several seconds, which is more efficient than the reconstruction methods. Unlike the previous CNN based architectures, which apply a single scale filter in the convolutional layer or perform

multi-scale concept through down/up-scale feature maps, we apply different filters during the convolution step to preserve both long and short contextual information. Additionally, we propose a separated residual learning strategy to predict the noise and generate a virtual flash image as one of the input scene feature buffers.

5.1 Residual Learning for Denoising Path Tracing

Denoising in image processing mostly challenges spatially-invariant Gaussian noise [78], which means evenly and globally distributed noise in an image. Figure 5.1 shows an example of path tracing image generated by limited samples per pixel. As shown in Figure 5.1, since the different local regions shows inhomogeneous noise in a image, conventional denoising in image processing is not efficient.

Although the noise level in path tracing is non-uniform and typically does not follow a Gaussian distribution over the whole image space, we observed that the residual noise within a local area is nicely approximated by a Gaussian distribution within the local area. As shown in Figure 5.1, the histograms of residual noise in different patches are near Gaussian distributions. Therefore, we show that this consistent representation of the residual is learned by the neural network more efficiently.

5.2 Dual Channel Residual Learning

Path tracing is a process to evaluate the light paths from camera to light sources by randomizing the direction of the ray at the surface intersection though the entire scene. During rendering, each sample ray within a pixel may not always successfully find a complete light path and contribute to calculating the final pixel value. Therefore, noise artifacts are caused by

Figure 5.1: In a noisy path tracing (the top), the noise level is various in different image regions; the noise level in the region of blue window is higher than the region of red window. In contrast, the area in the green window contains less noise. The bottom figure is the histogram distributions of the residual from the three different patches, showing near Gaussian distributions.

insufficient numbers of complete light paths during the rendering phase. Pixel value integrated by the insufficient samples would be either over or under exposed than the reference image. In Figure 5.2, the scene is rendered with 8 samples per pixel (spp), some pixels in the noisy rendering are much brighter or darker than the corresponding pixels in the reference image. For example, the pixel in the green square obviously has a higher value than the reference rendering, while the pixel value in the yellow square looks lower.

Figure 5.2: An example of noise artifacts in image rendered with only 8 samples per pixels (left image). The noise artifacts are caused by either extreme larger (green square) or smaller (yellow square) pixel value compared to the corresponding pixel in the reference rendering.

Given a noisy rendering y and a corresponding reference image G , we define the over and under exposed signals in the noise rendering as bright channel residual R_b and dark channel residual R_d respectively:

$$R_b = \begin{cases} y_i - G_i, & y_i \geq G_i \\ 0, & y_i < G_i \end{cases} \quad (5.1)$$

$$R_d = \begin{cases} G_i - y_i, & G_i \geq y_i \\ 0, & G_i < y_i \end{cases} \quad (5.2)$$

where i is the pixel position in the rendering.

The objective of our method is to remove the noise artifacts from the image $y = \{y_r, y_g, y_b\}$ rendered with a few samples per pixel (e.g., 8 spp), while aiming to have the denoised output with visual quality as close as possible to the reference image G that rendered with many samples (e.g., 5k to 64k spp). We apply a convolutional neural network to learn the mapping g between the noisy input and the reference image. Given the inputs of network include a noisy rendering y and a set of feature images f (i.e. surface normal, depth information, virtual flash image), our learning pro-

cess can be described as:

$$\Theta^* = \arg \min_{\Theta} \ell(G; g((y, f); \Theta)) \quad (5.3)$$

where ℓ is an error metric between the denoised result and the input noisy rendering. Θ is network parameters. Θ^* is the optimum ones we aim to estimate during the training of the network. We use the residual learning technique to learn a mapping of the residual:

$$R(y, f) \approx G - y \quad (5.4)$$

Specifically, we separate the residual R into two channels: bright channel residual R_b and dark channel residual R_d . By applying the separated networks to predict the two channels' residuals, we show that dual channel residual learning can boost the training process to learn efficient denoising filter for path tracing. Therefore, based on Eq.5.3, learning process would be:

$$\begin{aligned} \Theta^* &= \arg \min_{\Theta} \ell(G; y + R(y; \Theta)) \\ &= \arg \min_{\Theta_{b,d}} \ell(G; y + R_b(y; \Theta_b) + R_d(y; \Theta_d)) \end{aligned} \quad (5.5)$$

where the network parameters Θ can be expressed into two parts: Θ_b and Θ_d , which represent the parameters for R_b and R_d respectively.

5.3 Multi-scale CNN

We present a novel multi-scale CNN for dual channel residual (R_b and R_d) learning. In order to extract features from the input patches (the noise and scene feature images), multi-scale kernels are developed in each convolutional layer. Compared with the conventional CNN using fixed size kernels, our multi-scale CNN provides more robust path tracing denoising for complex scene with various contextual details.

Figure 5.3: Overview of the proposed denoising pipeline. The input of the network consists of a noise rendering, normal vector, depth information and virtual flash image buffer. The input is fed to two residual networks for prediction of the bright and dark channel residual separately. The denoised image is achieved by combining the residual images with the noise rendering.

5.3.1 Network Architecture

Two separated CNNs were set up to predict the bright channel residual and the dark channel residual. Figure 5.3 illustrates the overall flow of our denoising pipeline. As shown in the overall architecture of our network (Figure 5.3), the bright and dark channel residual networks have the same structure, therefore we only explain details for the bright channel residual.

The inputs to the network are patches of a noisy rendering and corresponding scene features (normal vector, depth, virtual flash image). For a N -layer CNN, the first $N - 2$ layers include a n types of multi-scale filters for multi-scale convolution. The multi-scale convolutional neural network has been applied for object detection [102], medical image processing [103]. However, it has not been applied to solve path tracing denoising. We use the Maxout function [104] as the activation function. For layer $(N - 1)$, a single linear convolution is applied, and then rectified linear units (ReLU)[105] is used as the activation function. The last layer contains a single scale convolution to reconstruct the denoised output.

Figure 5.4: The multi-scale structure in the convolutional layer of our network. Outputs from the previous layer are convolved with different size filters to generate new features maps. Then, group the new feature maps using Maxout activation to generate the outputs feature maps.

We presents a novel concept that present a multi-scale filters inside each convolutional layer rather than using the multi-scale strategy for re-sizing input data [102, 103]. Since the important details of the inputs could be lost through multi-scale resizing, our multi-scale filters operate inside the convolutional layers extracting the feature details from the different sized kernels. An optimal selection for denoising filter is to use a small scale filter for high frequency area, and a large scale filter for the smooth area.

As illustrated in Figure 5.4, supposing there are n types of filter k_i in the multi-scale convolutional layer, and each type of filter contains m filters. Thus, each type of filter generates m feature maps, and there will be $n \times m$ feature maps in total. Then, all feature maps will be separated into m groups.

Figure 5.5: Evaluation of the learning performance with different loss functions . The network is trained with L_1 loss and L_2 loss separately to compute the average PSNR (peak signal-to-noise ratio) on 7 tested images. With a comparison, L_1 loss is more efficient than L_2 loss.

5.3.2 Loss Function

To estimate the error between the outputs from the CNN and target reference images, a loss function is calculated while training the network. In image processing, L_2 loss is generally applied, to computes the mean squared error between the predicted and the target reference image. However, we apply L_1 loss as our loss function:

$$L_1 = \frac{1}{N} \sum_i^N |D_i - R_i| \quad (5.6)$$

here, D_i and R_i represent the i th pixel in the denoised and reference image respectively.

L_1 loss can better mitigate the splotchy artifacts in the denoised image comapre with L_2 loss [106]. Figure 5.5 shows the average PSNR estimated using L_1 and L_2 loss function. After 10 epochs, the network trained by the L_1 loss shows a better result than the one using L_2 loss. The bright and dark channel residual networks are trained independently with L_1 loss in

our networks. Figure 5.7 shows the training error of the proposed denoising network.

5.3.3 Training Data

The training data set in our experiment consists of 140K patches for each input including noise colour, normal, depth, and virtual flash images. All data has been generated from 30 scenes that contain various effects such as motion blur, global illumination, and depth of field. To increase the diversity of the training dataset, each scene is rendered with 4, 8, 16, 32, 64 spp. The reference images are rendered with a huge number of spp (e.g. 5k to 64k). Additionally, each scene is rendered four times at each sampling rate to avoid overfitting to specific noise patterns. After generating the training data, the images are cropped into 65×65 patches. During training, rotation and flip operations are randomly applied to each training patch pair.

The input is generated using PBRT [107]. Besides these commonly used scene features (noise colour, normal, depth), we utilized a virtual flash image as an additional input.

5.3.4 Virtual Flash Image

We utilize a virtual flash image as the additional scene feature in our approach in order to provide various important details in the scene; benefits of using a flash image has been demonstrated in few previous denoising methods [108, 109, 110]. An example of our virtual flash image is shown in the top row in Figure 5.6. To generate the virtual flash image, a virtual point light is set up at the camera position and then the rendering image is generated without original light sources capturing the high-frequency scene details. Figure 5.6. (bottom row) shows the pixel histogram distributions of the reference rendering, virtual flash image, and noisy rendering

Figure 5.6: Top row (from the left): a noisy rendering produced with limited samples, and the corresponding virtual flash rendering. Bottom row: histograms of the reference, noise, and virtual flash image.

Table 5.1: Denoising results of multi-scale CNN in different kernel size.

Multi-scale Kernel Sizes	PSNR
$\langle 3, 5, 7 \rangle$	27.68
$\langle 3, 5, 9 \rangle$	29.24
$\langle 5, 7, 9 \rangle$	28.12
$\langle 5, 9, 13 \rangle$	27.83
$\langle 7, 9, 13 \rangle$	24.56

of the scene. The virtual flash image could better approximate the distribution pattern of the ground truth rendering. Also, the virtual flash image can depict scene features where the shadow area is under the original light sources. Additionally, rendering the virtual flash image is quick, since we only consider the direct illumination without additional ray samples. The virtual flash feature image contributes to preserve high-frequency information in our denoised results.

5.3.5 Training Details

For the bright and dark channel residual networks, we apply six convolutional layers with 64 filters in each layer except the final layer, which contains three kernels. From the first layer to the fourth layer, three different size kernels are used: 3×3 , 5×5 and 9×9 respectively. We denote the setting of these kernel sizes as $\langle 3, 5, 9 \rangle$. The kernel size for the fifth layer and the last layer is set to 5×5 and 3×3 respectively. The kernel size of the multi-scale convolutional layer have been experimentally selected. As shown in Table 5.1, we modify the multi-scale kernel size into other options (e.g. $\langle 5, 7, 9 \rangle$) and compute the average PSNR on seven tested scenes. Our selection of $\langle 3, 5, 9 \rangle$ achieves best performance. For

Figure 5.7: Training error of the proposed bright and dark channel network trained on L_1 loss function.

efficient network training, mini-batch training is used, which uses a subset of the training data during computing the back-propagation of each iteration. The size of the mini-batch is set to 64 spp for our training. As common setting with other deep learning methods, we chose Adam solver [111] as optimizer in TensorFlow [112] with a learning rate of 0.0001, and trained on NVIDIA GTX1080 graphics card. We trained 100 epochs for two networks, the training time is around 3 days.

5.4 Results

5.4.1 Denoising Performance

This paper presents a series of contributions to enhance denoising quality for the path tracing image using CNN. These include dual channel residual learning, multi-scale CNN, and an additional scene features using the virtual flash image. We tested our result in several CNNs with different combinations to clarify the performance gain from each contribution. The first step is to show the performance difference between single and dual

Figure 5.8: We introduce dual channel residual learning with multi-scale convolutional neural networks for denoising path tracing renderings. We trained networks for single channel residual learning (SRS) and dual channel residual learning (DRS) using a single fixed scale CNN, a dual channel residual using a multi-scale CNN with (DRMF) and without the virtual flash image (DRM). The noisy input is rendered with 32 spp (samples per pixel), while the reference image is rendered with 64k spp. Our proposed DRMF closely resembles the reference.

channel residual learning. We trained networks for a single channel residual learning (SRS) and a dual channel residual learning (DRS) using a single fixed scale CNN. Then, we trained a network to learn the dual channel residual using a multi-scale CNN with (DRMF) and without the virtual flash image (DRM).

Figure 5.8 shows the results of each setup. Although SRS shows good denoising for the changing scene generated by highly limited samples, it shows various artefacts including over/under exposed noisy signals. Our dual channel residual learning (DRS) shows outstanding performance to denoise over/under exposed noisy signals. Then, our multi-scale CNN (DRM) further improve denoising in the low frequency area while preserving the high frequency details. Finally, our virtual flash image fea-

ture (DRMF) contributes to enhance overall denoising performance while adding extra scene details limited by the noisy rendering input. Our DRMF shows outstanding performance to denoise path tracing image even for the very challenge case as shown in Figure 5.8 (a noisy complex scene rendered by a highly limited number of samples). The final result of our denoising for the noisy rendering with limited samples (32 spp) shows comparable visual quality with the reference rendering having 64K spp.

5.4.2 Performance Comparison

We compared our DRMF with state-of-the-art denoising methods for path tracing. Our method was compared with two recent learning-based methods using the neural networks, LBF [80] and KPCN [81]. Then, compare our DRMF with colour based denoising, RHF [113], and scene feature based denoising, NFOR [114]. All methods are based on the original implementation with default settings described by the papers. Two most commonly used error metrics PSNR and SSIM are used to evaluate the denoising result of the noisy image (rendered by 8 to 32 spp) with respect to the reference image (rendered by 5k to 64K spp).

Denoising comparison with CNN based methods: Figure 5.9 shows comparison results with the CNN based approaches, LBF and KPCN. We tested across four challenging scenes such as Kitchen, Livingroom1, Living room2 and Bathroom (noise image with 8 spp, reference image with 64k spp). Our method (DRMF) shows better result compared with LBF and KPCN. In particular, our method outperform in the selected regions; both LBF and KPCN shows artefacts such as over blurring the high frequency areas (taps of the Bathroom scene) or under smoothing for low frequency areas (table of the Livingroom 2). Table 5.2 shows the error metrics (PSNR and SSIM) of the entire image to compare denoising results of our methods (SRS, DRS, DRM, and DRMF) with others.

Denoising comparison with colour/feature based methods: Figure

Figure 5.9: Comparison of LBF, KPCN and our proposed denoising network (DRMF).

Figure 5.10: A visual quality comparison of RHF, NFOR and our proposed denoising network (DRMF).

Table 5.2: PSNR/SSIM results of our methods (SRS, DRS, DRM, and DRMF), LBF and KPCN.

	SRS	DRS	DRM	DRMF	LBF	KPCN
Kitchen	29.86/0.9316	30.59/0.9458	31.79/0.9502	37.58/0.9873	32.13/0.9518	34.33/0.9648
Livingroom1	30.38/0.9326	34.84/0.9511	35.17/0.9525	41.22/0.9800	35.33/0.9572	36.98/0.9666
Livingroom2	28.04/0.9035	28.39/0.9248	29.17/0.9325	40.19/0.9783	29.68/0.9327	31.60/0.9567
Bathroom	26.59/0.8426	28.84/0.8639	31.24/0.8857	40.59/0.9747	32.75/0.8914	35.51/0.9315

Table 5.3: PSNR/SSIM results of our methods (SRS, DRS, DRM, and DRMF), RHF and NFOR.

	SRS	DRS	DRM	DRMF	RHF	NFOR
Dragonfog	40.15/0.9463	41.86/0.9636	43.07/0.9731	45.96/0.9884	43.41/0.9770	44.28/0.9869
Sanmiguel	32.96/0.8357	33.12/0.8401	33.24/0.8479	34.84/0.8864	34.15/0.8540	34.36/0.8755
Sibneik	39.82/0.9211	40.46/0.9376	41.07/0.9411	45.83/0.9815	41.23/0.9485	44.74/0.9816
Teapot	32.57/0.7051	33.09/0.7218	33.64/0.7455	35.43/0.8605	32.89/0.4304	34.24/0.8056

5.10. shows comparisons with the colour (RHF) and scene feature (NFOR) based approaches. We tested across four challenging noisy scene including Dragonfog, Sanmiguel (rendered by 32 spp) and Sibneik, Teapot (rendered by 16 spp). Our method (DRMF) shows better result compared with RHF and NFOR in all scenes. In the selected closeups, RHF tends to generate blurry artifices, and NFOR missed few local details although it provides better geometry and texture edge compared with RHF. Our DRMF outperform the previous methods in both preserving the high frequency details and smoothing the noisy artefacts. The error metrics (PSNR and SSIM) of the entire image to compare denoising results of our methods (SRS, DRS, DRM, and DRMF) with RHF and NFOR are in Table 5.3.

5.4.3 Efficiency comparison

To denoise an image of 1024*1024 pixels, our denoising network (DRMF) requires around 16 seconds to produce the final result. The running time is linearly related with the image resolution. The processing timing for RHF

is approximately 2 minutes, 20 seconds for LBF, 12 seconds for KPCN, and 5 minutes for NFOR. It is worth noting that the reference rendering of the test scenes requires hundreds of hours rendering time using 5k to 64K samples per pixel. However our denoising filter only needed significantly shorter time (16 second) to denoise images generated by highly limited samples (8 spp to 32 spp) and achieved almost similar visual quality compared with the reference rendering.

5.5 Summary and Discussion

In this chapter, we have presented a pilot study of using deep convolutional networks for denoising path tracing. Specifically, we introduced a multi-scale filter into convolution layer, which can better preserve short and long contextual information within the rendering. By adopting the residual learning strategy, we implemented a separated residual learning manner, and the result shows better denoising quality. To exploit and utilize the auxiliary scene features of the rendering, a virtual flash scene feature is generated for providing detailed information about the scene. Our results demonstrated that a CNN can represent the underlying, fundamental relationship between the noisy and reference rendering. Although it uses a relatively straightforward architecture, our solution is fast and it shows better or similar performance compared with other state-of-the-art denoising techniques for path tracing.

Based on the experiments, the proposed denoising network can generate promising results for various scene effects, but it is still hard to denoise some challenging scenes like the Villa scene. As discussed in Chapter 3, the light source in this scene is located outside and a few samples per pixel could not successfully compute the connection between the indoor surface point and the outside light source, therefore it fails to provide enough information for the denoising network. In Figure 5.11, under a fixed time budget (1.8 hours), the pixel recovery method in previous

chapter outperformed the proposed denoising network.

Figure 5.11: Under a fixed time budget, pixel recovery (in the middle) can generate better visual quality results than the proposed denoising network(on the right).

Looking ahead, one potential task is a complete exploration of the error metrics, which are quite different from each other. Also, combining the perceptual loss into the current loss function in our network, which may reveal better performance during the training. Additionally, different setting of the hyperparameters have been explored for an optimal performance of the network, but a complete investigation for the parameter selection can achieve better solutions. Additional data is always helpful for better network training but our procedure is limited by the huge computation time to render reference images (e.g., days to complete a single image with 64K spp). Recently proposed Noise2Noise learning [115] can relax this limitation by allowing network parameters to be trained using noisy image pairs instead of noise and clean data pairs. Finally, the proposed network only handles the single image denoising, but it would also be helpful for dealt with animation frames, especially in feature film production.

Chapter 6

Conclusions and Future Work

This thesis seeks to address the problem of the increasing time budget in path tracing and tackle this problem using reconstruction and denoising based approaches. We began by laying out the theoretical background of our research in Chapter 2 by first introducing the concept of path tracing. We also pointed out the limitation of path tracing, which is its slow convergence. Given this limitation, we then reviewed related work in reconstruction and denoising based methods.

Given the fact that path tracing usually takes a long time, especially with complex scene settings. Visualizing intermediate results of path tracing is challenging since the synthesized image with limited samples or improper sampling usually suffers from distracting noise. In Chapter 3, we presented a novel reconstruction method that efficiently recovers missing data in incompletely rendered images using matrix completion. Specifically, we present novel methods to construct low-rank matrices for incomplete images including missing pixel, missing sub-pixel, and multi-frame scenarios. A convolutional neural network provides fast pre-completion for initializing missing values, and subsequent weighted nuclear norm minimization with a parameter adjustment strategy efficiently recovers missing values even in high-frequency details. The result shows better visual quality than recent methods including compressed sensing based

reconstruction. Our method shows promising recovery results even in challenging scenes including high-frequency details.

The second work of this thesis is built upon Chapter 3. In Chapter 4, we extend the previous method by applying a block Toeplitz structure while forming an enhanced low-rank matrix of pixel recovery, and tensor structure for multi-frame restoration. In this manner, the reconstruction time can be further reduced for both tasks. Besides that, by exploiting temporal coherence of multi-frame with a tensor structure, better recovery quality can be achieved compared to our previous approach.

For the last work of the thesis, we looked into denoising based technique for path tracing. In Chapter 5, we introduce a convolutional neural network based filter for denoising rendered images. The deep neural network has demonstrated success in filtering noise of path tracing rendering under a low sample budget, and the recovery speed is 30 times faster than the proposed reconstruction method in Chapter 4. Unlike a plain CNN that applies fixed kernel size in each layer, we propose a multi-scale residual network with various auxiliary scene features to leverage a new efficient denoising filter for path tracing. Our experimental results show better or similar performance in terms of denoising quality and, speed compare to the state-of-the-art path tracing denoiser.

In the context of efficient solutions for previsualization of path tracing, this thesis covers both reconstruction and denoising based approaches. The methods we presented in this thesis made several novel contributions to intermediate rendering visualization or production-ready application. We believe this research enhanced our understanding of two post-processing paths for efficient path tracing and will give insights into future rendering research. Contributions of this thesis are provided as follows, along with limitations and the potential research areas that this work enables.

6.1 Summary of Contributions

A number of specific research objectives were stated in the introduction. Here we restate these objectives and relate them to the achievements in the thesis:

Develop Reconstruction based Method for Previsualization

The first contribution of the thesis is a new efficient and accurate reconstruction based method to recover the missing samples of path tracing. Chapter 3 formalizes the recovery of missing data in incompletely rendered images as a matrix completion problem. Specifically, we adopt the low-rank matrix completion scheme for previsualization of path tracing. Although matrix completion and separation have been applied to many lights rendering [22] and inpainting [4] recently, they have not yet been applied to recover sub-pixel samples of rendering or pixel samples in multiple frames. To form a low-rank matrix, we show how to formulate pixel, sub-pixel, and multi-frame sample problems as matrix completion. Our approach contributes to reducing the spatial and temporal noise in reconstructed renderings. Additionally, we have developed novel nuclear norm minimization methods using a multi-weighted approach with a progressive adjustment strategy. Our method has been evaluated against state-of-the-art methods and shows better results in terms of visual quality including CS based reconstruction. In particular, our method outperforms with limited samples.

Our second contribution is based on Chapter 3, we present novel methods in Chapter 4 to construct enhanced low-rank matrices and low-rank tensor for dealing with missing pixel and multi-frame scenarios respectively. Specifically, we adopted the latest enhanced matrix completion in pixel recovery and tensor completion for multi-frame case. Then we apply weighted nuclear norm minimization with a parameter adjustment strategy, which can more efficiently recover missing values even in high-

frequency details.

Develop Denoising based Method for Previsualization

Another contribution of this thesis is the novel denoising based filter technique in Chapter 5. Motivated by the recent deep learning-based works, we propose to apply a multi-scale residual learning based denoising network. Unlike the previous architectures, which apply a single scale filter in the convolutional layer or perform multi-scale concept through down/up scale the feature maps, we use different filters during the convolution step to preserve both long and short contextual information. Additionally, we propose a separated residual learning strategy to predict the noise and generate a virtual flash image as one of the input scene feature buffers. Our method shows on par or better denoising quality compare to the state-of-the-art methods.

6.2 Limitations

Although our proposed works in this thesis show promising results compared with other state-of-the-art techniques, our methods do have some limitations.

Limitations of Our Reconstruction based Methods

Since the performance of matrix completion and tensor completion relies on proper parameters, such as the size and number of low-rank matrices, the 3-D dimensions of a tensor structure and related hyper-parameters of the algorithm. Over several experiments, we determined appropriate parameters for different applications, but careful analysis of scene information may provide better optimal choices. However, it is challenging to do this because, in general, detailed scene information cannot be known be-

fore a significant number of samples have been taken. Our reconstruction based method is challenged when attempting to recover values under very low sampling rates ($< 10\%$). Previous recovery methods, such as the CS based approaches, suffer from similar issues. Our method shows better and more reliable results than CS based approaches at very low sampling rates.

On the other hand, the current implementation of the sub-pixel recovery is not efficient compared with previous filtering methods in terms of the total processing time. The reconstruction of unknown ray samples mainly rely on uniform sampling pattern, which is not the optimal choice to deal with various scenes.

Limitations of Our Denoising based Method

Our neural network based denoising filter can generate noise-free results efficiently and requires few parameters to manually set, which can be easily integrated into production application. Our learning based approach also have several inherent limitations. Firstly, our denoised result may lack the scene details which are not captured in the input scene features, and it will be missing in our training set. Also, due to these details didn't appear in the training dataset, the network cannot approximate them with mere use of the color information. But the problem can be relieved through adding similar examples to the training set. Similarly, examples with various distributed effects of the test set should be included during training, otherwise the network will fail to remove the corresponding noise.

6.3 Future Work

The promising results from this thesis suggest that further development is a worthy cause. These methods are not production ready due to the scenes used in all experiments are not from the visual effects industry, and

should naturally undergo further research, development, and testing with real production data set. The most interesting avenue for future work in reconstruction based method would be to explore GPU based implementation for better efficiency, since our reconstruction based methods are fully independent across patches or frames, and therefore each recovery step could be computed in parallel on the GPU. Also, integrating neural network techniques in Nvidia's latest ray racing based hardware (e.g., RTX 2080 Ti) for previzualization of path tracing is worth exploring in future.

Finally, to further make improvement on the performance of our denoising network, several network design choices could be investigated. The first straightforward direction is to explore different network structure and concepts such as autoencoders, generative adversarial networks and U-net, which have demonstrated great success for natural image denoising. All these novel network architectures can potentially improve the performance. Secondly, integrating the perceptual loss into the standard error metrics, and more potential loss functions can be considered. Moreover, it would be helpful in production to extend the proposed denoising network in animation frames noise removal, since the results from Chapter 5 only target for single image denoising.

Appendix A

Pre-completion with TV and CNN

A visual quality comparison of pre-completion in Section 3.2.1 between TV and our CNN for seven different scenes, all the inputs are only 10% pixels of the corresponding scene.

Bamboo Scene

Reference

Input 10% pixels

Pre-completion by TV
(psnr=23.01) Pre-completion by CNN
(psnr=26.03)

Bedroom Scene

Reference

Input 10% pixels

Pre-completion by TV
(psnr=23.91) Pre-completion by CNN
(psnr=31.82)

Bottles Scene

Reference

Input 10% pixels

Pre-completion by TV
(psnr=19.73) Pre-completion by CNN
(psnr=30.17)

Conference Scene

Referenece

Input 10% pixels

Pre-completion by TV
(psnr=25.54) Pre-completion by CNN
(psnr=33.67)

Killines Scene

Reference

Input 10% pixels

Pre-completion by TV
(psnr=23.34) Pre-completion by CNN
(psnr=33.95)

Sanmiguel Scene

Reference

Input 10% pixels

Pre-completion by TV
(psnr=15.78) Pre-completion by CNN
(psnr=25.65)

Toasters Scene

Reference

Input 10% pixels

Pre-completion by TV
(psnr=30.59) Pre-completion by CNN
(psnr=39.74)

Appendix B

PAWNNM for Inpainting

A visual quality comparison of NLI, GSR, and the proposed PAWNNM in Chapter 3. Five standard test images were selected and compared the recovery results with 20% and 30% sampling.

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