**Investigating a real-options approach to modelling public policy
investments, and implications for public administration**

BY

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Abstract

This thesis investigates the application of real-options modelling to policy investment coordination problems across government departments, with the aim of informing high-level institutional and system design choices in public administration. This analysis is motivated by recent and ongoing reforms to the legislative framework underpinning New Zealand's public service which, unlike the last major public administration reforms of the 1980s, are not expressly driven by or obviously supported by economic theory. We find that a simple real-options framework allows for a compelling illustration of the coordination problems that the current reforms are in part aiming to solve, and supports analysis of the types of mechanism that might be employed to address them. Such mechanisms include the promotion of structural integration, the broadening or alignment of departmental policy objectives, a relaxation of public finance restrictions, and joint decision-making over policy investments.

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Part I

Introduction and Policy Context

1 Introduction

New Zealand’s system of public administration is currently undergoing the most significant set of reforms it has experienced since the 1980s. While reviews are underway in a number of areas, the most significant set of changes to date are those recently progressed through the Public Service Legislation Bill.¹ This bill was passed in July 2020 and replaced the State Sector Act 1988 with a new Public Service Act, with key features including a clearer high-level framework for the operation of the public service as a whole and legislative models for facilitating cross-agency work. The primary stated objective of this bill was to achieve a public service that has the incentives and tools to operate more effectively as a unified institution, rather than individual departments operating as autonomous entities with distinct and separate goals.

The changes made through the bill respond to generally held perceptions of the shortcomings of the existing model of public administration, as reflected in a variety of reviews and reports released over the past three decades and in the public administration literature. Prior to the current reforms, New Zealand’s system of public administration had been underpinned by a legislative framework that had been in place since the 1980s. Reforms at that time drew heavily on economic theory and management theory, and resulted in a series of key pieces of legislation that fundamentally altered the way public institutions operate and the nature of the relationships that they have with elected officials.

For the core public service in particular – those departments which are part of the Crown and have a close relationship with Government ministers – assessments of the impact of the 1980s reforms has been mixed. Commentaries on the reforms have generally concluded that the changes made individual agencies more responsive to the Government of the day, and more efficient in their delivery of specific activities that they are directed to undertake. However, a common criticism of the existing model has been that it is not effective at incentivising or

¹Reforms have been discussed in relation to other key pieces of legislation such as the Public Finance Act 1989 and the Crown Entities Act 2004, but neither have progressed beyond the early policy development phase.

facilitating effective cross-agency coordination, which is considered necessary for achieving more complex policy objectives that cross portfolio boundaries.

The importance of aligning incentives across agencies to achieve more effective coordination of activities and policies is also reflected in public administration literature, and other parallels are present between this literature and the current reforms. However, unlike the reforms of the 1980s, the current reform proposals are not explicitly informed by economic concepts or theories. While the same basic theories that underpinned the 1980s reforms — including principal-agent and public choice theories — may not be as readily applied to evaluate the current changes, it is possible that other areas of economic theory may be relevant.

This thesis explores the application of real-options modelling to the coordination of policy investment decisions across public agencies. Real-options approaches originated in finance and are still primarily used to price options or to evaluate other financial investment decisions. However, there has also been a recent trend in research extending real-options analysis from financial decisions to micro-economic problems more broadly. We first show that coordination problems between public agencies can be effectively modelled in a real-options setting, and subsequently extend the model to analyse different approaches to incentivising better coordination.

We expect that there are a range of possible audiences for this research, including policymakers and academics in fields relating to both public administration and economics. With this in mind, this thesis is divided into three parts. The remainder of Part I sets out the public policy context with a focus on the New Zealand perspective, and provides an overview of the relevant academic literature across both public administration and finance disciplines. This Part also provides a high-level intuitive description of the modelling and results contained in later parts of the thesis, for the benefit of policymakers and academics not familiar with real-options models.

Part II of the thesis sets out a basic model, which shows that a real-options approach can be used to illustrate the types of inefficiencies that may arise when incentive structures for public institutions are too narrowly focused on achieving outcomes within a specific portfolio. Finally, Part III of the thesis considers the addition of a range of different inducements to the model, showing how the basic framework can be used to evaluate different mechanisms which might be employed by public administration practitioners in order to promote better coordination between departments.

2 Policy context and literature

2.1 Public administration in New Zealand

In the late 1980s and early 1990s, New Zealand overhauled its system of public administration, a process which involved significant reforms to the legislation governing the State sector. This included the passage of the State Sector Act 1988, the Public Finance Act 1989 and the State Owned Enterprises Act 1986.² These changes in New Zealand were part of a broader shift in approaches to public administration globally known as New Public Management — though New Zealand is often characterised as being the purest example of the new model due to the extensiveness and internal coherence of the reforms undertaken (Lodge and Gill, 2011), a fact which has resulted in the New Zealand case drawing a relatively high level of interest in public administration literature. The moniker ‘New Public Management’ was intended to distinguish the new model from the previous paradigm, now known as ‘Traditional Public Administration,’ which was heavily influenced by Weberian theories of bureaucracy (see Waters (2015)) and focused on the efficient delivery of public services through politically-neutral bureaucratic hierarchies.

New Public Management reforms in New Zealand arose in response to poor economic circumstances and falling standards of living, and a growing bureaucracy which was perceived as unresponsive to political direction (Matheson, 1997). The design of these reforms drew in large part on economic theory; this is evident in the writings of Treasury officials of the time (see, for example, Scott and Gorringer (1989)) which reference literature on principal-agent problems, public choice theory, and New Institutional Economics (with a particular focus on the work of Williamson (1985) on institutional form and contracts). In addition to economic theory, the reforms drew on management literature which emphasised increased autonomy and accountability (Schick, 1996) and methods of performance improvement that had seen success in the private sector (Osborne and Gaebler, 1992). The reforms adapted these ideas for the State sector, and adjusted them based on experiences of officials working in the New Zealand context (Scott,

²These three pieces of legislation, along with the Crown Entities Act 2004, have provided the overarching framework for State sector organisation in New Zealand over the last three decades by, respectively: providing for the operation and governance of functions within the Crown (i.e. through core public service departments and their ministers); providing for the appropriation and management of public finances; and, providing for the governance of commercial functions through arms-length entities with majority Crown ownership. The addition of the Crown Entities Act 2004 provided a framework for management of non-commercial functions through arms-length bodies, which had previously relied only on separate pieces of legislation to specify the framework for operation and governance of individual entities.

1997).

The reforms as implemented have been characterised as comprising four overarching shifts: a decentralisation of the core public service, with increased autonomy for chief executives over the management of departments; a shift towards managerialism, emphasising general management skills over specialist knowledge; an embedding of contractualism, with the establishment of quasi-contractual arrangements between agencies and ministers and an increased tendency to contract out the delivery of public services; and an increase in agencification, where functions were divided into a greater number of more focused agencies including many operating at arms-length from the Crown (Boston et al., 1996). These changes resulted in a system comprised of a greater number of agencies, each with a strong accountability to a minister for the performance of specific functions and the achievement of specific objectives in a narrowly-defined area.³

It is generally accepted that the New Public Management reforms achieved the purpose for which they were designed, and increased the responsiveness and efficiency of individual government agencies in New Zealand (Scott, 2001). At the same time, these reforms have been subject to a number of criticisms; for example that their emphasis on rational self-interest has crowded out intrinsic motivation of public servants (Bouckaert et al., 2016) or that their emphasis on accountability for short-term results has reduced focus on maintaining and safeguarding institutions (DeLeon and Denhardt, 2000). The most well-trodden and enduring critique of the New Public Management paradigm in New Zealand, however, has been that the strength of individual agency accountabilities incentivises a focus on individual agency policy priorities over the collective good, and has consequently suppressed coordination across agencies and collaboration on issues that do not fall within the remit of any single agency (Gregory et al., 2006).

This particular critique — that the State sector in New Zealand emphasises pursuit of individual agency priorities at the expense of a coordinated effort to improve outcomes overall — has been reiterated in a number of internal reviews of the New Zealand public administration

³We find it necessary here to clarify our use of the various terms referring to public organisations. The term ‘department’ has a specific meaning in New Zealand law, and indicates an entity that is part of the Crown and is responsible directly to a minister for the performance of its functions. The term ‘agency’ is a general term which refers to public organisations more broadly, including both departments and other types of entity (e.g. those that operate at arms-length from ministers). Our use of each of these terms is therefore context-dependent. The term ‘Ministry’ is not defined in current New Zealand law, but is often used in the titles of departments. Consequently, we avoid using this term unless it appears in the name of a specific department to which we refer.

system. Each of these identified a need to improve cross-agency coordination and consequently the performance of the system of government agencies as a whole, and recommended various initiatives aimed at achieving this objective. Early examples include the ‘Review of the Centre’ (State Services Commission, 2001), which recommended a strengthened mandate for central agencies⁴ and a shift in the focus of accountability documents towards the achievement of outcomes, and ‘Mosaics’ which considered the barriers to coordination at a regional level (Ministry of Social Development, 2003). More recently the ‘Better Public Services’ review (State Services Commission, 2011) built on the progress made through these earlier reviews, recommending further measures to strengthen collective leadership, trialling more flexible approaches to the governance and funding of cross-agency work, and enhancing the focus on outcomes by emphasising the achievement of a small number of important and well-defined government priority ‘results’ (Scott and Boyd, 2016).

While not all recommendations were implemented in each case, this series of reviews resulted in incremental changes to the existing public administration system, aimed at mitigating the shortcomings of New Public Management while retaining its benefits. Some of these changes were legislative (in the form of amendments to the foundational legislation of the 1980s), but the majority relied on Government policy or convention up until the passage of the Public Service Act 2020. This new Act replaced the State Sector Act 1988 as the legislative foundation for the operation of the core public service (comprised of the departments that act as administrative units of the Crown). A key objective of this new Act is to improve the operation of the public service as a unified whole. It aims to achieve this by shifting emphasis from individual agencies to the collective — thereby adjusting the balance of incentives away from the ‘self-interested’ behaviour promoted under New Public Management — and providing a number of mechanisms which better support coordinated deployment of resources where required (State Services Commission, 2019).⁵

It remains to be seen whether the current reforms will in practice be effective in further mitigating the coordination challenges attributed to New Public Management, and whether the Government will continue to push for reform in other areas of the public administration system

⁴Those agencies focused on the performance of government as a whole, including the Treasury, the Department of the Prime Minister and Cabinet, and the Public Service Commission (formerly State Services Commission).

⁵These mechanisms include more flexible deployment of the workforce between departments, and formal models for joint policy advice and expenditure decisions by groups of departments.

— such as the public finance system, or New Zealand’s approach to the governance of functions at arms-length from the Crown (as set out in the Crown Entities Act and State Owned Enterprises Act). The system of administration established through the legislative reforms of the 1980s has been described by some commentators as being underpinned by a clear and internally coherent theoretical framework (Lodge and Gill, 2011); and it is more generally accepted that the reforms implemented in New Zealand were the most holistic among New Public Management reforms internationally. In contrast, the changes made through the new Public Service Act continue the incremental approach to reform that New Zealand has taken over the intervening three decades by building on, rather than replacing, the foundations laid by those earlier reforms. While this pragmatic and measured adjustment is arguably a much more sensible approach to administrative reform — at least in the sense of mitigating the impacts of any unintended consequences, such as those we have experienced as a result of the fragmentation that accompanied the 1980s reforms — it does mean that the current set of reforms does not have as coherent a theoretical foundation as its predecessors.⁶ Also as a result, where New Zealand’s approach to the New Public Management reforms was underpinned by various economic theories (as discussed above) there is a relative lack of micro-economic theory informing the current changes. In particular the challenge of facilitating more effective coordination between departments — which is both one of the core aims of the reforms and, on the face of it, the most amenable to economic analysis — has not been explored through an economic lens. It is the issue of policy investment coordination between agencies, and how this can be understood and analysed through a micro-economic approach, that is the focus of this thesis.

2.2 Real-options modelling

There are a number of possible ways in which the modelling of policy investment coordination problems could be approached. We choose to employ a real-options approach, which is a category of modelling focused primarily on decisions relating to the timing of investments. We consider

⁶A variety of different theories of a post-New Public Management paradigm have been offered, some of which have elements that align with the direction of the current New Zealand reforms. For example, ‘Digital Era Governance’ (Dunleavy et al., 2006) which emphasises cross-agency service integration, and ‘New Public Service’ (perhaps the theory to which the current reforms are most closely aligned) which emphasises collective effort in pursuit of the public interest (Denhardt and Denhardt, 2000). However, the current reforms were not explicitly designed to embody these theories.

that using investment timing as a choice variable — over, for example, the scale of an investment — provides a realistic approximation of resource allocation processes in the public sector, which generally involve allocating funding or other resources (such as staff effort) from a limited budget to specific initiatives or policies with relatively inflexible costs. Where an initiative is identified as an immediate priority (for example, where there is immediate community need) it will be funded, with further consideration of remaining initiatives delayed to subsequent budget rounds. This prioritisation of investments over time aligns intuitively with the fundamental characteristics of a real-options approach.

Real-options modelling applies techniques traditionally used for valuing derivative financial instruments to decision-making in a wider range of different contexts. The fundamental components of a real-options model are uncertainty (usually over the path of an exogenous state variable), flexibility (usually as to whether to make an investment of some kind, and the timing of that investment) and irreversibility (of the investment once it has been made) (Dixit and Pindyck, 1994). Provided that these three assumptions can be made, techniques employed in real-options analysis can be applied to the analysis of a wide range of timing decision problems.⁷ Dixit and Pindyck (1994) provides an excellent summary of the theoretical foundations of real-options analysis, as well as an overview of early literature. Guthrie (2009) provides a more recent overview of the foundations of the approach, and describes how it can be applied to support practical decision-making.

Building on the mathematical foundations employed in option pricing models, such as that developed by Black and Scholes (1973), early real-options literature investigated the application of similar modelling techniques in a corporate finance setting. More recently, however, the approach has been extended to a wider range of economic applications. Of particular interest to us, a substantial literature exists on the combination of real-options and game theoretical approaches to analyse strategic interactions in a dynamic setting. There are two specific subsets of this literature that the model presented in this thesis draws on and contributes to.

The first is the literature that uses a real-options framework to study the interactions between two or more competing firms (i.e. the study of investment timing in a duopolistic or

⁷We explicitly consider these assumptions when setting up our model in Part II of the thesis, and argue that they can be reasonably made in a public policy setting.

oligopolistic setting). The earliest example of such a model was developed by Smets (1991), with a simplified version presented in Dixit and Pindyck (1994). This provided the basic framework for subsequent models, with two identical firms determining when to enter a market where a first-mover advantage is available. Another important work is Pawlina and Kort (2006), which analyses the effect of asymmetric investment costs. The basic model presented in Part II of this thesis has some important elements in common with these two models. However, in modelling policy coordination problems we assume that the departments' investments are complementary (i.e. each party's investment generates greater benefit when the other party has also invested) which results in different equilibrium behavior and welfare analysis from the traditional competitive setting.⁸ A number of other works exist with less direct relevance to our approach, with a good summary of the literature prior to 2011 provided by Chevalier-Roignant et al. (2011). Extensions to models of market entry (which are most comparable to our approach) include consideration of an increase in the number of firms (Bouis et al., 2009), the ability for firms to vary the scale of their investments (Huisman and Kort, 2015), and the effect of information asymmetries (Lambrecht and Perraudin, 2003). Wider literature considers a range of other types of timing decisions in a duopolistic or oligopolistic market, including exit from a declining market (Sparla, 2004), technology investments in an active market (Huisman et al., 1999), and the building of additional capacity both in an active duopoly (Boyer et al., 2012) and as a mode of entry deterrence by an incumbent monopolist (Huberts et al., 2019; Guthrie, 2020).

The second area of real-options literature to which this thesis contributes concerns the analysis of joint decisions in a real-options setting. These models investigate the behaviour of two or more parties who may generate a shared surplus through some joint timing decision. Such an approach is of interest to us when we investigate possible inducements to improve investment coordination in Part III of the thesis, one of which involves giving agencies joint control of investment decisions. The earliest example of a model involving jointly-held real-options is that presented by Lambrecht (2004), which considers a merger between two firms. Other relevant literature considers joint decision-making over the timing of a cooperative investment (such as

⁸Note that 'complementary' investments are discussed by Mason and Weeds (2010) but in a relatively trivial way: where two identical firms which would otherwise compete in a standard (competitive) duopolistic entry game may access an alternative payoff function if investing simultaneously (which may be assumed to be preferable to payoffs in the pre-emption equilibrium that otherwise exists in such models). It is also assumed in that model that respective benefit flows following simultaneous investment are lower than those experienced by a monopolist, which does not align with our meaning of complementary.

a joint venture) between firms with different characteristics (Cvitanic et al., 2011) and the timing of investment in a supply chain by a supplier and a retailer (Chen, 2012; Zormpas, 2020). Banerjee et al. (2014) provides a generalised model of joint exercise of a real option, where terms for division of the surplus are determined using a bargaining solution of the kind developed by Nash (1950) and one party is delegated the exercise decision (which may be taken before or after bargaining occurs). This two-stage approach — where the investment timing decision is separated from other negotiated terms, such as the division of profits following a merger or contributions to a joint investment — is common throughout the literature discussed above. Like Banerjee et al. (2014) we use a Nash bargaining approach to analyse joint investment decisions, but instead allow agencies to simultaneously bargain over both the allocation of investment costs and the timing of the investment, which in our opinion better avoids the potential for dynamic inconsistency.

A final area of the real-options literature worth mentioning is that which analyses the exercise of options in the context of agency problems. Examples include Grenadier and Wang (2005), Gryglewicz and Hartman-Glaser (2014), Kruse and Strack (2015) and Grenadier et al. (2016). In this thesis we focus specifically on strategic interactions between government agencies. To keep our analysis tractable and to ensure that we are able to clearly identify and address the coordination problems we are interested in, we do not also simultaneously model interactions between agencies and their ministers. However, in reality these interactions are a crucial part of the context in which agencies operate, and furthermore addressing agency problems in the relationship between ministers and government agencies was a core focus of New Zealand’s previous public administration reforms in the 1980s. Consequently, investigating how the results of this thesis are affected by the extension of our model to include minister-agency relationships could be an interesting opportunity for further research.⁹ The examples of literature on real-options in the presence of agency problems listed above provide a template for how such an extension could be approached.

⁹Possible extensions to the model are discussed further in the concluding remarks at the end of thesis.

3 Discussion of modelling and findings for policymakers

This section summarises the model and results presented in this thesis for those less familiar with real-options analysis, or indeed those less comfortable with theoretical micro-economic approaches to modelling more broadly. We intentionally keep this discussion as non-technical and intuitive as possible. A more concise summary of the approach taken and results obtained can be found in our concluding remarks at the end of the thesis.

Part II of this thesis, which follows immediately from this section, presents the basic model which we use to illustrate the coordination problems between departments that we are interested in understanding and addressing. This model involves two departments which are each considering investment in a policy investment of some description. To keep application of the results as broad as possible, we do not specify the exact nature of the policy investment — this could be a decision to deliver a new service to the community, an infrastructure investment, or any other type of expenditure commonly made in the public sector. Each department’s policy investment has a (different) fixed cost that is incurred at the time of investment¹⁰ and a stream of benefits that are generated from the time of investment onward. The size of the benefit generated by each investment at a given point in time is determined by a number of factors. First, the benefits from investment depend on a random variable (referred to as the ‘state variable’) that is outside the control of the departments and which changes unpredictably over time — this could be interpreted, for example, as some key determinant of the demand for or effectiveness of the new service that the department wishes to invest in (in an illustrative scenario offered later we use the example of the number of children in poverty, which affects demand for teacher aide and nutrition programmes offered by the Ministries of Education and Health respectively). Second, the benefits generated by each department’s investment depend on whether or not the other department has also invested in its policy programme. Because we are specifically interested in the coordination of policy investments to achieve outcomes across departmental boundaries, we assume that at a given time each programme generates a greater benefit if both investments have been made.¹¹

¹⁰Though this could also be interpreted as a constant stream of fixed operating costs incurred from the time of investment onward — an interpretation that is consistent with the baseline budgeting approach to public finance employed in New Zealand.

¹¹That is, the investments are complementary. As noted earlier, the assumption of complementary investments represents a key distinction between our model and other two-player investment timing games in the literature.

As is the case in all real-options analysis, the key decision for the departments (often referred to as the ‘choice variable’) is the timing of their respective investments. Because the departments only care about time to the extent that it determines the value of the state variable (which in turn determines the level of benefits generated by their investments), the investment timing decision of each department is represented by a threshold value of this state variable — that is the value of the state variable which, once reached or exceeded, will trigger immediate investment by the department. The departments choose the value of the investment threshold which maximises the expected present value of their investment, alternatively described as the value of the option to invest. Because the benefits generated by a given department’s investment depend on whether the other department has also invested, each department has a different threshold depending on whether it is the investment leader or the investment follower. Once we have established the objective functions of the departments (i.e. the expression for the option value which they aim to maximise) we determine how departments will behave in equilibrium – that is, the set of outcomes that may occur when the departments seek to maximise the values of their respective investments in a strategic setting. It turns out that there are two possible equilibrium outcomes, depending on the specific values of the benefits and costs of the departments’ respective investments. If the departments’ investments have relatively similar benefit-cost ratios then in equilibrium they will invest simultaneously. In this case the department with the higher benefit-cost ratio would be willing to engage in simultaneous investment slightly sooner, but must wait for the point at which the department with the lower benefit-cost ratio is also willing to invest simultaneously. If the benefit-cost ratios of the departments’ investments differ significantly the departments will instead invest sequentially; the department with the higher benefit-cost ratio will invest first, and the department with the lower benefit-cost ratio will follow at a later time.

Once we have described the equilibrium behaviour of the departments, we assess this behaviour from the perspective of a social planner — that is, a hypothetical individual who is interested in selecting investment timings that maximise the total expected value of both investments (often described as social welfare) rather than the value of one investment or the other individually. In comparing the socially optimal investment timings that would be chosen by a planner to those selected in equilibrium, we find that at least one department will invest

inefficiently late (that is, late from the perspective of the social planner). Intuitively, this occurs because the department with the lower benefit-cost ratio (the ‘follower’ department) does not take into account the additional social benefits that its investment will create indirectly by increasing the benefits generated by the other (‘leader’) department’s investment. This distortion takes different forms depending on the cost and benefit parameters of the model. Where departments invest sequentially in equilibrium, in some cases the efficient outcome would involve sequential investment with the follower investing earlier, and in others would involve both departments investing earlier and simultaneously. Where departments invest simultaneously in equilibrium, the efficient outcome also involves simultaneous investment but at a lower threshold. We consider that this comparison of the equilibrium and socially optimal investment timing decisions provides a compelling illustration of the coordination problems between agencies that have been identified in the public administration literature and reviews of the New Zealand public administration system. Because the departments in our model are concerned only with the benefits and costs associated with their own investments, and not the public benefits that may be generated more broadly, they undervalue complementary policy investments (i.e. those which generate positive externalities when active alongside other policy programmes of other departments) and consequently invest later than desirable from the perspective of the general public interest.

Part III of the thesis shows how the simple model presented in Part II can be extended to analyse different types of mechanism that might be employed by Governments or public administration practitioners to address the investment coordination issues identified. We first note that structural integration of departments where appropriate (or, the reversal of agencification resulting from the public administration reforms of the 1980s) may address coordination problems by creating a single department with identical incentives to the social planner; however, we stress that this will not be a practical response in every situation. We subsequently consider three extensions to the model that illustrate different possible (non-structural) mechanisms that could be employed to mitigate the problem identified. The first type of mechanism that we consider involves the adjustment of the departments’ objective functions to (either fully or partially) take into account the effect of their decision on the benefits generated by the investment of the other department. In practice such mechanisms might involve driving culture change or adjusting the

(arguably excessive) accountability arrangements to which the coordination problems between agencies are often attributed.¹² We model such a mechanism by allowing a planner to select the degree to which the follower department internalises the positive externality of their investment (i.e. its effect on the benefits generated by the leader’s investment). In the absence of any costs or frictions, a planner will prefer to implement a mechanism of this nature such that the incentives on the departments result in selection of the socially optimal investment timing derived in Part II. However, we show that if implementing the mechanism results in a ‘distraction cost’ — where increasing a department’s focus on the wider impacts of its decisions reduces its focus on cost-minimisation and consequently raises the cost of investment — the planner will choose to only partially implement the mechanism. Intuitively, the planner must trade off the positive effect of bringing the investment timing decision closer to the socially optimal level against the additional distortion caused by the distraction from cost-minimisation activities.

The second variation we make to the basic model considers a relaxation of public finance settings such that departments may decide to contribute to the costs of each other’s investments.¹³ In equilibrium, we find that the leader department will contribute to the cost of the follower’s investment, though this contribution is not large enough to implement the socially optimal investment timing derived in Part II. We subsequently investigate how the underlying parameters of the model (i.e. the benefits and costs of the two departments’ investments, and the characteristics of the state variable) affect the gap between the outcome achieved via this transfer mechanism and the socially optimal outcome (indicative of the mechanism’s effectiveness). Using numerical analysis we find that the transfer mechanism is more effective (i.e. achieves an outcome closer to the social optimum) when the state variable is relatively stable over time, or when the benefit-cost ratio and/or scale of the leader’s investment is significantly greater than that of the follower’s investment. The third and final variation to the model that we consider involves allowing departments to jointly determine investment timing. In practice, this would involve weakening the strong individual decision-rights and accountabilities that are the status quo in New Zealand’s public administration system, and facilitating more models

¹²The new Public Service Act 2020 attempts to achieve shifts of this nature by setting out a common framework for the operation of the public service (including a shared purpose, principles and values) and by introducing collective leadership arrangements to promote shared accountability for the work of the public service as a whole.

¹³We note that some additional flexibility was introduced through reforms to the Public Finance Act in 2013, but that financial accountabilities and non-legislative rules and guidelines continue to dis-incentivise investment across portfolio boundaries.

of joint decision-making over policy investments.¹⁴ We model joint decision-making using the Nash bargaining approach that is commonly employed in economic models involving negotiation to reach a joint decision. We allow the departments to bargain over both the timing of their (simultaneous) investment, and how the costs of investment will be allocated (again modelled through a transfer between departments).¹⁵ We find that imposing joint decision-making on the departments implements the socially optimal investment timing derived in Part II. Intuitively, the departments choose an investment timing to maximise the total expected value generated by their investments, and determine a transfer through the bargaining process that appropriately compensates the follower department (the department which must be incentivised to invest earlier) given the respective bargaining powers of the departments. This is consistent with results commonly obtained from such bargaining models, where parties agree on terms to maximise the total ‘surplus’ from bargaining and use direct transfer mechanisms (if available) to facilitate the allocation of this surplus between them.

¹⁴For example, the Public Service Act 2020 provides for more formal models of joint control over and accountability for resources than have previously existed, in the form of ‘interdepartmental executive boards’ and ‘interdepartmental ventures.’ However, these models have yet to be used extensively in practice.

¹⁵The ability to make such a transfer is necessary to facilitate an improved outcome through bargaining, as without access to a transfer mechanism the follower cannot be incentivised to agree on an adjusted investment timing.

Part II

Basic Model

4 Model set-up

4.1 General set-up and notation

We consider the following scenario. Two departments (denoted 1 and 2) are each considering investment in separate policy initiatives. Department n chooses the timing of its investment, and incurs a one-off investment cost I_n at the time investment occurs.¹⁶ Both departments take as given an exogenous state variable X , which evolves according to geometric Brownian motion (GBM) with drift μ and volatility σ (the mean annual percentage change in X and relative standard deviation of the change, respectively):

$$dX_t = \mu X_t + \sigma X_t d\xi_t \quad (1)$$

If at time t only one department has invested, its investment generates a continuous flow of social benefit $B_n^s X_t$, where B_n^s is a constant (here ‘ s ’ denotes solitary investment). If, however, at time t both departments have invested, their respective investments generate social benefit flows of $B_n^c X_t$, where B_n^c is a constant (with ‘ c ’ denoting collaborative investment) and $B_n^c > B_n^s$ for $n = 1, 2$ — that is, the policy programmes of the two departments are complementary and generate greater social benefit when both departments have invested.¹⁷ In the base model we assume that each department cares only about the social benefit generated by its own investment; that is, department 1 weighs the investment cost I_1 against the anticipated social benefit

¹⁶It is common in economic theories of bureaucracy to assume a budget constraint for public agencies (see, for example, Chapter 16 of Mueller (2003)). In our model I_n could be interpreted either as an investment outlay (i.e. assuming no budget constraint) or as a shadow price which also incorporates the opportunity cost of investing in the next-best alternative project, provided that the value of this other opportunity is constant over time. Alternative investment options with time-dependent values would need to be explicitly introduced to the model, for example through an additional state variable. Such extensions are not addressed in this thesis but could be investigated through future research.

¹⁷We focus on complementary investments due to our specific policy interest in such cases. We recognise, however, that in reality there are also cases where public sector investments are substitutes. While the inducements modelled in later sections of this thesis are designed to address a specific distortion in the context of two investments that are complements, it is worth noting that any generic (i.e. non-targeted) inducement would also need to take into account the existence of investments that are substitutes.

flows $B_1^s X_t$ and $B_1^c X_t$ and does not take into account the investment cost or benefit flows associated with department 2's investment (and vice versa). This is reflective of the strong focus of individual agency priorities that has been characteristic of New Zealand public administration system since the 1980s, and to which policy coordination problems have been attributed to in practice.

We assume that the two departments share a subjective discount rate ρ which reflects the government's time preferences. This could be interpreted, for example, as the Treasury's social discount rate, which is published and used in the valuation of government policy projects.¹⁸ As is standard, we assume that $\rho - \mu > 0$ to ensure that the solution to each department's investment problem is finite. Note that it is common in real-options modelling to use the notation $\delta = \rho - \mu$, but we avoid this here due to challenges in interpreting δ outside a financial market setting.

4.2 Illustrative example

Before we continue, we offer a simple illustrative example to help the reader understand the intuition behind this set-up. Suppose that the Ministries of Health and Education are both considering investment in policy programmes to help impoverished children in a particular region. The number of children in poverty at time t is given by X_t (i.e. the state variable as described above, which cannot be influenced by either department). The Ministry of Health can invest in a programme providing food to children in poverty, with investment cost I_H and benefits of $B_H^s X_t$ at each time t that the Ministry of Health is the only department to have invested (intuitively, the programme generates a benefit B_H^s for each hungry child, and therefore generates greater benefit if more children are in poverty and are helped by the programme). Similarly, the Ministry of Education can invest in a programme providing additional teacher aide resources to help impoverished children in the classroom, with investment cost I_E and benefits of $B_E^s X_t$ at each time t that the Ministry of Education is the only department to have invested.

At each time t that both Ministries have invested in their respective programmes, suppose that the Ministry of Health's investment generates benefit $B_H^c X_t$ where $B_H^c > B_H^s$. This can be interpreted as the teaching staff having more time (due to the presence of teacher aides through the Ministry of Education's programme) to spend time with individual students, identify whether

¹⁸This is discussed further in the sections on assumptions and feasible parameter values for the model.

they need additional nourishment and liaise with health programme staff to make sure that the children are getting the food that they need.¹⁹ Likewise, suppose that when both Ministries have invested the Ministry of Education’s investment generates benefit $B_E^c X_t$ at each time t , where $B_E^c > B_E^s$. This can be interpreted as the investment in additional teaching effort being more effective if the children have been properly fed and are therefore able to concentrate and participate more effectively in class.

4.3 Discussion of assumptions

As is necessary in any economic analysis, a number of assumptions are made (whether explicitly or implicitly) in the basic set-up described above. These assumptions are discussed below, along with a brief discussion of any relevant considerations or justifications.

Recall that the three fundamental assumptions required for a real-options approach are uncertainty over the path of the state variable, flexibility of investment timing, and irreversibility of the investment decision. We consider that each of these assumptions can be reasonably made in the context of public policy investments, for the following reasons:

- *Uncertainty* — The policy problems faced by departments often involve factors that have complex determinants, making them difficult to predict. These factors will often also be influenced by broader economic conditions, making them subject to shocks. For example, we used the number of children in poverty as the state variable in our illustrative example above. This variable is influenced by, among other things, the state of the labour market and the wider economy, and broader government policy settings (i.e. those outside the control of the departments in question). From the perspective of the individual departments referenced this makes it a variable that is unpredictable and effectively exogenous, and which affects demand for their policy initiatives.
- *Flexibility* — Public administration involves the prioritisation of a wide range of competing policy objectives in the context of limited public resources. This involves implicit investment timing decisions, whereby high priority initiatives (i.e. those where there is an immediate need, which can be understood in our model as a high value of the relevant

¹⁹This could also be interpreted as teaching staff having capacity to help administer the nutrition intervention during school hours, ensuring that the relevant children receive a lunchtime meal.

state variable) are invested in immediately and other initiatives are reconsidered at a later date when more funding becomes available. Another consideration is the extent to which a department can be considered to have flexibility over investment decisions in the context of wider Government policy and ministerial relationships. In the past ministers had much more control over the operation of departments. However, under current conventions ministers set high-level policy objectives and departmental chief executives are responsible for day-to-day operational decisions (Department of the Prime Minister and Cabinet, 2017), which include decisions on operational policy matters and how departmental funding is spent in order to achieve the high-level objectives set by ministers.²⁰

- *Irreversibility* — In economics and finance, irreversibility is usually a concept closely associated with capital investments (such costs are considered fixed and difficult to recover, as distinct from labour costs which are assumed to be variable and can therefore be reversed). However, in the context of public administration we consider that a wider range of investments can be considered irreversible. The public finance system in New Zealand uses a baseline approach to budgeting, meaning that once a policy investment with ongoing costs is funded it is incorporated into the budget of a department in subsequent years. As departments are reluctant to suggest decreases to their budgets, initiatives are very infrequently de-funded voluntarily by the departments overseeing them. De-funding initiatives can also have a significant political reputational cost — particularly in the case of initiatives which provide a service to or otherwise benefit communities. As these factors provide significant barriers to reversibility, we consider that initiatives involving capital expenditure or staff costs can be treated as irreversible for the purpose of our model.²¹

A number of additional assumptions are necessary to ensure that the solutions of the model can be easily interpreted, and that the basic model is simple enough that our subsequent introduction and analysis of public management interventions remains tractable. These simplifying

²⁰Bureaucratic discretion is discussed further in Shumavon and Hibbeln (1986) and Meier (1987). Modern public administrators are characterised as having a significant degree of authority to make policy and operational decisions under delegation from the executive and legislative branches of government.

²¹That there are significant barriers to de-funding existing programmes is of course also an issue for public administration systems, as in some cases this results in the continuation of ineffective or inefficient initiatives. Research into this issue could be undertaken separately, and could draw on existing real-options literature relating to abandonment options.

assumptions may be relaxed as extensions of the basic model to widen its application, a possibility which is discussed further in a separate section at the end of this thesis.

- *Discount rate* — The discount rate ρ is assumed to be subjective, as we are not considering a financial investment with returns that must be compared to the market. As noted above, a suitable interpretation of this parameter could be the Treasury’s social discount rate.
- *One-off fixed-cost investment* — In the basic model we assume for simplicity that investment involves a one-off fixed cost. Consequently the model is most compelling when applied to investment scenarios involving an up-front capital expenditure. However, the investment cost I_n could alternatively be interpreted as the present value of a perpetual stream of constant per-period costs beginning at the time of investment, provided that the department’s ability to cease incurring these costs in future is restricted.²² Variable or staged costs could be investigated as extensions of the model.
- *Linear benefit flows* — As for fixed investment costs, this assumption simplifies analysis for the base model, though could be relaxed as an extension by making benefits a more general function of the state variable.
- *Geometric Brownian motion* — GBM is one of the most commonly used stochastic processes in basic real-options models. It is likely that GBM is a good approximation of the evolution of some exogenous variables, while others would be better modelled by another of the random processes used in real-options analysis (e.g. a mean-reverting process). This may restrict the application of the model to a degree, but it remains useful in cases where GBM provides a compelling approximation of a particular state variable. Future extensions of the model could involve analysing investments where the state variable follows different random processes. It is also worth noting that the assumption that the state variable follows GBM implies that neither department can influence the state variable itself, as the evolution of this variable is determined only by the exogenous parameters μ and σ . This implies that for any practical application it must be possible to identify a state variable that is reflective of economic conditions more broadly or is not the focus

²²This could be due, for example, to features of the public finance system such as the baseline budgeting approach discussed above.

of the specific policy interventions in question, such that the effect of any policy decision endogenous to the model on the state variable itself can be assumed to be negligible.

- *No benefits without investment* — The final assumption implicit in our model set-up is that neither department experiences any benefit when it has not yet invested in its own policy programme, regardless of whether the other department has invested (i.e. investment by department 1 alone does not generate benefit for department 2, nor vice versa). Depending on the practical ease of attribution (i.e. the ability to isolate and attribute benefits associated with a specific investment through measurement and evaluation programmes) this may not always hold in practice, particularly since we are interested in investments that are inherently synergistic and are therefore more likely to have cross-portfolio effects. Making this assumption in the basic model helps us to keep our analysis tractable and to isolate the specific distortion that we are interested in from a policy perspective. We address this assumption specifically, and the implications of relaxing it, in the concluding section of this thesis.

Related to this final assumption, it is also worth noting here that a special case of the general model we have described is the scenario when $B_1^s = B_2^s = 0$; that is, when no benefits are experienced by either department unless both departments have invested in their respective policy programmes. This special case can be interpreted as a pure collaborative policy investment, where a specific programme cannot be effectively implemented at all without the contribution of two departments. This will occur, for example, when departments have specific resources, capabilities, powers or information which cannot be easily (if at all) accessed by other departments but are necessary for the success of the policy initiative. For example, an intervention focused on improving youth justice outcomes for children in state care might not be possible without participation by both the New Zealand Police and Oranga Tamariki Ministry for Children; the latter has responsibility for these children and considerable influence over and knowledge of their individual circumstances, while the former is likely their first point of contact with the justice system (and therefore will likely be best-placed to initiate an intervention in any specific case). Cases of this nature are often of particular interest from a policy perspective, particularly when one department is responsible for an outcome area where an initiative would have the greatest

impact (i.e. generate the greatest benefits) but another department controls a significant proportion of the specific resources necessary to deliver the initiative (i.e. will incur a relatively significant proportion of the investment costs). Focusing on this special case will help us to keep our analysis tractable when considering some of the more complex inducements available to a social planner later in the thesis.

4.4 Discussion of parameter values

The basic model set-up above involves nine underlying parameters that are exogenously determined. These are the shared subjective discount rate ρ , the drift μ and volatility σ of the state variable, the benefits generated by the departments' policy programmes when either one or both departments have invested (B_1^s, B_2^s and B_1^c, B_2^c , respectively) and the costs of the respective policy investments (I_1 and I_2). For the majority of our analysis we aim to find analytic results such that assigning specific parameter values is not necessary. However in some cases numerical evaluation will help us to better understand the behaviour of the model, and specific numerical examples may be used to illustrate results graphically (and otherwise). For this reason it is useful to discuss feasible values that these underlying parameters might take in practice.

As discussed above, a suitable figure for ρ in the context of public policy investments is an appropriate social discount rate. As noted in a 2017 working paper from the New Zealand Treasury (Creedy and Passi, 2017), there are a number of different approaches to estimating such a rate and different results have been offered in different jurisdictions. Given our focus on the New Zealand context, an appropriate estimate is the Social Opportunity Cost of Capital (SOC) figure offered by the Treasury. The working paper referenced suggests a range between 0.04 and 0.07 depending on the type of project, with a default rate of 0.06.²³ Consequently, when conducting numerical analysis we will consider values for ρ between 0.04 and 0.07 (consistent with the range offered by the Treasury) and will use a baseline rate of 0.06 for illustrative examples.

Values for μ and σ are conditional on the particular policy initiative being considered, as both depend on the specific state variable. We avoid specifying a particular state variable to

²³At the time our modelling was undertaken, this default rate aligned with the most recent rate for cost-benefit analysis provided by the Treasury. This rate is revised periodically, with updates published on the Treasury website.

keep application of the model as broad as possible, but can make some observations about the characteristics of the state variables likely to be of concern to government agencies. In standard financial settings where the state variable is often tied to asset prices, σ may take values as high as 0.5 depending on the nature of the asset in question. μ can theoretically take any value less than ρ (necessary to satisfy the assumption $\rho - \mu > 0$, which as noted earlier is required to ensure a finite solution to the model). In our public policy setting, however, relevant state variables are likely to be those that reflect underlying economic conditions, such as unemployment or poverty levels, which can be expected to change at a relatively slow and stable rate compared to asset prices. Consequently when conducting any numerical analysis we will consider values for μ of $0 - 0.04$ and values for σ of $0.02 - 0.08$. For illustrative examples we use baseline values of $\mu = 0.02$ and $\sigma = 0.05$.

Values for the costs and benefits of the departments' policy investments can take any values. As we work through the solution to the basic model it will become apparent that only relative values of these parameters matter (as opposed to absolute values). This is because departments' objective functions and welfare are both linearly homogeneous in the parameters $B_1^s, B_2^s, B_1^c, B_2^c, I_1$ and I_2 , and equilibrium and optimal policy decisions (i.e. investment timing decisions) do not change if we scale each of these parameters by the same factor. Note that this also means that the unit of measurement for these parameters is arbitrary. For illustrative examples we use baseline values of $B_1^c = B_2^c = 1, I_1 = 100$ and $I_2 = 200$. We vary B_1^s and B_2^s depending on the specific example to allow for illustration and analysis of the different types of equilibrium that may arise.

5 The basic model

5.1 Investment when policies are not complementary

We begin by considering the investment problem for a single department when there is no opportunity for a complementary investment to be made by another agency (i.e. either the other department does not have the option to invest, or the policy programme of each department does not impact the effectiveness of the other's programme, so that $B_n^s = B_n^c = B_n$). Considering this simple case will help to illustrate, for those unfamiliar with real-options modelling, the solution

approach which we employ in solving the general model and in our subsequent analysis of welfare and inducements. In particular, we establish two key results (Proposition 1 and Proposition 2 below) which are applied throughout the thesis. We cover the high-level steps here, with a more complete proof of the solution provided at **Appendix A**.

In the simple single-department case, department n 's investment has a cost I_n (incurred at the time of investment) and generates benefit $B_n X_t$ at each time t from the time of investment onward. Given an expected incremental percentage change in the state variable of μ and the department's discount rate of ρ , the expected present value of the (perpetual) benefit flow generated by the investment is given by $\frac{B_n X_t}{\rho - \mu}$. The investment payoff evaluated at the time of investment — or the standard net present value (NPV) of the programme — is therefore:

$$P(X_t) = \frac{B_n X_t}{\rho - \mu} - I_n \quad (2)$$

This gives us our first important result:

Proposition 1. *An investment with cost I_n incurred at the time of investment and which generates benefit $B_n X_t$ at every time t from the time of investment onward has an expected present value of $P(X_t) = \frac{B_n X_t}{\rho - \mu} - I_n$ at the time of investment. Note that this present value depends only on the value of the state variable at the time of investment (and not on any future values).*

As $P(X)$ is an increasing function, we consider investment rules involving a minimum threshold for X ; that is, we assume that the department will invest when the state variable exceeds some constant $\hat{X}$. Let $V(X)$ be the value of the option to invest in the policy programme — or, equivalently, the discounted present value of an investment in the policy programme occurring at the first time the state variable reaches an arbitrary $\hat{X}$. Using Itô's lemma it can be shown that $V(X)$ satisfies the following ordinary differential equation for all values of $X < \hat{X}$:

$$\frac{1}{2}\sigma^2 X^2 V''(X) + \mu X V'(X) - \rho V(X) = 0 \quad (3)$$

Where X follows geometric Brownian motion, if $X_t = 0$ then $X_\tau = 0$ for all $\tau > t$; that is, once X falls to 0 it will stay there indefinitely. This implies that if $X_t = 0$ the department's

investment will never generate any benefit, meaning that $V(0) = 0$. Also note that by definition investment will occur immediately when $X = \hat{X}$, meaning that $V(\hat{X}) = P(\hat{X})$. That is, the value of the option to invest is equal to the payoff from investment at the investment threshold. It is well established that after imposing these two boundary conditions, the solution to the differential equation takes the form $V(X) = A_1 X^{\beta_1}$, where:²⁴

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\frac{2\rho}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2} \quad (4)$$

and

$$A_1 = \frac{P(\hat{X})}{\hat{X}^{\beta_1}} \quad (5)$$

Substituting this expression for A_1 into our above expression for $V(X)$ tells us that the value of the option to invest in the policy programme is given by $V(X) = P(\hat{X}) \left(\frac{X}{\hat{X}}\right)^{\beta_1}$ when the state variable is below the investment threshold (i.e. $X < \hat{X}$).²⁵ This gives us our second important result:

Proposition 2. *The discounted present value at time t of a payoff $P(\hat{X})$ to be received at the first time that the state variable X reaches an arbitrary threshold value $\hat{X}$ is given by $V(X_t) = P(\hat{X}) \left(\frac{X_t}{\hat{X}}\right)^{\beta_1}$ for any $X_t < \hat{X}$. Note that here the term $\left(\frac{X_t}{\hat{X}}\right)^{\beta_1}$ is interpreted as a stochastic discount factor which is equal to the expected present value at time t of \$1 received when the state variable reaches the threshold value $\hat{X}$.*

Note that the value of the option $V(X)$ is also equal to $P(X)$ when $X > \hat{X}$, as the option will be exercised immediately for any X above the threshold value. Combined with Propositions 1 and 2 above, this implies the following general expression for the value of the department's

²⁴As the equation in question is a second-order linear homogeneous differential equation, its general solution can be expressed as a linear combination of any two independent solutions, in this case $V(X) = A_1 X^{\beta_1} + A_2 X^{\beta_2}$. It can be shown that $\beta_1 > 1$ and $\beta_2 < 0$ (that is, the quadratic obtained by substituting the identified functional form of the solution into the differential equation has one negative root and one root that is greater than unity; see Chapter 5 of Dixit and Pindyck (1994)). Imposing the boundary condition $V(0) = 0$ requires that $A_2 = 0$, implying that $V(X) = A_1 X^{\beta_1}$. The expressions for β_1 and A_1 are obtained, respectively, by substituting the functional form of the solution into the differential equation and obtaining the positive root of the resulting quadratic, and by imposing the boundary condition that $V(\hat{X}) = P(\hat{X})$.

²⁵Note that this result does not require optimality of the department's investment timing decision, and applies for any arbitrary threshold $\hat{X}$. Consequently, while here we have described $V(X)$ in terms of a policy investment option this result can be applied more generally to the valuation of any future payoff to be received at the first time the state variable reaches a certain value.

investment option:

$$V(X_t) = \begin{cases} \left(\frac{B_n \hat{X}}{\rho - \mu} - I_n \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} & \text{if } 0 \leq X_t < \hat{X} \\ \frac{B_n X_t}{\rho - \mu} - I_n & \text{if } \hat{X} \leq X_t \end{cases} \quad (6)$$

The department's optimal investment threshold will be the threshold that maximises the option value $V(X)$. In the region where investment is delayed, the option value $V(X) = A_1 X^{\beta_1}$ is maximised by choosing the value of $\hat{X}$ that maximises A_1 . Taking a first-order condition yields the following expression for the optimal $\hat{X}$:

$$\hat{X} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_n}{B_n} \quad (7)$$

This tells us that the department will invest in the policy programme as soon as:

$$\frac{B_n X_t}{\rho - \mu} \geq \frac{\beta_1 I_n}{\beta_1 - 1} \quad (8)$$

Note that the left hand side of this condition represents the expected present value of the benefits from investment, and the right hand side is the cost of investment multiplied by a factor $\frac{\beta_1}{\beta_1 - 1}$. As $\beta_1 > 1$ this term exceeds 1, meaning that the department waits longer to invest than a simple NPV rule would suggest. This is a core result of real-options analysis — that irreversibility of investment and uncertainty of future benefit flows increase investment thresholds beyond those seen in more traditional valuation approaches.

Note also that there is exactly one level of $\hat{X}$ which satisfies the department's first-order condition (i.e. a unique turning point). Note also that because $\beta_1 > 1$, $V(X) \rightarrow 0$ as $\hat{X} \rightarrow \infty$ and the coefficient $A_1 \rightarrow -\infty$ as $\hat{X} \rightarrow 0$.²⁶ Consequently, we know that the optimal $\hat{X}$ above yields a global maximum.

²⁶Intuitively, as the investment threshold becomes arbitrarily high investment never occurs so the option to invest has no value. Conversely, as the investment threshold becomes arbitrarily low the department invests immediately but the benefits of investment $\frac{B_n \hat{X}_t}{\rho - \mu}$ are arbitrarily small and outweighed by the cost of investment, making the option value negative.

5.2 Investment when policies are complementary

We now turn to the general case when both departments may invest, and their policy programmes are complementary. Before turning to the general case where either department may invest at any time, we consider a scenario where a specified department must invest first.

5.2.1 Pre-determined leader

Without loss of generality, we assume that it is department 1 which must be the first to invest (i.e. department 2 is restricted from investing until department 1 has invested). Suppose that department 1 has already invested, and that department 2 will invest when $X_t \geq \hat{X}_2^F$ (where F here stands for ‘follower’) for some constant $\hat{X}_2^F$ that we will determine. Applying Propositions 1 and 2, the value of department 2’s option to invest in its policy programme is:

$$F_2(X_t) = \begin{cases} \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{B_2^c X_t}{\rho - \mu} - I_2 & \text{if } \hat{X}_2^F \leq X_t \end{cases} \quad (9)$$

Taking a first-order condition and solving for the optimal $\hat{X}_2^F$ yields:

$$\hat{X}_2^F = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c} \quad (10)$$

Noting that we have assumed department 2 cannot invest before department 1 (i.e. $\hat{X}_2^F > \hat{X}_1^L$, where L here stands for ‘leader’ and $\hat{X}_1^L$ is department 1’s investment threshold) the general expression for department 2’s optimal behaviour in this case is given by:

$$\hat{X}_2^F = \max \left\{ \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c}, \hat{X}_1^L \right\} \quad (11)$$

Figure 1 plots the option value of investing in department 2’s policy programme when department 1 has already invested, for a specified set of parameter values.

Now we consider the behaviour of department 1, the assumed investment leader. Department 1 will invest when $X_t \geq \hat{X}_1^L$. First, we construct an expression for the value of department 1’s policy programme immediately before it invests for an arbitrary value of $\hat{X}_1^L$. If $X_t \geq \hat{X}_2^F$

Figure 1: This graph plots the option value for department 2's policy programme when department 1 has already invested, as a function of X . The dotted line plots department 2's payoff at the time of investment. Parameter values are $I_1 = 100$, $I_2 = 200$, $B_1^s = B_2^s = 0.7$, $B_1^c = B_2^c = 1$, $\mu = 0.02$, $\sigma = 0.05$ and $\rho = 0.06$. Department 2's optimal investment threshold is $\hat{X}_2^F = 12.68$.

(i.e. X is above the level at which department 2 would invest as the follower) then department 1 knows that department 2 will invest immediately. Department 1 will therefore receive the benefit flow $B_1^c X_t$ in perpetuity, and the investment payoff is therefore just the present value of this benefit flow as per Proposition 1:

$$L_1(X_t) = \frac{B_1^c X_t}{\rho - \mu} - I_1, \quad X_t \geq \hat{X}_2^F \quad (12)$$

If instead $X_t < \hat{X}_2^F$, there will be a delay before department 2 invests. Department 1 will initially receive a benefit flow of $B_1^s X_t$, and this will increase to $B_1^c X_t$ after department 2 invests. The present value of these benefits can be expressed as the sum of two perpetual flows; the initial flow $B_1^s X_t$ beginning immediately, and the incremental benefit flow $(B_1^c - B_1^s) X_t$ beginning as soon as $X_t = \hat{X}_2^F$. Applying Propositions 1 and 2 to value the second delayed benefit flow, it follows that the investment payoff in this case is:

$$L_1(X_t) = \frac{B_1^s X_t}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} - I_1, \quad 0 \leq X_t < \hat{X}_2^F \quad (13)$$

Combined, these expressions give the following expression for the value of department 1 immediately before investment, assuming it is the leader:

$$L_1(X_t) = \begin{cases} \frac{B_1^s X_t}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} - I_1 & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{B_1^c X_t}{\rho - \mu} - I_1 & \text{if } X_t \geq \hat{X}_2^F \end{cases} \quad (14)$$

Again applying Proposition 2, the value of the option to invest in department 1's policy programme prior to investment is given by $L_1(\hat{X}_1^L) \left(\frac{X}{\hat{X}_1^L} \right)^{\beta_1}$. Given that the function $L_1(X)$ is piecewise, we must consider two possibilities when solving for the optimal $\hat{X}_1^L$: either $\hat{X}_1^L < \hat{X}_2^F$ and the optimal $\hat{X}_1^L$ is in the low- X region of the value function, or $\hat{X}_1^L \geq \hat{X}_2^F$ and the optimal $\hat{X}_1^L$ is in the high- X region. First consider the case where $\hat{X}_1^L < \hat{X}_2^F$, and the value of department 1's option to invest is therefore:

$$L_1(\hat{X}_1^L) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} = \left(\frac{B_1^s \hat{X}_1^L}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} \left(\frac{\hat{X}_1^L}{\hat{X}_2^F} \right)^{\beta_1} - I_1 \right) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} \quad (15)$$

Taking a first order condition and solving for $\hat{X}_1^L$ yields:

$$\hat{X}_1^L = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^s} \quad (16)$$

Note that this is the same investment threshold for department 1 as in the single department case (i.e. the presence of a second department does not alter the decision of the investment leader). Intuitively, once department 1 has invested the decision of department 2 is independent of department 1's actions, meaning that department 1's decision does not take into account the response of department 2. Recognising that this threshold assumes that the constraint $\hat{X}_1^L < \hat{X}_2^F$ holds, the general expression for department 1's optimal behaviour in this case is given by:

$$\hat{X}_1^L = \min \left\{ \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^s}, \hat{X}_2^F \right\} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \min \left\{ \frac{I_1}{B_1^s}, \frac{I_2}{B_2^c} \right\} \quad (17)$$

Now consider the case where $\hat{X}_1^L \geq \hat{X}_2^F$, and the value of department 1's option to invest is

therefore:

$$L_1(\hat{X}_1^L) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} = \left(\frac{B_1^c \hat{X}_1^L}{\rho - \mu} - I_1 \right) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} \quad (18)$$

Taking a first order condition and solving for $\hat{X}_1^L$ in this case yields:

$$\hat{X}_1^L = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c} \quad (19)$$

By comparing to the expression for $\hat{X}_2^F$, and noting the symmetry of the model, we can see that this is the same investment threshold that department 1 would have if it was the follower. Again noting that we have assumed a constraint $\hat{X}_1^L \geq \hat{X}_2^F$, the general expression for department 1's optimal behaviour in this case is given by:

$$\hat{X}_1^L = \max \left\{ \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c}, \hat{X}_2^F \right\} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \max \left\{ \frac{I_1}{B_1^c}, \frac{I_2}{B_2^c} \right\} \quad (20)$$

As can be inferred from the expressions for $\hat{X}_1^L$ above, the general solution to department 1's optimisation problem depends on the relative values of the cost-benefit ratios $\frac{I_1}{B_1^s}$, $\frac{I_1}{B_1^c}$ and $\frac{I_2}{B_2^c}$. Noting that our base assumption $B_n^s < B_n^c$ implies that $\frac{I_n}{B_n^s} > \frac{I_n}{B_n^c}$, there are three possible orderings of these ratios as depicted in Figure 2. In general, our approach to determining the optimal behaviour of department 1 in each of these cases is to evaluate the above expressions for $\hat{X}_1^L$ given our assumption about the relative values of parameters. If a local interior solution is available for either section of the piecewise-defined function $L_1(X)$ (i.e. $\hat{X}_1^L \neq \hat{X}_2^F$ for one of the above expressions) this constitutes the general solution to department 1's problem; otherwise if both local maxima are corner solutions (i.e. $\hat{X}_1^L = \hat{X}_2^F$ in both cases) the general solution is also a corner solution. Following this approach, department 1's optimal investment thresholds for each possible ordering of $\frac{I_1}{B_1^s}$, $\frac{I_1}{B_1^c}$ and $\frac{I_2}{B_2^c}$ are summarised in Table 1.

Figure 2: This diagram shows the possible orderings of the ratios $\frac{I_1}{B_1^s}$, $\frac{I_1}{B_1^c}$ and $\frac{I_2}{B_2^c}$, which determine the departments' optimal investment decisions and ultimately the equilibrium of the pre-determined leader game. Note that the ordering of $\frac{I_1}{B_1^s}$, $\frac{I_1}{B_1^c}$ is fixed by the assumptions of the model, so the key consideration in determining the outcome of the game is the value of $\frac{I_2}{B_2^c}$ relative to these.

Parameter values	Low-X region	High-X region	General solution
$\frac{I_1}{B_1^s} < \frac{I_2}{B_2^c}$	$\hat{X}_1^L = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^s}$	$\hat{X}_1^L = \hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$	$\hat{X}_1^L = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^s}$
$\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c} < \frac{I_1}{B_1^s}$	$\hat{X}_1^L = \hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$	$\hat{X}_1^L = \hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$	$\hat{X}_1^L = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$
$\frac{I_2}{B_2^c} < \frac{I_1}{B_1^c}$	$\hat{X}_1^L = \hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$	$\hat{X}_1^L = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c}$	$\hat{X}_1^L = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c}$

Table 1: Department 1's investment threshold under different conditions

The optimal investment thresholds for department 1 in Table 1, along with the expression for $\hat{X}_2^F$ derived earlier, describe the different possible equilibria of the pre-determined leader game. Depending on parameter values, there are therefore three possible equilibria:

1. Department 1 is a natural leader ($\frac{I_1}{B_1^s} < \frac{I_2}{B_2^c}$, depicted in diagram A in Figure 2) — in this case there are values of X for which department 1 will invest (when $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^s}$) and department 2 will not immediately follow (but instead waits until $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$). Intuitively, here department 1's investment is significantly more attractive than department 2's (either coming at lower cost or yielding greater benefits) such that at some point it will no longer be willing to wait for department 2's cooperation and will invest

alone. Figure 3 plots department 1's leader investment payoff in this case (i.e. $L_1(X)$) for specified parameter values.

2. Department 1 waits and then invests as soon as it knows department 2 will follow immediately ($\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c} < \frac{I_1}{B_1^s}$, depicted in diagram B in Figure 2) — in this case there are no values of X where department 1 will invest while knowing that department 2 will not immediately follow. Department 1 is willing to engage in joint investment as soon as $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c}$ but must wait until department 2 is also willing to invest (i.e. when $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$), at which point department 1 invests and department 2 immediately follows. Intuitively, as joint investment is more attractive than investment alone for both departments, when the characteristics of the departments' respective investments are relatively similar both will be willing to invest together before department 1 is willing to invest alone.
3. Department 1 waits past the point where it knows department 2 would invest immediately ($\frac{I_2}{B_2^c} < \frac{I_1}{B_1^c}$) — again there are no values of X where department 1 will invest while knowing that department 2 will not immediately follow. Department 2 would be willing to follow when $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$ but department 1 will not invest until $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c}$, at which point it invests and department 2 immediately follows. Again, intuitively joint investment is more attractive than investment alone, so when the characteristics of their investments are relatively similar we see the departments wait until the point at which both are willing to invest provided the other will invest also.

The key characteristics of the equilibria described above are summarised in Table 2. The optimal thresholds for the respective departments in equilibrium, $\hat{X}_1^L$ and $\hat{X}_2^F$, are depicted graphically in Figure 4 for different parameter values.

Parameter values	Optimal thresholds	Description
$\frac{I_1}{B_1^s} < \frac{I_2}{B_2^c}$	$\hat{X}_1^L = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^s}, \hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$	Dept. 1 invests before dept. 2
$\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c} < \frac{I_1}{B_1^s}$	$\hat{X}_1^L = \hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$	Dept. 1 waits for dept. 2
$\frac{I_2}{B_2^c} < \frac{I_1}{B_1^c}$	$\hat{X}_1^L = \hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c}$	Dept. 2 must wait for dept. 1

Table 2: Possible equilibria for the pre-determined leader game

Figure 3: This graph plots the option value for department 1 in the single department case ($V(X)$), and department 1's leader investment payoff $L_1(X)$ assuming that department 2 enters according to its optimal investment policy. The steeper dotted line is department 1's investment payoff assuming that department 2 immediately invests, and the flatter dotted line is the investment payoff in the single department case. The dotted curve is the option value for department 1, which meets $L_1(X)$ at department 1's optimal investment threshold $\hat{X}_1^L = 9.06$. The kink in the curve $L_1(X)$ at $\hat{X}_2^F = 12.68$ arises because this is where the follower invests. Parameter values are the same as for Figure 1.

Conversely, we could consider the case where department 2 is the leader and department 1 is the follower. Given the symmetry of the model, this is identical to the scenario described in this section, only with the subscripts reversed.

5.2.2 No pre-determined leader

Finally we consider a game where either department may invest first. In such a game the leader and follower value functions are identical to those in the pre-determined leader scenario, implying that for both departments the threshold for investment alone (i.e. knowing that the other department will not immediately follow) is given by $\hat{X}_n^L = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_n}{B_n^s}$ and the investment threshold when the other department has already invested is given by $\hat{X}_n^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_n}{B_n^c}$. As for the pre-determined leader game the behaviour of the departments in equilibrium depends on the relative values of the ratios $\frac{I_1}{B_1^s}$, $\frac{I_1}{B_1^c}$ and $\frac{I_2}{B_2^c}$; however now that department 2 is not

Figure 4: This diagram depicts the optimal thresholds for the respective departments in equilibrium, $\hat{X}_1^L$ and $\hat{X}_2^F$, for different values of $\frac{I_2}{B_2^c}$ (given fixed values for $\frac{I_1}{B_1^c}$ and $\frac{I_1}{B_1^s}$). The three segments of the diagram correspond to the three different types of equilibrium that result for different parameter values, as described above. For example the right hand segment (where $\frac{I_1}{B_1^s} < \frac{I_2}{B_2^c}$) corresponds to the case where department 1 is a natural leader, and therefore $\hat{X}_1^L < \hat{X}_2^F$.

restricted from investing before department 1, in this case the value of $\frac{I_2}{B_2^s}$ is also relevant. The two equilibria from the pre-determined leader game for which $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^s}$ are also equilibria of this game — in these cases, parameter values are such that the possibility for department 2 to invest first does not alter the behaviour of departments in equilibrium. However, where $\frac{I_2}{B_2^c} < \frac{I_1}{B_1^c}$ parameter values may be such that department 2 will choose to invest before department 1. In this case there are two further possible equilibria, depending on the relative values of $\frac{I_2}{B_2^s}$ and $\frac{I_1}{B_1^c}$.

1. Department 2 waits and then invests as soon as it knows department 1 will follow immediately ($\frac{I_2}{B_2^c} < \frac{I_1}{B_1^c} < \frac{I_2}{B_2^s}$) — in this case there remain no values of X for which department 2 is willing to invest while knowing that department 1 will not immediately follow. De-

partment 2 is willing to engage in joint investment as soon as $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$ but must wait until department 1 is also willing to invest (i.e. when $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c}$), at which point department 2 invests and department 1 immediately follows.

2. Department 2 is a natural leader ($\frac{I_2}{B_2^s} < \frac{I_1}{B_1^c}$) — in this case there are values of X for which department 2 will invest (when $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^s}$) and department 1 will not immediately follow (but instead waits until $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c}$).

There are therefore four possible equilibria for the game without a pre-determined leader, as discussed above. These are summarised in Table 3.

Parameter values	Optimal thresholds	Description
$\frac{I_1}{B_1^s} < \frac{I_2}{B_2^c}$	$\hat{X}_1^L = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^s}, \hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$	Dept. 1 invests before dept. 2
$\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c} < \frac{I_1}{B_1^s}$	$\hat{X}_1^L = \hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$	Dept. 1 waits for dept. 2
$\frac{I_2}{B_2^c} < \frac{I_1}{B_1^c} < \frac{I_2}{B_2^s}$	$\hat{X}_2^L = \hat{X}_1^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c}$	Dept. 2 waits for dept. 1
$\frac{I_2}{B_2^c} < \frac{I_1}{B_1^c}$	$\hat{X}_2^L = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}, \hat{X}_1^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c}$	Dept. 2 invests before dept. 1

Table 3: Possible equilibria for the general game (without a pre-determined leader)

Given the symmetry of the model, and the fact that in this general case we have removed any restriction on the investment timing of either department, it is not surprising to note that the four equilibria described in Table 3 are symmetric also and can be categorised as one of two general types:

1. **Natural leader** ($\frac{I_1}{B_1^s} < \frac{I_2}{B_2^c}$ or $\frac{I_2}{B_2^s} < \frac{I_1}{B_1^c}$) — one of the two departments is willing to invest knowing that the other will not follow immediately. If department 1 is the natural leader, it invests when $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^s}$ and department 2 follows when $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$ (or vice versa with subscripts reversed). This type of equilibrium will occur when the departments are dissimilar such that there are large differences in the investment costs incurred or benefit flows received between the two (e.g. department 1 will be more likely to be a natural leader if I_1 is substantially lower than I_2 , or B_1^s is larger than B_2^c). Intuitively, when the benefits from investment are substantially lower or investment cost substantially higher for one department than the other, the department with more favourable investment characteristics will not be willing to wait for the other to invest (despite joint investment

being preferable for both departments). The equilibria described in rows 1 and 4 of Table 3 are equilibria of this type.

2. **Waiting game** ($\frac{I_1}{B_1^c}, \frac{I_2}{B_2^c} < \frac{I_1}{B_1^s}, \frac{I_2}{B_2^s}$) — neither department wants to invest before the other; both wait until they know the other will also invest, and then they invest simultaneously. This simultaneous investment occurs when $X \geq \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \max\{\frac{I_1}{B_1^c}, \frac{I_2}{B_2^c}\}$. This type of equilibrium will occur when the departments are sufficiently similar in the ratio of their investment costs to the benefit flows received following investment (i.e. this type of equilibrium will occur when the ratios $\frac{I_1}{B_1^c}$ and $\frac{I_2}{B_2^c}$ are similar). Note that this may occur either when the two investments are similar in both costs and benefits, or when the two investments are comparable but of a different scale (e.g. department 1's programme is a scaled-up version of department 2's programme). Intuitively, when the ratios of the costs and benefits for the two departments' investments are sufficiently similar, the two departments have similar preferences for investment timings and therefore will both be willing to invest jointly before either will be willing to invest alone. The equilibria described in rows 2 and 3 of Table 3 are equilibria of this type.

There are clear differences between the equilibria that we obtain here, and those which are obtained in a competitive duopoly setting where $B_n^c < B_n^s, n = 1, 2$. While such games may also have both equilibria where the firms invest sequentially and equilibria where firms invest simultaneously, these have different characteristics to the equilibria derived in our model above. In simple models of duopolistic market entry of the type developed by Smets (1991) and simplified in Dixit and Pindyck (1994), simultaneous-investment equilibria of the type identified in our model do not occur.²⁷ Simultaneous equilibria are present in Pawlina and Kort (2006), but these are explicitly attributed to the fact that in that model the firms are already operating in a market (i.e. some level of investment has already occurred) and consequently will under certain conditions delay investment in a new innovation beyond the follower thresholds of both firms in

²⁷The authors note that in theory simultaneous investment may occur by “mistake” in the setting where firms have homogeneous investment costs — the firms are worse-off if both invest immediately, but independently select a probability of doing so as a result of indifference between being the leader and the follower at the pre-emptive investment threshold. In discrete time settings such a mixed-strategy equilibrium would result in simultaneous investment occurring with some positive probability. However, the authors argue that in a continuous time setting the probability of such mistakes occurring goes to zero in accordance with arguments presented in Fudenberg and Tirole (1985).

order to protect existing profits (i.e. tacit collusion).²⁸ Those models of competitive investment timing also invariably involve sequential ‘pre-emption’ equilibria — that is, equilibria where the leader invests earlier than they would have otherwise to avoid being preempted by the follower. Such equilibria do not occur in our model because our assumption that $B_n^s < B_n^c$ for $n = 1, 2$ implies that $\frac{I_n}{B_n^s} < \frac{I_m}{B_m^c}$ cannot hold for $n = 1, m = 2$ and $n = 2, m = 1$ simultaneously, so it is not possible for both departments to be simultaneously willing to invest knowing the other will not immediately follow. Intuitively, the assumption $B_n^s < B_n^c, n = 1, 2$:

- makes entry by a second department more profitable for the leader than having invested alone, meaning a ‘threat’ of pre-emption is not threatening to the leader; and
- implies that following (or joint investment) is weakly preferable to leading for the follower department in equilibrium, as $F(X) \geq L(X) \forall X$. This is shown graphically in Figure 5.

6 Social planner’s problem

6.1 Solving for the socially optimal outcome

We now consider the problem of a social planner who has an interest in the outcomes of the policy programmes of both departments. Suppose that the social planner cares about the investment costs of both programmes (I_1, I_2) as well as the benefits ($B_1^s, B_2^s, B_1^c, B_2^c$), and is able to choose the investment timing for both departments ($\hat{X}_1, \hat{X}_2$).²⁹ The planner will choose investment timings that maximise the sum of the option values for the two policy programmes. Because the option values are dependent on which department is the investment leader and which the follower, there are two cases to consider: where $\hat{X}_1 \geq \hat{X}_2$ and where $\hat{X}_1 \leq \hat{X}_2$.

We begin by considering the case where department 1 is the leader and department 2 the follower (i.e. $\hat{X}_1 \leq \hat{X}_2$). In this case, the planner will choose $\hat{X}_1^L, \hat{X}_2^F$ to maximise the sum of

²⁸This is not comparable to our model, where no benefits are generated until at least one department has invested. Note that in the competitive case delay of investment past the firms’ follower thresholds also implies a simultaneous investment threshold that is greater than both firms’ leader thresholds (which are greater than their respective follower thresholds due to the presence of a first-mover advantage). This is also in contrast to the simultaneous equilibria in our model, which occur as soon as both departments’ follower thresholds have been reached and before either department is willing to invest alone.

²⁹There is a tradition in public choice theory which posits that public agencies overstate the benefits and understate the costs of policy programmes, relative to social efficiency (see, for example, Chapter 16 of Mueller (2003)). To keep our analysis tractable we do not incorporate this further distortion here, but note that the effects of its addition to our model could be investigated through future research.

Figure 5: This graph plots the department 2's follower option value $F_2(X)$, and its leader investment payoff $L_2(X)$. Note that $F_2(X)$ is weakly greater than $L_2(X)$ for all values of X , meaning that when department 2 is the follower it will never prefer to preempt department 1. Parameter values are the same as for Figures 1 and 3.

the option values of the two departments' policy investments. From the above analysis of the respective departments' investment problems, the value of the individual policy investments at the time of department 1's (the leader's) investment are:

$$L_1(X_t) = \begin{cases} \frac{B_1^s X_t}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} - I_1 & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{B_1^c X_t}{\rho - \mu} - I_1 & \text{if } X_t \geq \hat{X}_2^F \end{cases} \quad (21)$$

$$F_2(X_t) = \begin{cases} \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{B_2^c X_t}{\rho - \mu} - I_2 & \text{if } \hat{X}_2^F \leq X_t \end{cases} \quad (22)$$

Prior to investment by department 1, we know from Proposition 2 that the value of the options to invest in the policies of the respective departments are given by $L_1(\hat{X}_1^L) \left(\frac{X}{\hat{X}_1^L} \right)^{\beta_1}$ and $F_2(\hat{X}_1^L) \left(\frac{X}{\hat{X}_1^L} \right)^{\beta_1}$, implying that the social planner's objective function is given by $W(X) = L_1(\hat{X}_1^L) \left(\frac{X}{\hat{X}_1^L} \right)^{\beta_1} + F_2(\hat{X}_1^L) \left(\frac{X}{\hat{X}_1^L} \right)^{\beta_1}$ (i.e. the sum of the two option values). When solving for

the planner's optimal investment timings, there are two cases to consider: an interior solution, where the constraint $\hat{X}_1 \leq \hat{X}_2$ is not binding (i.e. $\hat{X}_1 < \hat{X}_2$) and a corner solution, where the constraint is binding and $\hat{X}_1 = \hat{X}_2$. We begin by considering an interior solution, where the objective function is given by:

$$\begin{aligned}
W(X_t) &= L_1(\hat{X}_1^L) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} + F_2(\hat{X}_1^L) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} \\
&= \left(\frac{B_1^s \hat{X}_1^L}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} \left(\frac{\hat{X}_1^L}{\hat{X}_2^F} \right)^{\beta_1} - I_1 \right) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} \\
&\quad + \left(\left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{\hat{X}_1^L}{\hat{X}_2^F} \right)^{\beta_1} \right) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} \\
&= \left(\frac{B_1^s \hat{X}_1^L}{\rho - \mu} - I_1 \right) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} + \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1}
\end{aligned} \tag{23}$$

Taking first-order conditions of this objective function with respect to the choice variables $\hat{X}_1^L$, $\hat{X}_2^F$ yields the following:

$$\begin{aligned}
0 &= \left(\frac{B_1^s}{\rho - \mu} + \frac{(B_1^c - B_1^s)(\hat{X}_2^F)^{(1-\beta_1)}}{\rho - \mu} \cdot \beta_1 (\hat{X}_1^L)^{(1-\beta_1)} \right) (\hat{X}_1^L)^{\beta_1} \\
&\quad - \beta_1 \left(\frac{B_1^s \hat{X}_1^L}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} \left(\frac{\hat{X}_1^L}{\hat{X}_2^F} \right)^{\beta_1} - I_1 \right) (\hat{X}_1^L)^{-(1+\beta_1)}
\end{aligned} \tag{24}$$

$$\begin{aligned}
0 &= \left(\frac{B_2^c}{\rho - \mu} (\hat{X}_2^F)^{-\beta_1} - \beta_1 \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - I_2 \right) (\hat{X}_2^F)^{-(1+\beta_1)} \right) \\
&\quad + (1 - \beta_1) \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} (\hat{X}_2^F)^{-\beta_1}
\end{aligned} \tag{25}$$

Solving these conditions to find the socially optimal values for the choice variables yields:

$$\hat{X}_1^{L*} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^s} \tag{26}$$

$$\hat{X}_2^{F*} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c + (B_1^c - B_1^s)} \tag{27}$$

Next, we consider a corner solution where the constraint $\hat{X}_1 \leq \hat{X}_2$ is binding and the planner chooses a single investment timing for both policy investments $\hat{X} = \hat{X}_1 = \hat{X}_2$. In this case, the

planner's objective function is:

$$\begin{aligned} L_1(\hat{X}) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} + F_2(\hat{X}) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} &= \left(\frac{B_1^c \hat{X}}{\rho - \mu} - I_1 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} + \left(\frac{B_2^c \hat{X}}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} \\ &= \left(\frac{(B_1^c + B_2^c) \hat{X}}{\rho - \mu} - (I_1 + I_2) \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} \end{aligned} \quad (28)$$

Note that this is identical to the investment problem faced by a single department when investments are not complementary, where $I = I_1 + I_2$ and $B = B_1^c + B_2^c$. The planner's optimal investment timing is therefore:

$$\hat{X}^{E*} = \hat{X}_1^* = \hat{X}_2^* = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1 + I_2}{B_1^c + B_2^c} \quad (29)$$

Now consider the case where department 2 is the leader and department 1 the follower (i.e. $\hat{X}_1 \geq \hat{X}_2$, and the planner chooses $\hat{X}_2^L, \hat{X}_1^F$ to maximise the sum of the option values of the policy investments $L_2(X) + F_1(X)$). This is just the problem above with subscripts reversed, meaning that in this case when the constraint is binding the optimal investment timings are given by the expression for $\hat{X}^{E*}$ above, and when the constraint is not binding the optimal investment timings are:

$$\hat{X}_2^{L*} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^s} \quad (30)$$

$$\hat{X}_1^{F*} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c + (B_2^c - B_2^s)} \quad (31)$$

Thus far, we have a range of candidates for a socially optimal choice of $\{\hat{X}_1, \hat{X}_2\}$ — labelled above as $\{\hat{X}_1^{L*}, \hat{X}_2^{F*}\}$, $\{\hat{X}_2^{L*}, \hat{X}_1^{F*}\}$ and $\{\hat{X}^{E*}, \hat{X}^{E*}\}$. We show below that for any given set of parameter values only one of these candidates can yield a local maximum and must therefore be the investment rule that yields the global maximum $\{\hat{X}_1^*, \hat{X}_2^*\}$. More specifically, we show that it is not possible for both $\hat{X}_1^{L*} < \hat{X}_2^{F*}$ and $\hat{X}_2^{L*} < \hat{X}_1^{F*}$ to be satisfied by the same set of parameter values, meaning that we cannot simultaneously have a local maximum involving department 1 investing before department 2 and a local maximum involving department 2 investing before department 1.

Proposition 3. *For a given set of parameter values, $\{\hat{X}_1^{L*}, \hat{X}_2^{F*}\}$ and $\{\hat{X}_2^{L*}, \hat{X}_1^{F*}\}$ cannot both be interior solutions, implying a unique local maximum for the planner's objective function (which must therefore be the global maximum).*

Proof. Suppose that both $\hat{X}_1^{L*} < \hat{X}_2^{F*}$ (required for $\{\hat{X}_1^{L*}, \hat{X}_2^{F*}\}$ to be a local maximum) and $\hat{X}_2^{L*} < \hat{X}_1^{F*}$ (required for $\{\hat{X}_2^{L*}, \hat{X}_1^{F*}\}$ to be a local maximum). Using the expressions derived above, these conditions can be restated as:

$$\frac{I_1}{B_1^s} < \frac{I_2}{B_2^c + (B_1^c - B_1^s)} \quad (32)$$

$$\frac{I_2}{B_2^s} < \frac{I_1}{B_1^c + (B_2^c - B_2^s)} \quad (33)$$

Rearranging to express each condition in terms of $\frac{I_1}{I_2}$ and applying the transitive property of inequalities, these combine to give:

$$\frac{B_1^c + (B_2^c - B_2^s)}{B_2^s} < \frac{B_1^s}{B_2^c + (B_1^c - B_1^s)} \quad (34)$$

This simplifies to $B_1^c + B_2^c < B_1^s + B_2^s$ which violates our base assumption that $B_n^s < B_n^c$, $n = 1, 2$, showing the original conjecture ($\hat{X}_1^{L*} < \hat{X}_2^{F*}$ and $\hat{X}_2^{L*} < \hat{X}_1^{F*}$) to be impossible to satisfy given the assumptions of the model. As the constrained optimum $\{\hat{X}^{E*}, \hat{X}^{E*}\}$ is identical regardless of which department is the leader and which the follower, an unconstrained local maximum (either $\{\hat{X}_1^{L*}, \hat{X}_2^{F*}\}$ or $\{\hat{X}_2^{L*}, \hat{X}_1^{F*}\}$) will therefore be unique if one exists, and will also be the global maximum. $\square$

Given Proposition 3 and the derivations of $\{\hat{X}_1^{L*}, \hat{X}_2^{F*}\}$, $\{\hat{X}_2^{L*}, \hat{X}_1^{F*}\}$ and $\{\hat{X}^{E*}, \hat{X}^{E*}\}$ above, we can specify a set of socially optimal investment decisions for any given set of parameter values, as set out in Table 4.

Parameter values	Optimal thresholds	Description
$\frac{I_1}{B_1^s} < \frac{I_2}{B_2^c + (B_1^c - B_1^s)}$	$\hat{X}_1^{L*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^s},$ $\hat{X}_2^{F*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c + (B_1^c - B_1^s)}$	Dept. 1 invests before dept. 2
$\frac{I_1}{B_1^s} > \frac{I_2}{B_2^c + (B_1^c - B_1^s)},$ $\frac{I_2}{B_2^s} > \frac{I_1}{B_1^c + (B_2^c - B_2^s)}$	$\hat{X}_1^* = \hat{X}^{E*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1 + I_2}{B_1^c + B_2^c}$ $\hat{X}_2^* = \hat{X}^{E*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1 + I_2}{B_1^c + B_2^c}$	Departments invest simultaneously
$\frac{I_2}{B_2^s} < \frac{I_1}{B_1^c + (B_2^c - B_2^s)}$	$\hat{X}_2^{L*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^s},$ $\hat{X}_1^{F*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c + (B_2^c - B_2^s)}$	Dept. 2 invests before dept. 1

Table 4: Socially optimal investment timing

6.2 Comparison with equilibrium outcomes

Now that we have defined both equilibrium investment decisions and socially optimal investment decisions for any given set of parameter values, we can evaluate the investment decisions of the departments from a welfare perspective by comparing equilibrium outcomes with the social planner's decisions.

First, consider the case where $\frac{I_n}{B_n^s} < \frac{I_m}{B_m^c + (B_n^c - B_n^s)}$ for either $n = 1, m = 2$ or $n = 2, m = 1$. That is, there are values of X for which it is socially optimal for one of the departments to invest in its policy programme without simultaneous investment by the other department. Without loss of generality, assume that parameter values are such that it is optimal for department 1 to invest first (i.e. the stated condition holds for $n = 1$). As per Table 4, the socially optimal investment thresholds in this case are $\hat{X}_1^{L*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^s}$ and $\hat{X}_2^{F*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c + (B_1^c - B_1^s)}$. Noting that $\frac{I_2}{B_2^c + (B_1^c - B_1^s)} < \frac{I_2}{B_2^c}$ implies that $\frac{I_1}{B_1^s} < \frac{I_2}{B_2^c}$, in equilibrium the departments' investment thresholds are $\hat{X}_1^L = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^s}$ and $\hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$ (i.e. department 1 is a natural leader and invests earlier than department 2; see Table 3). Comparing the equilibrium outcome with the socially optimal investment timings, we can see that $\hat{X}_1^L = \hat{X}_1^{L*}$ and $\hat{X}_2^F > \hat{X}_2^{F*}$; that is, the leader department's investment decision is socially optimal, but the follower department delays investment past the socially optimal threshold. Intuitively, when making its investment decision the leader's objective is aligned with that of the social planner — as the leader is a 'natural leader' both in equilibrium and from a welfare perspective, neither its equilibrium nor socially optimal investment timing decision will impact the decision of follower and consequently

both the leader and the planner would make investment decisions taking into account only the benefits and costs of the leader's policy programme (i.e. B_1^s and I_1). Conversely, the follower department does not take into account the increased benefit from the leader's policy programme (denoted $B_1^c - B_1^s$) that results from its investment, so it invests late from a welfare perspective.

Next, consider the case where $\frac{I_n}{B_n^s} > \frac{I_m}{B_m^c + (B_n^c - B_n^s)}$ for both $n = 1, m = 2$ and $n = 2, m = 1$, such that it is socially optimal for the departments to invest simultaneously when X exceeds the threshold $\hat{X}_1^* = \hat{X}_2^* = \hat{X}^{E*} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1 + I_2}{B_1^c + B_2^c}$. Without loss of generality (given the symmetry of the model), assume that $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}$ such that department 1 is willing to engage in joint investment before department 2. There are two possible scenarios to consider:

1. First, suppose that $\frac{I_1}{B_1^s} < \frac{I_2}{B_2^c}$ such that, while not socially optimal, in equilibrium department 1 will invest for some values of X knowing that department 2 will not immediately follow. In this case equilibrium thresholds are $\hat{X}_1^L = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^s}$ and $\hat{X}_2^F = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c}$ (see Table 3). As we have assumed that $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}$ it must be the case that $\frac{I_2}{B_2^c} > \frac{I_1 + I_2}{B_1^c + B_2^c}$, and the condition $\frac{I_1}{B_1^s} > \frac{I_2}{B_2^c + (B_1^c - B_1^s)}$ implies that $\frac{I_1}{B_1^s} > \frac{I_1 + I_2}{B_1^c + B_2^c}$. Consequently, $\hat{X}_1^L > \hat{X}_1^{L*}$ and $\hat{X}_2^F > \hat{X}_2^{F*}$; that is, both departments delay investment past the socially optimal threshold. Intuitively, the follower department does not take into account the increased benefit from the leader's policy programme, and the leader department must wait until it is profitable to invest alone (even though it would have been willing to engage in joint investment at the socially optimal threshold).
2. Second, suppose that $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}, \frac{I_2}{B_2^c} < \frac{I_1}{B_1^s}$ such that in equilibrium neither department will be willing to invest while knowing the other will not immediately follow. Given our assumption that $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}$, in equilibrium department 1 waits until department 2 is willing to engage in joint investment, and then both departments invest simultaneously. In this case equilibrium thresholds are $\hat{X}_1 = \hat{X}_2 = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c}$ (see Table 3). As $\frac{I_2}{B_2^c} > \frac{I_1 + I_2}{B_1^c + B_2^c} > \frac{I_1}{B_1^c}$, it follows that $\hat{X}_1^L > \hat{X}_1^{L*}$ and $\hat{X}_2^F > \hat{X}_2^{F*}$; that is, both departments delay investment past the socially optimal threshold. Intuitively, department 1 must wait until department 2 is willing to engage in joint investment, and department 2's threshold is greater than the socially optimal threshold because it does not take into account the impact of its investment on the benefits of department 1's policy programme.

While each of these cases described above differ in equilibrium and/or socially optimal outcome, they do have a common theme; that is, in equilibrium at least one of the two departments delays investment past the point at which investment would be socially optimal. The intuition for the departments investing late (from a welfare perspective) is also common across each of the cases described — the respective departments do not internalise the positive effect of their investments on the benefits resulting from the other department's policy programme. In other words, investments have a positive externality that is not accounted for in departments' investment decisions.

Note that we have so far only considered the case where the investment leader is not pre-determined. If an investment leader is pre-determined, a further inefficiency is possible in the case that the restriction on the follower's investment timing is binding. Without loss of generality, assume that department 1 is the investment leader, and that $\frac{I_2}{B_2^s} < \frac{I_1}{B_1^c}$ (i.e. parameter values are such that department 2 would be a natural leader if investment timing was unrestricted). In this scenario department 2 would be willing to invest without department 1 immediately following as soon as $X = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^s}$ (i.e. department 2's unrestricted investment threshold). However, it is unable to invest at this threshold due to the requirement that department 1 invests first and must instead wait until department 1 is willing to engage in joint investment, with equilibrium thresholds therefore $\hat{X}_1^L = \hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^c}$. If we assume that $\frac{I_2}{B_2^s} < \frac{I_1}{B_1^c + (B_2^c - B_2^s)}$, then department 2's unrestricted investment threshold is equal to the optimal investment threshold $\hat{X}_2^{L*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^s}$. Consequently, in this case department 2 is forced to delay investment past the socially optimal threshold by the restriction on its investment timing where it would otherwise choose the socially optimal investment threshold. If we assume that $\frac{I_2}{B_2^s} > \frac{I_1}{B_1^c + (B_2^c - B_2^s)}$ then department 2's unrestricted investment threshold is above the optimal investment threshold for both departments $\hat{X}^{E*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1+I_2}{B_1^c+B_2^c}$, meaning that in this case department 2's unrestricted choice would be late from a welfare perspective. However, as the restriction on its choice causes department 2 to delay investment even further past the optimal threshold, an additional distortion is also introduced in this case. Note that department 1 also delays investment past the socially optimal threshold in both cases, but its investment thresholds are consistent with those selected in the absence of any restrictions on department 2 (i.e. no further distortion is introduced).

Part III

Inducements and Conclusions

7 Inducements

The basic model we have developed describes the behaviour of the departments in equilibrium, and compares this with the behaviour that would be desirable from the perspective of a social planner. The model shows that in general, where policy investments are complementary, a failure to take into account the impact of their investment timing decisions on the benefits generated by the other department's investment results in departments investing too late from the planner's perspective.

In order for our analysis to usefully contribute to public administration decisions in practice, we next consider the types of mechanisms that might be used by a planner to induce an outcome that improves on the equilibrium decisions made by the departments in the absence of intervention. While the model could be modified in any number of ways to evaluate different hypothetical public management interventions that respond to the issue we have identified, here we consider four different types of inducement that have some relevance to current or recent issues and reforms in New Zealand public administration:

1. **Structural change** — commonly referred to as ‘Machinery of Government’ change in the New Zealand context, the first and historically the most common approach to influencing behaviour of public institutions is through structural change to merge or separate departmental functions;
2. **Internalisation of cross-benefits** — more recently, public management practices have shifted towards softer (non-structural) mechanisms aimed at incentivising departments to internalise the impacts of their decisions not only within their own sphere of responsibility but across other portfolios;
3. **Transfers between departments** — a further possibility, though one which has less support in current public management practice in New Zealand, is manipulation of the

Treasury rules regarding departmental expenditure to allow for transfers between departments where this could result in a mutually beneficial outcome;

4. **Joint decision-making** — finally, recent innovations in the New Zealand public service’s approach to cross-agency work include models of joint accountability and decision-making for policy advice and delivery of services in areas where multiple departments have an interest.

We consider each of these possible inducement approaches in turn. Ultimately we will aim to evaluate each based on its effectiveness (i.e. ability for the planner to achieve a socially optimal outcome in the context of our model) as well as its feasibility as a solution to the problem we have identified (a more qualitative assessment of whether the political and practical realities of the public sector would allow for such a mechanism to be implemented and in what circumstances).

7.1 Structural change

In theory, a structural change to merge the two departments we consider in our basic model could address the issue that we have identified. To understand this, note that in such a scenario (where the two departmental functions were merged such that both investment decisions were considered by a single entity) the entity resulting from the merger would care about the benefits and costs of both policy programmes and would consequently have an objective function identical to that of the planner in our basic model. This larger department would therefore make investment decisions consistent with the social optimum. In practice, the use of structural change to align incentives across portfolios has also been observed in specific instances — for example through the creation of ‘super ministries’ such as the Ministry of Social Development and the Ministry of Business, Innovation and Employment, which at their creation each brought together a number of different functions that had previously been housed within separate departments.

However, there are limitations on the extent to which such changes can be used to address coordination problems across portfolios. If followed to its logical conclusion, this approach would result in merging all departmental functions into a single entity in order to eliminate any potential coordination problems. This would be politically and practically infeasible in New Zealand — and likely also in most other systems of Government internationally — as our

system of Government involves multiple ministers (often from multiple political parties since the introduction of Mixed Member Proportional Representation) who each require departmental chief executives to devote a certain amount of time and focus to engaging with them and delivering the priorities in their respective portfolios. Such increases in agency span of control are also likely to have costs (beyond the transaction costs of the initial structural change) in terms of an agency’s ability to ensure that it maintains the expertise and focus necessary to deliver on each of its individual functions — which in many cases can be quite technical and require very different skill-sets and approaches.³⁰

Consequently, structural change to merge departments is unlikely to be an appropriate solution in all cases, and in many cases the costs of doing so will exceed the benefits of any improvement in the coordination of investment decisions. There will always be some number of departments in existence and consideration of how their investment decisions might be improved from a welfare perspective (through means other than structural integration of functions) remains relevant. However, the fact that merging departments results in the first-best solution in the context of our model (i.e. the decision that the planner would have selected) does have clear implications for public management decisions — namely that any increase in agency focus and specialisation that may be afforded by agencification (a proliferation of agencies to deliver specific functions or address new problems, one of the fundamental tenets of New Zealand’s approach to public administration since the 1980s) must be weighed against the greater coordination of policy decisions that may be afforded by keeping like-functions within a smaller number of public agencies.³¹

7.2 Internalisation of cross-benefits

Assuming that the two departments in question cannot be merged for one of the reasons outlined in the preceding section, one possible mechanism available to a planner is to take steps to

³⁰For example, the scenario we offered to illustrate our basic model set-up involved a set of complementary policy investments between the Ministries of Education and Health. Despite there being very plausible examples of positive externalities between the policies of these two departments, there is very little overlap between the education and health sectors in regards to the technical knowledge that is required to deliver on their particular functions, and merging the two departments would bring together two of the largest areas of public investment in New Zealand. This would likely be politically complicated, and would bring together functions that in many other cases would not overlap to the same extent as in the specific example we offer, and would no longer benefit from the increased focus of dedicated Health and Education agencies.

³¹The notion that high levels of agencification result in decreased public sector performance has some empirical support in the public administration literature — see Overman and Van Thiel (2016).

incentivise the departments to internalise the effects of their respective investments on the other department's policy programme. That is, inducements may be available which alter the incentives of the departments, modelled as a change in their respective objective functions and consequently their decision-making. In practice, a variety of such mechanisms have been introduced recently or are being introduced through ongoing State sector reform programmes which aim to achieve this. For example, the Better Public Services Results programme beginning in 2012 used the setting of and reporting on common targets to drive cross-agency investment in a shared goal, and a 'State Services Leadership Team' has been in operation for a number of years with the goal of fostering a shared public service identity across departments at the chief executive level (formalised in the new Public Service Act 2020 as the 'Public Service Leadership Team'). The new Public Service Act also sets out for the first time a shared purpose and values for the public service in legislation. Each of these interventions is aimed at shifting the focus of public agencies from achieving objectives within their individual portfolio area, to achieving objectives that benefit the public more broadly (regardless of which portfolio these fall under).

Recall that in our basic model both the decisions of departments in equilibrium and the decisions of the planner varied depending on the parameters of the model — that is, in some cases it may be preferable for departments to invest sequentially, while in others it may be preferable for investment to occur simultaneously. First, consider the case where $\frac{I_n}{B_n^s} < \frac{I_m}{B_m^c + (B_n^c - B_n^s)}$ for either $n = 1, m = 2$ or $n = 2, m = 1$. That is, there are values of X for which it is socially optimal for one of the departments to invest in its policy programme without simultaneous investment by the other department. Without loss of generality, assume that it is optimal for department 1 to invest first (i.e. the stated condition holds for $n = 1$). As outlined in the section on the social planner's problem in our basic model, for these parameter values the leader department (department 1) chooses to invest at the socially optimal threshold, $\hat{X}_1^L = \hat{X}_1^{L*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1}{B_1^s}$, while the follower department (department 2) invests later than would be socially optimal at $\hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^s} > \hat{X}_2^{F*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c + (B_1^c - B_1^s)}$. Consequently, in intervening the social planner would aim to alter the incentives of the follower to take into account the impact of its investment on the leader's policy programme, such that the objective function for the follower

is:

$$F_2(X_t) = \begin{cases} \left(\frac{(B_2^c + \theta(B_1^c - B_1^s)) \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{(B_2^c + \theta(B_1^c - B_1^s)) X_t}{\rho - \mu} - I_2 & \text{if } \hat{X}_2^F \leq X_t \end{cases} \quad (35)$$

where θ captures the degree to which the impact on department 1's investment is internalised. Given this objective function, the follower's investment threshold is $\hat{X}_2^F = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c + \theta(B_1^c - B_1^s)}$. Consequently, if implementing the mechanism is costless then $\theta = 1$ will implement the first-best social optimum.

Next, consider the case where $\frac{I_n}{B_n^s} > \frac{I_m}{B_m^c + (B_n^c - B_n^s)}$ for both $n = 1, m = 2$ and $n = 2, m = 1$, such that it is socially optimal for the departments to invest simultaneously when $\hat{X}_1^* = \hat{X}_2^* = \hat{X}^{E*} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1 + I_2}{B_1^c + B_2^c}$. Without loss of generality (given the symmetry of the model), assume that $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}$ such that department 1 is willing to engage in joint investment before department 2. As is outlined in the section on the social planner's problem in our basic model, in this case department 1 would be willing to invest simultaneously, and in some cases alone, before department 2 is willing to invest; however in any of these equilibria both departments invest later than is socially optimal. Given our assumption that $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}$, it must be the case that $\frac{I_1}{B_1^c} < \frac{I_1 + I_2}{B_1^c + B_2^c} < \frac{I_2}{B_2^c}$, meaning that department 1 would be willing to invest simultaneously at the socially optimal threshold and it is department 2's delay that causes the departments to behave sub-optimally from a welfare perspective. Consequently, in intervening the social planner would again aim to alter the incentives of the follower to take into account the total impact of welfare of its investment decision, such that the objective function for the follower is:

$$F_2(X_t) = \begin{cases} \left(\frac{(B_2^c + \theta B_1^c) \hat{X}_2^F}{\rho - \mu} - I_2 - \theta I_1 \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{(B_2^c + \theta B_1^c) X_t}{\rho - \mu} - I_2 - \theta I_1 & \text{if } \hat{X}_2^F \leq X_t \end{cases} \quad (36)$$

where θ captures the degree to which the follower department internalises the total welfare impact of its investment decision. Given this objective function, the follower's investment threshold is $\hat{X}_2^F = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{\theta I_1 + I_2}{\theta B_1^c + B_2^c}$. Consequently, if implementing the mechanism is costless then again $\theta = 1$ will implement the first-best social optimum.

The simple mechanism discussed above is designed to implement the first-best social optimum in the absence of any implementation or transaction cost. In reality, however, it is likely that mechanisms of this nature will have either a direct cost of implementation (for example, the time spent by central agencies or senior ministers developing and communicating revised expectations for departments and chief executives) or indirect costs resulting from tradeoffs within the department attempting to internalise the impact of its decisions across other portfolios (for example, reduced efficiency in other areas of the department's operation due to bounded rationality or agency costs). As there are a variety of possible consequences of such a mechanism, there will inevitably be a variety of ways in which costs could be introduced to the model. As an illustrative example, we consider a case where the department being asked to internalise the impact of its actions across portfolios (the follower) has bounded rationality, and expanding focus to a broader set of objectives reduces the department's focus on cost minimisation. This is modelled by the department's investment costs being an increasing function of θ .

Consider first the case where it is optimal for the leader to invest before the follower. In this case, the planner wants the follower to invest after the leader, but just not as late as the follower invests in equilibrium. As in the simple costless example above, we let the planner pick a level of θ (between 0 and 1) that describes the degree to which the mechanism results in the follower internalising the benefits of their investment to the leader's policy programme. However, this time we assume that the investment costs associated with the follower's project are also impacted by the mechanism and become $I_2(1 + \delta\theta)$, where δ is an exogenous parameter that determines the magnitude of the additional cost of the mechanism resulting from inefficiencies (i.e. the cost of distraction from a department's specific area of responsibility). Given these assumptions, the follower's objective function becomes:

$$F_2(X_t) = \begin{cases} \left(\frac{(B_2^c + \theta(B_1^c - B_1^s)) \hat{X}_2^F}{\rho - \mu} - I_2(1 + \delta\theta) \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{(B_2^c + \theta(B_1^c - B_1^s)) X_t}{\rho - \mu} - I_2(1 + \delta\theta) & \text{if } \hat{X}_2^F \leq X_t \end{cases} \quad (37)$$

The follower's equilibrium investment threshold in this case is $\hat{X}_2^F = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2(1 + \delta\theta)}{B_2^c + \theta(B_1^c - B_1^s)}$. Note that, unlike the case where the mechanism is costless, setting $\theta = 1$ will no longer implement

the first-best social optimum $\hat{X}_2^{F*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^s + (B_1^c - B_1^s)}$.

We now set up the planner's problem and solve for the optimal θ . From the basic model, the welfare function is:

$$\begin{aligned} W(\theta) &= L_1(\hat{X}_1^L) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} + F_2(\hat{X}_1^L) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} \\ &= \left(\frac{B_1^s \hat{X}_1^L}{\rho - \mu} - I_1 \right) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} + \left(\frac{B_2^s \hat{X}_2^F(\theta)}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F(\theta)}{\rho - \mu} - I_2(1 + \delta\theta) \right) \left(\frac{X_t}{\hat{X}_2^F(\theta)} \right)^{\beta_1} \end{aligned} \quad (38)$$

Note that the effects of θ on the cost function of the follower are included directly in the above objective function, but θ does not appear directly in relation to the investment benefits. Intuitively, the term $\theta(B_1^c - B_1^s)$ in the follower's objective function reflects the extent to which that department internalises benefits accrued outside its portfolio, but does not have any real impact on the benefits realised from the planner's perspective (other than indirectly through the follower's investment timing decision, $\hat{X}_2^F(\theta)$). In contrast, the impact of the mechanism on the follower's investment cost $I_2(1 + \delta\theta)$ does have a real effect on investment costs incurred, so is reflected directly in the welfare function. Taking a first-order condition of the planner's objective function with respect to θ yields:

$$\left(\frac{X_t}{\hat{X}_2^F(\theta)} \right)^{\beta_1} \left[\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta} \left((1 - \beta_1) \frac{B_2^c + (B_1^c - B_1^s)}{\rho - \mu} + \frac{\beta_1 I_2(1 + \delta\theta)}{\hat{X}_2^F(\theta)} \right) - \beta_1 I_2 \delta \right] = 0 \quad (39)$$

where $\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \left(\frac{I_2 \delta}{B_2^s + \theta(B_1^c - B_1^s)} - \frac{(B_1^c - B_1^s) I_2(1 + \delta\theta)}{(B_2^s + \theta(B_1^c - B_1^s))^2} \right)$ and $\hat{X}_2^F(\theta) = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2(1 + \delta\theta)}{B_2^s + \theta(B_1^c - B_1^s)}$.

By dividing by $\left(\frac{X_t}{\hat{X}_2^F(\theta)} \right)^{\beta_1} \beta_1 I_2(1 + \delta\theta)$ this condition can be restated as:

$$-\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta} \left(\frac{\beta_1 - 1}{\beta_1(\rho - \mu)} \cdot \frac{B_2^c + (B_1^c - B_1^s)}{I_2(1 + \delta\theta)} - \frac{1}{\hat{X}_2^F(\theta)} \right) - \frac{I_2 \delta}{I_2(1 + \delta\theta)} = 0 \quad (40)$$

The first term of this expression can loosely be interpreted as the marginal benefit of the mechanism; $\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta}$ captures the sensitivity of investment timing to an increase in θ and the bracketed expression is a measure of the gap between the equilibrium investment timing and optimal timing (adjusted for the cost distortion introduced), meaning that this term captures the benefit

gained via the impact of θ on the follower's investment timing. Likewise, the second term of the expression can be interpreted as the marginal cost of the mechanism; this is just the direct marginal cost of an increase in θ (i.e. $I_2\delta$) as a proportion of the total cost of the follower's investment. Note also that:

1. The expression in the brackets is equal to 0 when $\theta = 1$. As the term $\frac{I_2\delta}{I_2(1+\delta\theta)}$ is positive for positive values of δ (i.e. when there is a cost associated with the mechanism), the first-order condition cannot be satisfied when $\theta = 1$. This confirms that the planner will not select $\theta = 1$ (i.e. the optimal θ in the costless case) in the case where implementing the mechanism has a cost.
2. The expression in the brackets is positive for $0 < \theta < 1$, meaning that the first-order condition can be satisfied only when $\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta} < 0$. Intuitively, we will have an interior solution when the mechanism reduces the followers investment timing $\hat{X}_2^F(\theta)$, as this brings it closer to the socially optimal timing. We can show that $\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta} < 0$ only when $B_1^c - B_1^s > \delta B_2^c$ — that is, when the marginal effect of the mechanism on the benefits internalised by the follower is greater than the direct marginal cost of the mechanism.
3. When $\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta} > 0$ the first-order condition cannot be satisfied for a positive θ and we therefore cannot have an interior solution. Intuitively, when the mechanism increases $\hat{X}_2^F(\theta)$ rather than decreasing it, the planner will not utilise the mechanism as doing so would not bring the follower's investment timing closer to the optimal level.

Evaluating the above first-order condition yields two possible solutions for the optimal θ^* , which are as follows:

$$\theta^* = \frac{-B_1^c + B_1^s - B_2^c\delta - \phi}{2\delta(B_1^c - B_1^s)}, \frac{-B_1^c + B_1^s - B_2^c\delta + \phi}{2\delta(B_1^c - B_1^s)} \quad (41)$$

where:

$$\phi = \sqrt{B_1^{c2}(4\delta + 1) - 2B_1^c(4B_1^s\delta + B_1^s + B_2^c\delta(2\delta - 1)) + B_1^{s2}(4\delta + 1) + 2B_1^sB_2^c\delta(2\delta - 1) - 3B_2^{c2}\delta^2} \quad (42)$$

Evaluating these two solutions numerically for plausible sets of parameter values indicates that

the second solution above may fall in the range $0 < \theta < 1$ required for an interior solution, provided that the direct cost of the mechanism (represented by δ) is not too high. This is illustrated in Figure 6, which graphs the ‘marginal benefit’ $-\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta} \left(\frac{\beta_1 - 1}{\beta_1(\rho - \mu)} \cdot \frac{B_2^c + (B_1^c - B_1^s)}{I_2(1 + \delta\theta)} - \frac{1}{\hat{X}_2^F(\theta)} \right)$ and ‘marginal cost’ $\frac{I_2\delta}{I_2(1 + \delta\theta)}$ of the mechanism, for three different values of δ . This series of examples highlights the (intuitive) result that the mechanism becomes less attractive as the cost of implementing it increases; where δ is low (e.g. $\delta = 0.01$, meaning fully implementing the first-best investment timing would result in only a 1 percent increase in investment costs for the follower) a relatively high value of θ is selected, while when δ is relatively high (e.g. $\delta = 0.05$) the chosen value of θ is close to zero. When δ becomes high enough that marginal cost exceeds marginal benefit for all allowable values of the mechanism $0 < \theta < 1$ (e.g. when $\delta = 0.1$) we have a corner solution of $\theta^* = 0$ and the mechanism is not employed at all. Consequently, as we would have anticipated, the effectiveness of a mechanism of this nature depends largely on the magnitude of any unintended costs resulting from its use.

We can conduct a similar exercise in the case where it is optimal for both departments to invest simultaneously. In this case, the follower’s objective function taking into account the mechanism and its impact on investment costs $I_2\delta\theta$ is:

$$F_2(X_t) = \begin{cases} \left(\frac{(B_2^c + \theta B_1^c) \hat{X}_2^F}{\rho - \mu} - (I_2(1 + \delta\theta) + \theta I_1) \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{(B_2^c + \theta B_1^c) X_t}{\rho - \mu} - (I_2(1 + \delta\theta) + \theta I_1) & \text{if } \hat{X}_2^F \leq X_t \end{cases} \quad (43)$$

In this case, the follower’s equilibrium investment threshold is $\hat{X}_2^F = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{\theta I_1 + I_2(1 + \delta\theta)}{\theta B_1^c + B_2^c}$, which in the simultaneous investment case is also the leader’s investment threshold (i.e. $\hat{X}_1^L = \hat{X}_2^F = \hat{X}$). From the basic model, the planner’s objective function in the case where simultaneous investment is optimal is:

$$W(\theta) = \left(\frac{(B_1^c + B_2^c) \hat{X}(\theta)}{\rho - \mu} - (I_1 + I_2(1 + \delta\theta)) \right) \left(\frac{X_t}{\hat{X}(\theta)} \right)^{\beta_1} \quad (44)$$

Figure 6: The above figures plot the marginal benefit and marginal cost of the mechanism as functions of θ , for three different values of δ (as labelled). Other parameter values are as used in previous numerical examples: $I_1 = 100$, $I_2 = 200$, $B_1^s = B_2^s = 0.7$, $B_1^c = B_2^c = 1$, $\mu = 0.02$, $\sigma = 0.05$ and $\rho = 0.05$. Where there is an interior solution, the optimal value of θ is given by the horizontal coordinate of the intersection of the two curves. In the first two cases these are, respectively, $\theta^* = 0.821$ and $\theta^* = 0.236$. In the final case, where $\delta = 0.1$, there is no interior solution as marginal cost exceeds marginal benefit for all allowable values of the mechanism $0 < \theta < 1$. Consequently in this case we have a corner solution $\theta^* = 0$.

Taking a first-order condition with respect to θ yields:

$$\left(\frac{X_t}{\hat{X}(\theta)} \right)^{\beta_1} \left[\frac{\partial \hat{X}(\theta)}{\partial \theta} \left((1 - \beta_1) \frac{B_1^c + B_2^c}{\rho - \mu} + \frac{\beta_1(I_1 + I_2(1 + \delta\theta))}{\hat{X}(\theta)} \right) - \beta_1 I_2 \delta \right] = 0 \quad (45)$$

where $\frac{\partial \hat{X}(\theta)}{\partial \theta} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \left(\frac{I_1 + \delta I_2}{\theta B_1^c + B_2^c} - \frac{B_1^c(\theta I_1 + I_2(1 + \delta\theta))}{(\theta B_1^c + B_2^c)^2} \right)$ and $\hat{X} = \hat{X}_2^F = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{\theta I_1 + I_2(1 + \delta\theta)}{\theta B_1^c + B_2^c}$. By dividing by $\left(\frac{X_t}{\hat{X}(\theta)} \right)^{\beta_1} \beta_1(I_1 + I_2(1 + \delta\theta))$ this condition can be restated as:

$$-\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta} \left(\frac{\beta_1 - 1}{\beta_1(\rho - \mu)} \cdot \frac{B_1^c + B_2^c}{I_1 + I_2(1 + \delta\theta)} - \frac{1}{\hat{X}_2^F(\theta)} \right) - \frac{\delta I_2}{I_1 + I_2(1 + \delta\theta)} = 0 \quad (46)$$

The same considerations apply in this case as for the discussion of the corresponding condition for the sequential investment case above; that is, the first and second terms of the expression can be loosely interpreted as the marginal benefit and marginal cost of the mechanism (respectively), and provided that $\delta > 0$ the first-order condition will not be satisfied at $\theta = 1$ and can only be satisfied for $0 < \theta < 1$ (i.e. an interior solution) when $\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta} < 0$ (which in this case requires $B_1^c I_2 > B_2^c(I_1 + \delta I_2)$).

In this case it turns out that the first-order condition has only one real solution for the optimal θ^* . A closed-form expression for the solution is obtainable, but it is a complicated and lengthy function of the model's parameters. For this reason we do not replicate the expression here. However numerical evaluation of the expression indicates that, as was the case for one of our solutions in the sequential case, it may fall in the range $0 < \theta < 1$ required for an interior solution provided that the direct cost of the mechanism (represented by δ) is not too high. This is illustrated in Figure 7, which graphs the 'marginal benefit' $-\frac{\partial \hat{X}_2^F(\theta)}{\partial \theta} \left(\frac{\beta_1 - 1}{\beta_1(\rho - \mu)} \cdot \frac{B_1^c + B_2^c}{I_1 + I_2(1 + \delta\theta)} - \frac{1}{\hat{X}_2^F(\theta)} \right)$ and 'marginal cost' $\frac{\delta I_2}{I_1 + I_2(1 + \delta\theta)}$ of the mechanism for three different values of δ , and has an equivalent interpretation to Figure 6.

For completeness, there is one final possibility to consider. By allowing selection of any $0 < \theta < 1$, the above analysis of the case where the first-best social optimum involves simultaneous investment implicitly assumed that departments would also invest simultaneously in equilibrium in the absence of any inducement (albeit later than would be socially optimal). However, we know from our earlier comparison of equilibrium and socially optimal behaviours that for

Figure 7: The above figures plot the marginal benefit and marginal cost of the mechanism as functions of θ , for three different values of δ (as labelled). B_1^s and B_2^s are set equal to zero such that simultaneous investment is both optimal and occurs in equilibrium. Other parameter values are as used in previous numerical examples: $I_1 = 100$, $I_2 = 200$, $B_1^c = B_2^c = 1$, $\mu = 0.02$, $\sigma = 0.05$ and $\rho = 0.05$. The optimal value of θ in each of the three cases are, respectively, $\theta^* = 0.813$, $\theta^* = 0.446$ and $\theta^* = 0.210$ (given by the horizontal coordinate of the intersection of the two curves).

some parameter settings it is possible for departments to invest sequentially in equilibrium where it would be optimal for them to invest simultaneously. In such a scenario, numerical evaluation of both the above cases is necessary to determine the optimal setting for θ . If $\hat{X}(\theta^*) = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{\theta^* I_1 + I_2(1+\delta\theta^*)}{\theta^* B_1^c + B_2^c}$ for the simultaneous case above is less than $\frac{\beta_1(\rho-\mu)}{\beta_1-1} \frac{I_1}{B_1^a}$ (i.e. the leader's threshold for investing alone), then the planner will select the optimal θ^* derived in the simultaneous case (as doing so will induce simultaneous investment as desired). However, if $\hat{X}(\theta^*)$ from the simultaneous case is greater than $\frac{\beta_1(\rho-\mu)}{\beta_1-1} \frac{I_1}{B_1^a}$ then a numerical comparison is required between: welfare achieved in the sequential case above; and welfare achieved in the simultaneous case where θ is set just high enough to induce simultaneous investment (i.e. $W(\bar{\theta})$, where $\hat{X}(\bar{\theta}) = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \frac{I_1}{B_1^a}$).

As indicated earlier, the addition of 'distraction' costs in the model above is only one example of the types of tradeoff that could be relevant when determining whether and how to implement a mechanism of this nature. In reality the costs of implementing such a mechanism might take other forms — for example, as a direct implementation cost to the planner rather than an additional distortion of the departments' objective functions. Mechanisms of this nature are also often likely to be aimed at adjusting the incentives on departments more broadly rather than in relation to a particular investment decision, in which case it would be appropriate to model the impact of the inducement across a larger portfolio of investments (rather than a single case as we have done here). While more detailed modelling could be undertaken to analyse these alternative approaches and extensions as part of further research, this relatively simple example serves to demonstrate how a real-options model can be used to illustrate and analyse the tradeoffs that may be associated with an inducement that adjusts the incentives on departments to better recognise the effects of their decisions across portfolios.

7.3 Transfers between departments

Next we consider a mechanism that facilitates transfers between the two departments, where these would be mutually beneficial (i.e. permitted at the discretion of the departments, requiring agreement by both). The public finance system in New Zealand does allow transfers of sorts from a legislative perspective, though these are not widely facilitated in practice. The norm for government expenditure is that funds are appropriated for a specific purpose and adminis-

tered by the department which will be using them (either directly or to procure services from non-government organisations). Recent changes to the Public Finance Act permit additional flexibility in the use of appropriations by other departments, though the new provisions are not widely used in practice and have not been actively promoted by the New Zealand Treasury.³² Departments attempting to use the provisions to fund investment with an inter-temporal element (i.e. where investment will be incurred in a subsequent budget cycle or is staged) may also face commitment problems and/or relatively high transaction costs, as the chief executive of the administering department retains ultimate authority over use of an appropriation (which can be withdrawn at any time and cannot be altered via agreement between departments) unless explicit and formal approval is sought from the appropriation minister for a second department to be given authority to use the appropriation.

We consider a simple mechanism where a transfer T may be made between departments at the time of investment, but investment timing decisions remain at the discretion of the department making the investment. We assume (without loss of generality) that $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}$ such that department 1 has a lower threshold for investment in the basic model, meaning that any potential transfer will be made from department 1 to department 2. To simplify the analysis we focus on the specific case where $B_1^s = B_2^s = 0$ such that there is no benefit to investment without the other department having invested also, meaning that in equilibrium investment will occur simultaneously once X_t exceeds the investment thresholds of both departments.³³

Suppose that department 1 (the leader) may offer a transfer to department 2 (the follower), and department 2 determines its investment timing $\hat{X}_2^F(T)$ on the basis of this transfer. Note that in this scenario we implicitly assume that the investment timing decision itself is non-contractible; while department 1 may influence department 2's decision via the transfer, the departments may not agree on an investment timing in advance. This is intended to reflect the inability of departments to enter into legally binding agreements due to being administrative units of the same legal entity (the Crown), which when combined with the strong legislative decision rights of chief executives over their respective departments can result in commitment

³²These provisions distinguish between the administration and use of appropriations, and can be found at Section 7C of the Public Finance Act 1989. The provisions were added through a 2013 Public Finance Amendment Act.

³³As discussed in the basic model set-up, this case can be interpreted as a pure collaborative investment and is of particular interest from a policy perspective, meaning that results will remain of interest even under these more restrictive assumptions.

problems when it comes to collaborative investment between departments.³⁴ This set-up also requires department 1 to be confident when making the transfer that department 2 will use the funds for the investment in question and not for some other purpose; this assumption is consistent with restrictions on use of funding in New Zealand's public finance system, where appropriated funds can only legally be used for activities that fall within the scope of the relevant appropriation.

In this case, we can solve for an equilibrium transfer value using simple backwards induction. As the transfer T effectively amounts to a reduction in department 2's investment costs at the time of investment, the value of department 2's policy investment option becomes:

$$F_2(X; T) = \begin{cases} \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - (I_2 - T) \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{B_2^c X_t}{\rho - \mu} - (I_2 - T) & \text{if } \hat{X}_2^F \leq X_t \end{cases} \quad (47)$$

making department 2's investment threshold in this case:

$$\hat{X}_2^F(T) = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2 - T}{B_2^c} \quad (48)$$

Given the assumptions $B_1^s = B_2^s = 0$ and $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}$ made above, in equilibrium department 1 will also invest when X_t reaches department 2's investment threshold (i.e. $\hat{X}_2^L = \hat{X}_2^F(T)$). By substituting our expression $\hat{X}_2^F(T)$ into department 1's objective function in place of $\hat{X}_1^L$ and recognising the additional transfer cost T incurred by department 1 at the time of investment, the leader's objective function can be expressed as:

$$\begin{aligned} L_1(X_t) &= \left(\frac{B_1^c \hat{X}_2^F(T)}{\rho - \mu} - (I_1 + T) \right) \left(\frac{X_t}{\hat{X}_2^F(T)} \right)^{\beta_1} \\ &= \left(\frac{B_1^c}{\rho - \mu} \left(\hat{X}_2^F(T) \right)^{1-\beta_1} - (I_1 + T) \left(\hat{X}_2^F(T) \right)^{-\beta_1} \right) (X_t)^{\beta_1} \\ &= \left(\frac{B_1^c}{\rho - \mu} \left(\frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2 - T}{B_2^c} \right)^{1-\beta_1} - (I_1 + T) \left(\frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2 - T}{B_2^c} \right)^{-\beta_1} \right) (X_t)^{\beta_1} \end{aligned} \quad (49)$$

³⁴These features of the public administration system are discussed further in the subsequent section on models of joint decision-making, where we relax our assumptions regarding contractibility and allow departments to bargain over the timing of their investments.

Taking a first-order condition with respect to T yields:

$$\left(\frac{X_t}{\hat{X}_2^F(T)} \right)^{\beta_1} \left[\frac{-\beta_1(\rho - \mu)}{(\beta_1 - 1)B_2^c} \left(\frac{(1 - \beta_1)B_1^c}{\rho - \mu} + \frac{\beta_1(I_1 + T)}{\hat{X}_2^F(T)} \right) - 1 \right] = 0 \quad (50)$$

Substituting in our expression for $\hat{X}_2^F(T)$ and solving for the equilibrium transfer yields:

$$T = I_2 - \frac{I_1 + I_2}{\frac{B_1^c}{B_2^c} + \frac{\beta_1 - 1}{\beta_1}} \quad (51)$$

In order to evaluate this equilibrium transfer from a welfare perspective, we must determine the efficient value of the transfer T^* that will implement the socially optimal investment timing. Recall from our analysis of the basic model that the socially optimal investment timing in the case where departments invest simultaneously is given by the expression:

$$\hat{X}^{E*} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1 + I_2}{B_1^c + B_2^c} \quad (52)$$

The optimal transfer value T^* is that which achieves this socially efficient investment threshold in equilibrium. To find this we equate the above expressions for $\hat{X}_2^F(T)$ and $\hat{X}^{E*}$ and solve for the optimal T^* , which yields:

$$T^* = \frac{B_1^c I_2 - B_2^c I_1}{B_1^c + B_2^c} \quad (53)$$

The value of the equilibrium transfer T relative to the efficient T^* — an indicator of effectiveness of the mechanism at correcting the distortion in the follower's investment timing decision — appears to be dependent on the parameters of the model. Specifically, it depends on the value of β_1 and on the relative values of the investment benefits and costs of the departments. By combining and manipulating the two expressions for T and T^* , we are able to obtain the following expression for the gap between the equilibrium and efficient transfer:

$$T^* - T = (I_1 + I_2) \left(\frac{1}{\frac{B_1^c}{B_2^c} + \frac{\beta_1 - 1}{\beta_1}} - \frac{1}{\frac{B_1^c}{B_2^c} + 1} \right) \quad (54)$$

This expression allows us to make some high-level observations about how the various param-

eters are likely to affect the mechanism's effectiveness. For example, it is simple to verify that as β_1 approaches ∞ , the above expression tends to 0 — that is, the equilibrium T approaches the efficient T^* (with $T < T^*$ for all finite levels of β_1). While such extreme levels of β_1 are not possible in reality, this result indicates that higher levels of β_1 are likely to result in the mechanism being more effective from the perspective of the planner. Likewise, the right hand (bracketed) term of the expression is decreasing in $\frac{B_1^c}{B_2^c}$, suggesting that an increase in B_1^c or a decrease in B_2^c will increase the mechanism's effectiveness.

However, while this analysis of the expressions for T and T^* may give us some insight into how well the mechanism performs in different circumstances, the approach ignores the wider effects that varying these parameters may have on the model — including the direct effects that these variables have on welfare. As it is more difficult to determine the overall effects of changes in these variables analytically, a numerical evaluation of the model for different parameter settings is necessary to further understand the effectiveness of the mechanism under different parameter settings.

In total there are seven underlying parameters whose effect we must consider. These include the investment costs associated with each department's policy investment (I_1 and I_2); the respective benefits resulting from investment (B_1^c and B_2^c); and the underlying determinants of β_1 , which is a function of three variables (some of which also appear directly in the objective functions of the departments and the planner). These are the departments' discount rate ρ , the drift of the state variable μ , and the volatility of the state variable σ .

We take a consistent approach across each of these parameters to analysing the effect of a change on the effectiveness of the transfer mechanism. This involves:

1. Calibrating the model using a realistic set of baseline parameter values (as described in our model set-up). These baseline values are $B_1^c = B_2^c = 1$, $I_1 = 100$, $I_2 = 200$, $\rho = 0.06$, $\mu = 0.02$ and $\sigma = 0.05$. Without loss of generality, the initial value for the state variable is set at $X_t = 1$;
2. Varying the parameters, either individually or in combinations with an intuitive economic interpretation, and recording graphically the effects of these changes on: transfer value, investment timing and welfare;

3. Repeating this analysis for three relevant cases: equilibrium without intervention (i.e. the base model), equilibrium with the transfer mechanism (as described above), and the social optimum; and
4. Using these results to determine how changes in the model's parameters impact the proportion of the welfare 'gap' between the equilibrium and socially optimal cases which is recovered by allowing for transfers between departments (discussed further below).

We begin by analysing the effects of a change in the underlying determinants of β_1 : ρ , μ and σ . The effects of such changes are shown in figures 8 to 10, respectively. The range of values of ρ , μ and σ considered are as discussed in the earlier section of the thesis on plausible parameter values (see the basic model set-up). Before we interpret these figures, it is important to note that β_1 is increasing in ρ , and is decreasing in both μ and σ ; that is, a relatively high value of ρ and low values of μ and σ will result in a higher value of β_1 , and vice versa.

The first graph in each figure shows the effect of a change in the relevant parameter on the equilibrium transfer T and the efficient transfer T^* . These graphs show that T is increasing in ρ and decreasing in μ and σ , and T^* is independent of all three parameters. This is consistent with our analysis of the expression for $T^* - T$ above, and exactly as we would expect given that the equilibrium transfer T is increasing in β_1 and the three parameters do not appear in our expression for T^* either directly or indirectly.

The second graph in each figure shows the effect of a change in the relevant parameters on investment timing for: equilibrium with the transfer ($\hat{X}_2^F(T)$), equilibrium of the basic model without the transfer ($\hat{X}_2^F$) and the socially efficient case ($\hat{X}^{E*}$). As expected, we can see that when the transfer is larger (i.e. when μ and σ are low or when ρ is high) the equilibrium investment timing moves closer to the efficient level.

Most importantly, the third and fourth graphs in each figure illustrate the effect of a change in the parameters on welfare. The third graph plots the welfare generated with the transfer mechanism in place ($W(\hat{X}_2^F(T))$), welfare without the transfer mechanism ($W(\hat{X}_2^F)$) and welfare when the socially optimal investment timing is selected ($W(\hat{X}^{E*})$). The fourth and final graph plots the proportion of the welfare 'gap' between the equilibrium and socially optimal cases that

is recovered by allowing for transfers between departments, which is measured by:

$$P = \frac{W(\hat{X}_2^F(T)) - W(\hat{X}_2^F)}{W(\hat{X}^{E*}) - W(\hat{X}_2^F)} \quad (55)$$

We can see that when the equilibrium transfer is larger (again, when μ and σ are low or when ρ is high) a greater proportion of the welfare gap is recovered, indicating that the mechanism is more effective under these conditions.³⁵

This analysis confirms that the effectiveness of the mechanism is sensitive to changes in β_1 and its determinants; when parameters are such that β_1 is low the mechanism is relatively ineffective, and vice versa. Intuitively, a low β_1 can be indicative of a state variable that is either volatile or has a high drift, or of a low rate of time preference for the leader department. Under conditions where μ and/or σ are high the state variable is more likely to reach the follower's investment threshold sooner without intervention, meaning that there is less of an incentive for the leader to lower the follower's investment threshold by subsidising its investment cost. Under conditions when ρ is low the leader does not discount future benefit flows as heavily, and so is more likely to be willing to wait for the follower to invest rather than incentivising earlier investment via a transfer. Conversely, a high β_1 indicates a relatively static state variable or a high rate of time preference, meaning the leader will have to wait for a significant period of time for the follower to be ready to invest if it does not offer a transfer (when μ and/or σ are low) or will have a stronger preference for bringing the follower's investment forward (when ρ is high), resulting in a greater incentive for the leader to offer a transfer.

We next assess effectiveness of the mechanism under different settings for the costs and benefits of the departments' policy investments. As noted in our earlier discussion of the model's parameters, only relative values of the costs and benefits of the departments' policy investments matter — as objective and welfare functions are linearly homogeneous in these parameters and investment timing decisions do not change as the parameters are scaled by a common factor. This has the following implications for our analysis:

³⁵While it is difficult to observe from the figures showing the effects of ρ and μ due to the direct effect of these variables on welfare, it is also the case that the size of the welfare gap relative to total welfare in the efficient case (measured by $\frac{W(\hat{X}^{E*}) - W(\hat{X}_2^F)}{W(\hat{X}^{E*})}$) is larger when μ and σ are low and when ρ is high. This indicates that the conditions under which the mechanism recovers a higher proportion of the 'lost' welfare are also the conditions when the need for intervention is greater, further supporting the assertion that the mechanism is more effective for these parameter values.

Figure 8: This figure illustrates the effect of varying the departments' discount rate ρ on the effectiveness of the transfer mechanism. The first three graphs respectively plot the transfer (where relevant), investment timing and welfare under each of the following cases: equilibrium without transfer, equilibrium with transfer, and the social optimum. The final graph plots the proportion of welfare lost in equilibrium that is recovered by the mechanism. Baseline parameter values are $B_1^c = B_2^c = 1$, $I_1 = 100$, $I_2 = 200$, $\rho = 0.06$, $\mu = 0.02$ and $\sigma = 0.05$.

Figure 9: *This figure illustrates the effect of varying the drift μ of the state variable on the effectiveness of the transfer mechanism. Dependent variables and baseline parameter values are as for Figure 8.*

Figure 10: *This figure illustrates the effect of varying the volatility σ of the state variable on the effectiveness of the transfer mechanism. Dependent variables and baseline parameter values are as for Figure 8.*

- The costs and benefits of the departments' policy investments can take any values, provided that the assumption $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}$ holds (as we have assumed, without loss of generality, that parameters are such that department 1 is the investment leader). Note that this condition can be alternatively stated as $\frac{I_1}{I_2} < \frac{B_1^c}{B_2^c}$ (that is, the ratio of benefits of department 1's policy investments to department 2's exceeds the ratio of investment costs) or as $\frac{B_2^c}{I_2} < \frac{B_1^c}{I_1}$ (i.e. department 1's investment is more 'profitable' than department 2's); and
- Because we have assumed for this section that $B_1^s = B_2^s = 0$, objective functions are linearly homogeneous in and investment timing decisions are invariant to scaling by a common factor the following four parameters: B_1^c , B_2^c , I_1 and I_2 (i.e. only relative values of these parameters matter). This means that understanding how the model varies in response to changes in any three of these parameters (or combinations of the parameters) will be sufficient to fully describe the performance of the transfer mechanism under different project characteristics.

Rather than varying cost and benefit parameters individually, we vary them in ways that have a compelling economic interpretation. The three cases which we consider are as follows:

1. Holding total 'profitability' ($\frac{B_1^c + B_2^c}{I_1 + I_2}$) constant while varying the profitability of individual projects (i.e. $\frac{B_1^c}{I_1}$ and $\frac{B_2^c}{I_2}$). This is achieved by varying B_1^c and I_1 while simultaneously adjusting B_2^c and I_2 (respectively) to hold $B_1^c + B_2^c$ and $I_1 + I_2$ constant. This allows us to assess the effect of the relative profitability of the two projects on the effectiveness of the mechanism;
2. Holding the relative profitability of individual projects ($\frac{B_1^c}{B_2^c}$ and $\frac{I_1}{I_2}$) constant while varying the overall profitability of the combined projects (i.e. $\frac{B_1^c + B_2^c}{I_1 + I_2}$). This is achieved by scaling B_1^c and B_2^c by the same factor, and I_1 and I_2 by the same factor. This allows us to assess the effect of the total profitability of the combined projects on the effectiveness of the mechanism; and
3. Holding the (absolute) profitability of individual projects ($\frac{B_1^c}{I_1}$ and $\frac{B_2^c}{I_2}$) constant while varying the 'size' of the projects. This is achieved by scaling B_1^c and I_1 by the same factor, and B_2^c and I_2 by the same factor. This allows us to assess the effect of relative

scale of the projects on the effectiveness of the mechanism.

The approach to analysing the effectiveness of the mechanism under different variations of the parameters as described above is identical to the approach taken for the determinants of β_1 . That is, we compare the effects of a variation in the parameters on transfer values, investment timing and welfare across the three relevant cases (equilibrium without transfer, equilibrium with transfer, and the social optimum), and finally on the proportion of the welfare gap that is closed by the mechanism. The results of this analysis are set out in figures 11 to 13. Note that in each case we restrict values of the parameters to those that satisfy the assumption $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}$.

Figure 11 shows the results of varying $\frac{B_1^c}{I_1}$ and $\frac{B_2^c}{I_2}$ while holding $\frac{B_1^c+B_2^c}{I_1+I_2}$ constant. The left column of graphs shows the effect of varying B_1^c and B_2^c while holding $B_1^c+B_2^c$ constant. We can see that as B_1^c increases and B_2^c decreases the mechanism becomes more effective — the value of the transfer, investment timing and welfare achieved all approach the efficient level, and the proportion of the welfare gap recovered approaches 1. The right column of graphs shows the effect of varying I_1 and I_2 while holding I_1+I_2 constant. This time we can see that as I_1 increases and I_2 decreases the mechanism becomes less effective — the value of the transfer, investment timing and welfare achieved become relatively closer to the level in equilibrium without transfer than the efficient level, and the proportion of the welfare gap recovered falls to 0. Both of these cases indicate that the mechanism is more effective when there is a larger gap between the profitability of the more profitable project $\frac{B_1^c}{I_1}$ and the profitability of the less profitable project $\frac{B_2^c}{I_2}$. Note that the welfare gap is also larger under these conditions (as illustrated by the third graph in each column) meaning that as we observed when varying the determinants of β_1 the mechanism is at its most effective when the need for intervention is greatest.

Figure 12 shows the results of varying B_1^c and B_2^c by the same scale, and varying I_1 and I_2 by the same scale (i.e. holding $\frac{B_1^c}{B_2^c}$ and $\frac{I_1}{I_2}$ constant). The left column of graphs shows the effect of varying B_1^c and B_2^c by the same scale, and the right column of graphs shows the effect of varying I_1 and I_2 by the same scale. These sets of graphs clearly show that the effectiveness of the mechanism is invariant to changes in the parameters B_1^c , B_2^c , I_1 and I_2 provided that $\frac{B_1^c}{B_2^c}$ and $\frac{I_1}{I_2}$ are held constant — the equilibrium transfer is a constant proportion of the efficient transfer, and the proportion of the welfare gap recovered is also constant. This indicates that

these ratios $\frac{B_1^c}{B_2^c}$ and $\frac{I_1}{I_2}$ are key determinants of the mechanism's effectiveness.³⁶

Finally, figure 13 shows the results of varying B_1^c and I_1 by the same scale and B_2^c and I_2 by the same scale (i.e. holding $\frac{B_1^c}{I_1}$ and $\frac{B_2^c}{I_2}$ constant). The graphs show the effect of scaling B_1^c and I_1 by $0 < \lambda < 1$ and simultaneously scaling B_2^c and I_2 by $1 - \lambda$ (such that for high values of λ the more profitable project of department 1 is relatively large, and for low values of λ the less profitable project of department 2 is larger). These graphs show that the mechanism is more effective when the more profitable project is large relative to the less profitable project (i.e. when λ is high) — as λ increases, the value of the transfer, investment timing and welfare achieved all approach the efficient level, and the proportion of the welfare gap recovered approaches 1. Again we can also see that the mechanism is more effective when the need for intervention is greater (i.e. when the welfare gap is larger).

This analysis indicates that the relative benefits and costs of the departments' investments $\frac{B_1^c}{B_2^c}$ and $\frac{I_1}{I_2}$ are key determinants of the effectiveness of the transfer mechanism. When $\frac{B_1^c}{B_2^c}$ is high (i.e. when B_1^c is large relative to B_2^c) the mechanism is more effective. This can occur, for example, when the departments have comparable investment costs but department 1's investment generates a much greater benefit (i.e. is significantly more profitable), or if department 1's policy investment is of a greater scale such that both costs and benefits are higher (even if the profitabilities of the two investments do not differ significantly). Intuitively, $\frac{B_1^c}{B_2^c}$ is indicative of the benefit-cost ratio for department 1 of incentivising department 2 to move its investment threshold forward; a higher B_1^c means the value of earlier investment to department 1 is greater, while a higher B_2^c makes department 2's investment timing $\hat{X}_2^F(T)$ less sensitive to the transfer T meaning department 1 must pay more to influence investment timing. Similarly, if $\frac{I_1}{I_2}$ is low due to a relatively wide profitability gap between the departments (i.e. the two investments generate similar benefits but department 1's investment cost is significantly lower) the mechanism will be more effective.³⁷

³⁶This is consistent with our earlier analysis of the expression for $T^* - T$, which also indicated the dependence of the mechanism's effectiveness on $\frac{B_1^c}{B_2^c}$.

³⁷Note, however, that if the relative scale of department 1's investment decreases such that $\frac{I_1}{I_2}$ falls and $\frac{B_1^c}{B_2^c}$ falls by the same proportion, the effectiveness of the mechanism declines (i.e. the effect of the change in $\frac{B_1^c}{B_2^c}$ dominates the effect of the change in $\frac{I_1}{I_2}$). This indicates that relative benefits $\frac{B_1^c}{B_2^c}$ are more important to the mechanism's effectiveness than relative costs.

Figure 11: This figure illustrates the effect of varying benefits and costs of the departments' investments while holding $\frac{B_1^c + B_2^c}{I_1 + I_2}$ constant. The left-hand column shows the effect of varying B_1^c , while simultaneously varying B_2^c in order to hold $B_1^c + B_2^c$ constant. The right-hand column shows the effect of varying I_1 , while simultaneously varying I_2 to hold $I_1 + I_2$ constant. Dependent variables and baseline parameter values are as for Figure 8.

Figure 12: This figure illustrates the effect of varying benefits and costs of the departments' investments while holding $\frac{B_1^c}{B_2^c}$ and $\frac{I_1}{I_2}$ constant. The left hand column shows the effect of varying B_1^c while simultaneously varying B_2^c by the same proportion. The right hand column shows the effect of varying I_1 while simultaneously varying I_2 by the same proportion. Dependent variables and baseline parameter values are as for Figure 8.

Figure 13: This figure illustrates the effect of varying benefits and costs of the departments' investments while holding $\frac{B_1^c}{I_1}$ and $\frac{B_2^c}{I_2}$ constant. This is achieved by scaling both B_1^c and I_1 by $0 < \lambda < 1$ and simultaneously scaling B_2^c and I_2 by $1 - \lambda$. Dependent variables and baseline parameter values are as for Figure 8.

The above considerations indicate that there are a range of plausible circumstances in which allowing transfers between departments can be beneficial — particularly when the state variable is relatively stable, or when the leader’s (department 1’s) investment is significantly more profitable or of a greater scale than the follower’s (department 2’s). As the mechanism described above is cost-neutral from the planner’s perspective and results in a weak improvement in welfare, this suggests that more flexible public finance settings should be supported in practice.

7.4 Joint decision-making

The final possible set of mechanisms that we consider are those which involve a shift from individual to joint decision-making by the departments involved. The traditional model of public administration involves division of decision rights into separate departmental silos. More recently, though, attempts have been made to introduce models of joint decision-making where a policy or investment decision does not sit clearly within the remit of one department — for example, when investments generate benefits across multiple portfolios as we have modelled here.

As noted in our earlier discussion of the relevant literature, models of joint decision-making in a real-options setting have previously been investigated in the context of mergers, joint-ventures and the establishment of supply chains. Lambrecht (2004) provides one of the earlier examples of joint decision-making in a merger setting, while Banerjee et al. (2014) provides a more recent and more generalised example using a Nash bargaining approach. Like the latter of these two examples we will employ a Nash bargaining approach, as one of the more familiar approaches to modelling joint decisions outside of the real-options literature.

In the bargaining model presented in this section we will also allow for a transfer between departments, as in the simple transfer model presented in the previous section. The core difference between the approach in the previous section and the bargaining model we present here relates to the contractibility of departments’ investment timing decisions. As noted earlier, the transfer model in the previous section assumed that departments’ investment timing decisions were non-contractible between departments, which we argued to be consistent with some core features of the public administration system in New Zealand. The bargaining model we present here relaxes this assumption and allows for departments to agree on (and reliably commit to)

a time at which both departments will simultaneously invest — hence the description of this model as one of joint decision-making. The circumstances under which this alternative set of assumptions regarding the contractibility of departments’ investment timing decisions might hold in practice are discussed at the end of this section, including recent legislative changes which allow for certain decision rights to be shared rather than held by departments individually.

We set up the bargaining model as follows. First, as we did for the preceding section we assume that $B_1^s = B_2^s = 0$ such that investment would occur simultaneously in the equilibrium of the basic model. As always, we also assume without loss of generality that $\frac{I_1}{B_1^c} < \frac{I_2}{B_2^c}$ so that department 1 has the lower investment threshold for simultaneous investment. Suppose that the two departments must jointly determine the timing at which they will both invest, denoted $\hat{X}$, and suppose that the departments may agree on a transfer T through the bargaining process — that is, we allow the departments bargain over both investment timing and terms of agreement in the form of a transfer.³⁸ We will show later that the ability to agree a transfer is required for the solution to the bargaining model to be non-trivial. Finally, suppose that department 1 has bargaining weight η and department 2 has bargaining weight $1 - \eta$.

We assume that the departments bargain over $\hat{X}$ and T simultaneously at some arbitrary time X_t . Note that this departs from the approach employed by Banerjee et al. (2014), who assume that investment timing and other terms of the agreement between the parties are determined sequentially. We avoid this approach both because it does not have as clear an intuitive application in our model as in a merger setting, and because of the potential for dynamic inconsistency under sequential bargaining. Finally, we assume that if negotiations break down the departments revert to the timing decisions that they would have made in the basic model, meaning that disagreement payoffs are just those that the departments would have received at $\hat{X} = \hat{X}_2^F$ and $T = 0$.

The standard Nash bargaining approach requires that the departments select $\hat{X}$ and T to maximise the product of the surpluses that each receive as a result of bargaining (the Nash product). Given the above assumptions, the bargaining problem for the two departments can

³⁸Alternative terms of agreement are also considered in the literature; we choose the transfer T as it is both relatively simple and allows for comparison with the pure transfer mechanism (i.e. without bargaining) considered earlier.

be stated as follows:

$$\max_{\hat{X}, T} \left(\left(\frac{B_1^c \hat{X}}{\rho - \mu} - T - I_1 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} - D_1 \right)^\eta \left(\left(\frac{B_2^c \hat{X}}{\rho - \mu} + T - I_2 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} - D_2 \right)^{1-\eta} \quad (56)$$

where the expressions $\left(\frac{B_1^c \hat{X}}{\rho - \mu} - T - I_1 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1}$ and $\left(\frac{B_2^c \hat{X}}{\rho - \mu} + T - I_2 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1}$ are the respective values of department 1 and department 2's policy investment options for an arbitrary investment timing $\hat{X}$ and transfer T agreed through the bargaining process, and $D_1 = \left(\frac{B_1^c \hat{X}_2^F}{\rho - \mu} - I_1 \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1}$ and $D_2 = \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1}$ are the disagreement payoffs for the two departments (i.e. the values of their respective investment options under equilibrium of the basic model).

We are able to show that the investment timing selected by the departments (i.e. as part of the $\hat{X}, T$ that solves the above bargaining problem) is the socially efficient investment timing. First, note that by isolating factors containing X_t the bargaining problem above can be expressed as:

$$\max_{\hat{X}, T} X_t^{(2\beta_1+1)} \left(\left(\frac{B_1^c \hat{X}}{\rho - \mu} - T - I_1 \right) \left(\frac{1}{\hat{X}} \right)^{\beta_1} - \left(\frac{B_1^c \hat{X}_2^F}{\rho - \mu} - I_1 \right) \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \right)^\eta \left(\left(\frac{B_2^c \hat{X}}{\rho - \mu} + T - I_2 \right) \left(\frac{1}{\hat{X}} \right)^{\beta_1} - \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \right)^{1-\eta} \quad (57)$$

Because the term $X_t^{(\beta_1+\eta)} X_t^{(\beta_1+(1-\eta))} = X_t^{(2\beta_1+1)}$ is independent of the choice variables $\hat{X}, T$, any solution to the bargaining problem is also a solution to the following maximisation problem that is independent of X_t :³⁹

$$\max_{\hat{X}, T} \left(\left(\frac{B_1^c \hat{X}}{\rho - \mu} - T - I_1 \right) \left(\frac{1}{\hat{X}} \right)^{\beta_1} - \left(\frac{B_1^c \hat{X}_2^F}{\rho - \mu} - I_1 \right) \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \right)^\eta \left(\left(\frac{B_2^c \hat{X}}{\rho - \mu} + T - I_2 \right) \left(\frac{1}{\hat{X}} \right)^{\beta_1} - \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \right)^{1-\eta} \quad (58)$$

³⁹Note that this implies that the solution to the problem will also be independent of X_t , and will therefore be dynamically consistent. That is, the investment threshold selected through the bargaining process does not depend on the point in time which the decision is made. This is important to ensure that the departments are able to reliably commit to an investment threshold in advance of the time at which investment occurs, and was also a feature of the solution to the basic model.

Taking a first-order condition with respect to T yields:

$$\begin{aligned}
& \left(\frac{1}{\hat{X}} \right)^{\beta_1} \left(\left(\frac{B_1^c \hat{X}}{\rho - \mu} - T - I_1 \right) \left(\frac{1}{\hat{X}} \right)^{\beta_1} - \left(\frac{B_1^c \hat{X}_2^F}{\rho - \mu} - I_1 \right) \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \right)^\eta \\
& \left(\left(\frac{B_2^c \hat{X}}{\rho - \mu} + T - I_2 \right) \left(\frac{1}{\hat{X}} \right)^{\beta_1} - \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \right)^{-\eta} \\
& \left((1 - \eta) - \frac{\eta \left(\left(\frac{1}{\hat{X}} \right)^{\beta_1} \left(\frac{B_2^c \hat{X}}{\rho - \mu} - I_2 + T \right) + \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \left(\frac{B_2^c \hat{X}_2^F}{\mu - \rho} + I_2 \right) \right)}{\left(\frac{1}{\hat{X}} \right)^{\beta_1} \left(\frac{B_1^c \hat{X}}{\rho - \mu} - I_1 - T \right) + \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \left(\frac{B_1^c \hat{X}_2^F}{\mu - \rho} + I_1 \right)} \right) = 0
\end{aligned} \tag{59}$$

Taking a first-order condition with respect to $\hat{X}$ yields:

$$\begin{aligned}
& \frac{1}{\mu - \rho} \left(\frac{1}{\hat{X}} \right)^{\beta_1 + 1} \left(\left(\frac{B_1^c \hat{X}}{\rho - \mu} - T - I_1 \right) \left(\frac{1}{\hat{X}} \right)^{\beta_1} - \left(\frac{B_1^c \hat{X}_2^F}{\rho - \mu} - I_1 \right) \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \right)^\eta \\
& \left(\left(\frac{B_2^c \hat{X}}{\rho - \mu} + T - I_2 \right) \left(\frac{1}{\hat{X}} \right)^{\beta_1} - \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \right)^{-\eta} \\
& \left[\frac{\eta \left((\beta_1 - 1) B_1^c \hat{X} - \beta_1 (I_1 + T)(\rho - \mu) \right)}{B_1^c \left(\left(\frac{1}{\hat{X}} \right)^{\beta_1 - 1} - \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1 - 1} \right) - (\rho - \mu) \left((I_1 + T) \left(\frac{1}{\hat{X}} \right)^{\beta_1} - I_1 \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} \right)} \right. \\
& \left(B_2^c \left(\left(\frac{1}{\hat{X}} \right)^{\beta_1 - 1} - \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1 - 1} \right) + (\rho - \mu) \left(I_2 \left(\frac{1}{\hat{X}_2^F} \right)^{\beta_1} - (I_2 - T) \left(\frac{1}{\hat{X}} \right)^{\beta_1} \right) \right) \\
& \left. + (1 - \eta) \left((\beta_1 - 1) B_2^c \hat{X} - \beta_1 (I_2 - T)(\rho - \mu) \right) \right] = 0
\end{aligned} \tag{60}$$

Solving the first-order condition with respect to T for the optimal transfer T as a function of $\hat{X}$ yields:

$$\begin{aligned}
T(\hat{X}) = & \left(\frac{\hat{X}_2^F (B_2^c \eta - B_1^c (1 - \eta))}{\rho - \mu} + (1 - \eta) I_1 - \eta I_2 \right) \left(\frac{\hat{X}}{\hat{X}_2^F} \right)^{\beta_1} \\
& - \left(\frac{\hat{X} (B_2^c \eta - B_1^c (1 - \eta))}{\rho - \mu} + (1 - \eta) I_1 - \eta I_2 \right)
\end{aligned} \tag{61}$$

Next we show that the first-order condition with respect to $\hat{X}$ is satisfied when $\hat{X} = \hat{X}^{E*}$

(i.e. $\hat{X}$ is set at the socially efficient level) and $T(\hat{X}) = T(\hat{X}^{E*})$ (i.e. the first-order condition with respect to T is satisfied also). We substitute our expression for $T(\hat{X})$ above into the first derivative of the objective function with respect to $\hat{X}$, and subsequently substitute in $\hat{X}_2^F = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_2}{B_2^c}$ and $\hat{X} = \hat{X}^{E*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1+I_2}{B_1^c+B_2^c}$. The resulting expression is a complicated function of the parameters of the model, and is described in **Appendix B**. This expression is the product of a number of factors, one of which is:

$$\left(\frac{\beta_1 B_2^c - B_2^c}{\beta_1 I_2 \rho - \beta_1 I_2 \mu} \right)^{\beta_1} - \left(\frac{B_2^c(I_1 + I_2)}{I_2(B_1^c + B_2^c)} \right)^{\beta_1} \left(\frac{(\beta_1 - 1)(B_1^c + B_2^c)}{\beta_1(I_1 + I_2)(\rho - \mu)} \right)^{\beta_1} \quad (62)$$

Through some simple manipulations this factor can be expressed as:

$$\left(\frac{(\beta_1 - 1)B_2^c}{\beta_1 I_2(\rho - \mu)} \right)^{\beta_1} - \left(\frac{(\beta_1 - 1)B_2^c}{\beta_1 I_2(\rho - \mu)} \right)^{\beta_1} = 0 \quad (63)$$

As this factor is equal to 0, the expression for the derivative in Appendix B is also equal to 0. This shows that when $\hat{X} = \hat{X}^{E*}$ and $T = T(\hat{X}^{E*})$, the first-order condition with respect to $\hat{X}$ is satisfied. We are able to verify that this solution is a maximum by evaluating the model numerically for a sensible set of parameter values, as illustrated in Figure 14.

We can therefore conclude that the solution to the bargaining model is as follows:

$$\hat{X} = \hat{X}^{E*} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1 + I_2}{B_1^c + B_2^c} \quad (64)$$

$$T = T(\hat{X}^{E*}) = \left(\frac{B_2^c(I_1 + I_2)}{I_2(B_1^c + B_2^c)} \right)^{\beta_1} \frac{B_2^c \eta I_2 - (1 - \eta)(\beta_1 B_1^c I_2 - (\beta_1 - 1)B_2^c I_1)}{(\beta_1 - 1)B_2^c} - \frac{\beta_1(I_1 + I_2)(B_2^c \eta - B_1^c(1 - \eta))}{(\beta_1 - 1)(B_1^c + B_2^c)} - (1 - \eta)I_1 + \eta I_2 \quad (65)$$

where the expression for the optimal transfer T is obtained by substituting expressions for $\hat{X} = \hat{X}^{E*}$ and $\hat{X}_2^F$ into the earlier expression for $T(\hat{X})$.

This analysis shows that the joint decision-making mechanism described above (i.e. Nash bargaining over both the timing of the departments' investments and the size of a cash transfer) results in the socially efficient investment timing decision in equilibrium. Intuitively, departments select $\hat{X}$ to maximise the total surplus from their investments (which is equivalent to maximising welfare) and split the surplus via the transfer mechanism T . The expression for the

Figure 14: *This contour plot depicts the objective function of the bargaining model for parameter values $B_1^c = B_2^c = 1$, $I_1 = 100$, $I_2 = 200$, $\rho = 0.06$, $\mu = 0.02$, $\sigma = 0.05$ and $\eta = 0.5$. Lighter grey regions of the plot correspond to higher values of the objective function and darker grey regions correspond to lower values. White areas indicate values of the choice variables for which the objective function is undefined (due to involving complex numbers). The function is maximised for $\hat{X} = \hat{X}^{E*} = 9.5082$ and $T = T(\hat{X}^{E*}) = 27.0729$, where $\hat{X}^{E*}$ is the socially optimal investment timing.*

optimal $T(\hat{X}^{E*})$ above differs from the expression for the ‘efficient’ T^* in the transfer model from the previous section, and depends on the departments’ bargaining weights η and $1 - \eta$; however this is of no concern to the planner, who cares only about the total investment cost $I_1 + I_2$ (this is unchanged by the transfer, which only alters how costs are allocated between the two departments).

It is important to note that the bargaining model only results in the efficient investment timing decision when departments have the ability to make a transfer T . To see this, consider

that the objective function for the model is the product of two factors, one of which is:

$$\left(\left(\frac{B_2^c \hat{X}}{\rho - \mu} + T - I_2 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} - \left(\frac{B_2^c \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} \right)^{1-\eta} \quad (66)$$

If departments do not have the ability to agree a transfer T (i.e. $T = 0$) then the left-hand term in the bracketed expression above is $\left(\frac{B_2^c \hat{X}}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1}$, which is just the objective function of department 2 in the basic model. By definition this term is maximised when $\hat{X} = \hat{X}_2^F$, meaning that the right-hand term in the bracketed expression above is weakly greater than the left-hand term and the entire expression (and consequently the Nash product) is ≤ 0 . Consequently, the best that the departments can do is to select $\hat{X} = \hat{X}_2^F$ as in the equilibrium for the basic model. Intuitively, without access to the transfer mechanism it is not possible to generate a positive surplus for department 2, which will therefore not agree to alter its individual investment timing decision. This shows that, in addition to being a relatively effective mechanism in its own right as shown in the previous section, the transfer is also crucial to the effectiveness of the joint decision-making mechanism considered here.

As noted earlier, the core difference between the simple transfer model described earlier and the joint decision-making model described here — which also involves a transfer between departments — is the ability for individual departments to commit to an investment timing (or, the contractibility of departments' individual investment timing decisions). Joint decision-making as modelled here is a relatively recent concept in New Zealand public administration. The 1980s model of public administration vested decision rights solely in individual departments and the individual ministers that they reported to, including decisions over public finances.⁴⁰ Because departments are administrative units of a single legal entity (the Crown) there is no ability for two departments to enter into a binding contractual agreement with one another, or to otherwise override the authority that the individual departments have over policy or financial decisions in their portfolio, which can result in commitment problems when it comes to collaborative investment between departments. Because in such a case neither department

⁴⁰The Public Finance Act 1989 allowed only the administering department to use a specific departmental appropriation, and even with 2013 amendments to the Act responsibility for funding remains with the administering department even if it allows another department to use an appropriation it administers (unless directed by the appropriation minister).

is legally obliged to abide by any agreement reached over investment timing and cost-sharing, the simple transfer model from the previous section (which requires only that departments behave consistently with their individual preferences at the time investment occurs) provides a suitable description of the behaviour of departments under these conditions. However, in the past decade we have seen a move towards joint decision-making in specific areas where multiple agencies or ministers have an interest; first with informal models relying on Cabinet direction and authority (usually relating to the provision of joint policy advice), but more recently by including in legislation formal models for joint decision-making and administration of appropriations.⁴¹ These models allow for legal decision rights over a certain set of resources and departmental functions to be held jointly by multiple departments, rather than being held by departments individually as is the default. The joint decision-making model presented in this section is intended to describe this latter set of scenarios.

The above considerations indicate that sharing decision rights between departments can be an effective mechanism for achieving the efficient investment timing for a collaborative investment requiring contributions from multiple departments, provided that those departments have the ability to determine how investment costs are allocated (i.e. departments also have access to a transfer mechanism). This suggests that models of joint decision-making by departments should be supported and further explored, in conjunction with more flexible public finance settings.

8 Conclusions

8.1 Summary of findings

The model presented in this thesis shows how a real-options approach can be used to illustrate policy investment coordination problems between government departments, and to analyse the types of institutional arrangements and public management settings that might be implemented to address such issues.

The basic model presented in Part II shows how, where two departments have respective options to invest in policy programmes that are complementary (i.e. generate greater benefit

⁴¹These models are the interdepartmental executive board and interdepartmental venture, which are both provided for in the Public Service Act 2020.

when both programmes are active), at least one department will delay investment past the socially efficient threshold. Intuitively this is because the department with the higher individual threshold for investment does not take into account the additional benefits that their investment will generate by increasing the benefits of the other department's investment.

The variations to the model in Part III show how this approach can be used to inform decisions about institutional design, including how the public service is structured, how incentives are structured, how public finances are managed and how decision-rights are allocated within the public service. At a high level, this part of the thesis highlights that real-options models like the one presented here can be used relatively flexibly to evaluate different types inducements that could be employed in practice to address the distortion illustrated by the basic model in Part II. More specifically, our analysis allows us to draw some general conclusions about how effective particular inducements are likely to be in practice.

Structural integration of departmental functions helps to address the coordination problem identified by resulting in a single decision-maker with identical incentives to those of the social planner (by bringing decision-rights over the respective policy investment decisions into a single entity). There are limits on the extent to which integration of departmental functions is practically and politically feasible, and however the public service is structured some agency boundaries will always exist. There are also often significant costs associated with the movement of functions between departments, particularly when establishment or disestablishment of entities is involved. However, this result indicates that where possible we should aim for agency configurations that keep functions together where there are likely to be cross-benefits between policy investments, and that we should aim to minimise fragmentation of the State sector (i.e. aim for fewer agencies).

Altering the incentives on departments to make decision-makers take into account the impacts of their department's investments of the effectiveness of other departments' policy programmes may in some cases be successful in improving investment coordination across departments. This will depend on whether there are any unintended consequences or costs associated with the inducement. One example which we illustrated was a distraction from cost minimisation activities, resulting in a new distortion that offset the positive effects of the mechanism on investment timing (and consequently prevented the first-best social optimum from being

achieved). However, other consequences or costs could also be possible, including direct costs of implementation. Implementing the mechanism in practice would require an assessment of whether the benefits of doing so (in terms of correcting the original distortion) outweighed any likely consequences or costs.

Finally, our analysis indicates that under certain circumstances relaxing public finance settings to facilitate transfers between departments and implementing models of joint decision-making between departments can also be effective methods of improving decision-making from a welfare perspective. Simply allowing and encouraging departments to contribute funding towards policy investments in other portfolios can shift investment timing decisions towards the efficient level, depending on the characteristics of the specific investments involved. In particular, this mechanism can be effective where the underlying state variable (which affects the expected benefits from investment) is relatively stable (i.e. has low volatility and drift) and where the benefits of the more ‘profitable’ investment (i.e. the investment of the department which we have identified as the ‘leader’) are high relative to those generated by the less profitable investment.⁴²

Shifting to a model of joint decision-making over policy investments (which we have modelled using a Nash bargaining approach) can implement the socially efficient investment timing provided that departments also have access to the ability to make transfers. In practice this suggests that the most effective mechanism for resolving policy investment coordination problems is to both support more flexibility in how departments may spend appropriated funds, and to move away from the traditional model of siloed decision-making where investments require the cooperation of a number of departments and generate benefits across portfolios.

8.2 Future research

As described above, the model we have presented offers some useful heuristics to guide public administration decisions. However, it is important to keep in mind that it relies on a number of assumptions, some of which were discussed explicitly when setting up the basic model in Part II. One topic of future research could be the implication of relaxing these assumptions for the

⁴²As we demonstrated, this can occur when the more profitable investment is also of a greater scale, or when there is a relatively large profitability gap between the two investments.

results we have obtained here. For example, a particularly interesting extension to the model would involve relaxing the assumption that a department does not experience benefits when it has not invested but the other department has invested. We briefly discuss the implications of relaxing this assumption in **Appendix C**.⁴³

Other possible extensions could involve altering features of the model which were not addressed in our explicit assumptions, but which would nevertheless extend the model’s application to a wider range of situations and broaden the range of inducements that could be assessed using the model. One such extension could be increasing the number of departments; here we have considered the interaction between the policy investments of two departments, but in reality a given policy area may be relevant to the portfolios of many different departments and agencies. Altering the model in this way would likely impact on our ability to obtain closed-form analytic solutions of the kind that we have presented in this thesis, but would result in a better approximation of the complexity of some cross-cutting policy issues.

Another possible extension would involve including ministers in the model as well as departments. We have assumed in our simple set-up that departments have full control over investment decisions within their portfolio, but in reality these decisions are made within an authorising environment and in accordance with high-level policy parameters set by the relevant minister for the portfolio area (or in some cases the Government as a whole). Including ministers in the model is likely to add significant complexity (i.e. by adding an additional layer of strategic interaction), but could also allow for consideration of a wider range of inducements — involving for example the adjustment of portfolio allocations or other changes to ministerial arrangements — and agency issues, where the objectives of agencies differ from those of their ministers even excluding issues of complementarity with the investments of other agencies.

⁴³In this Appendix we show that relaxing the assumption in question complicates the model by increasing the number and complexity of possible scenarios that must be considered when designing and assessing a given inducement. The model that we have considered in this thesis becomes a special case of the more general model (i.e. where departments may experience benefits without having invested), which could be further investigated through future research.

Part IV

Appendices

A Basic model proofs

For those less familiar with real-options modelling, this appendix provides a more complete description of the mathematics underpinning our analysis. Proofs and explanations in this section draw on those set out in Chapters 3-5 of Dixit and Pindyck (1994).

In our analysis we assume an exogenous state variable X , which evolves according to geometric Brownian motion (GBM) with (percentage) drift μ and volatility σ :

$$dX_t = \mu X_t + \sigma X_t d\xi_t \quad (67)$$

where both μ and σ are constants, and $d\xi_t$ is the increment of a Wiener process (Brownian Motion). The stochastic differential equation (SDE) above has the following properties:

1. The SDE has an analytic solution $X_t = X_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma \xi_t}$, where X_0 is an (arbitrary) initial value;
2. This solution is a log-normally distributed random variable with expected value $\mathbb{E}(X_t) = X_0 e^{\mu t}$ and variance $Var(X_t) = X_0^2 e^{2\mu t} (e^{\sigma^2 t} - 1)$

Consider the simple case in which a department n incurs an initial investment cost I_n at the time of investment, and experiences a social benefit $B_n X_t$ for each period t from the time of investment onward (i.e. the benefits and costs of the department's policy investment are unaffected by behaviour of other departments). The expected present value of the flow of benefits at a given time t post investment is given by:

$$\mathbb{E} \int_t^\infty B_n X_\tau e^{-\rho(\tau-t)} d\tau \quad (68)$$

where ρ is the department's discount rate. Let $P(X_t)$ denote the expected investment payoff at time t if the department invests at that time (i.e. the present value of the benefits less the

investment cost, analogous to the standard net present value of the policy programme). Given the above expression for the expected present value of benefits, $P(X_t)$ is given by:

$$P(X_t) = \mathbb{E} \int_t^\infty B_n X_\tau e^{-\rho(\tau-t)} d\tau - I_n \quad (69)$$

Noting the expression for $\mathbb{E}(X_t)$ above, this expected present value at the time of investment simplifies to:

$$\begin{aligned} P(X_t) &= \int_t^\infty B_n \mathbb{E}(X_\tau) e^{-\rho(\tau-t)} d\tau - I_n \\ &= B_n X_t \int_t^\infty e^{-(\rho-\mu)(\tau-t)} d\tau - I_n \\ &= \frac{B_n X_t}{\rho - \mu} - I_n \end{aligned} \quad (70)$$

The relevant decision for the department is when to invest, a timing decision which is expressed as a threshold value of the state variable X_t . As $P(X_t)$ is an increasing function, we consider investment rules involving a minimum threshold for X ; that is, we assume that the department invests when the state variable exceeds some constant $\hat{X}$.

Let $V(X_t)$ be the value of the option to invest in the policy programme. There are a number of methods used in the literature to derive this value function and consequently solve for the optimal threshold $\hat{X}$, with two of the most common being dynamic programming and contingent claims analysis. As the latter approach relies on comparisons with traded financial assets, it does not have a strong intuitive analogy in our specific scenario. Consequently, we employ a dynamic programming approach.

Consider that at all times before investment is made (known as the ‘continuation region’) the department experiences no benefits or costs — investment costs are only incurred at the time of investment, and benefits only experienced from the time of investment onwards. Consequently, before investment has occurred the value of the option to invest in the department’s policy programme $V(X_t)$ can be expressed as the discounted present value of the option value at time $t + dt$. This observation allows us to create the following (somewhat trivial) Bellman equation:

$$V(X_t) = e^{-\rho dt} \mathbb{E}[V(X_t + dX)] = e^{-\rho dt} (V(X_t) + \mathbb{E}(dV)) \quad (71)$$

As $dt \rightarrow 0$, this can be expressed as:

$$\begin{aligned}
V(X_t) &= (1 - \rho dt) (V(X_t) + \mathbb{E}(dV)) + o(dt) \\
&= V(X_t) + \mathbb{E}(dV) - \rho V(X_t)dt + o(dt) \\
&\Rightarrow 0 = \mathbb{E}(dV) - \rho V(X_t)dt
\end{aligned} \tag{72}$$

where $o(dt)$ denotes terms which go to zero faster than dt . We make use of Itô's lemma to define dV .

Consider a time-dependent function f of an Itô process⁴⁴ x_t which satisfies the SDE $dx_t = \mu(x_t, t)dt + \sigma(x_t, t)dz_t$, where z_t is a Brownian motion. Itô's lemma states that the differential df of such a function is given by:

$$df = \left(\frac{\partial f}{\partial t} + \mu(x, t) \frac{\partial f}{\partial x} + \frac{\sigma^2(x, t)}{2} \frac{\partial^2 f}{\partial x^2} \right) dt + \sigma(x, t) \frac{\partial f}{\partial x} dz_t \tag{73}$$

Note that in our model $\frac{\partial f}{\partial t} = 0$, as $\mu(X, t) = \mu X$ and $\sigma(X, t) = \sigma X$ (meaning t does not appear in the SDE) and t does not appear in the stopping payoff $P(X_t)$. Noting also that $E(d\xi) = 0$, Itô's lemma tells us that $E(dV)$ can be stated as:

$$E(dV) = \mu X \frac{\partial V}{\partial X} dt + \frac{1}{2} \sigma^2 X^2 \frac{\partial^2 V}{\partial X^2} dt \tag{74}$$

Substituting this expression for dV in our manipulated Bellman equation above and dividing by dt yields the following ordinary differential equation (ODE):

$$\frac{1}{2} \sigma^2 X^2 \frac{\partial^2 V}{\partial X^2} + \mu X \frac{\partial V}{\partial X} - \rho V = 0 \tag{75}$$

As this is a second-order differential equation, we also require two boundary conditions to identify a solution. First, note that because the state variable follows GBM if $X_t = 0$ for an arbitrary t then $X_\tau = 0 \forall \tau > t$ (i.e. once X goes to zero it will stay at zero). This implies that once X falls to zero the threshold $\hat{X}$ will never be reached, giving us our first boundary

⁴⁴An Itô process is a stochastic process that can be written as the sum of the integral of a process over time and the integral of a process over a Brownian motion. Geometric Brownian motion, the stochastic process followed by the state variable in our model, is an example of an Itô process.

condition:

$$V(0) = 0 \quad (76)$$

The next boundary condition involves the optimal investment threshold $\hat{X}$. For $X_t \geq \hat{X}$ investment will by definition occur immediately, implying that $V(X_t) = P(X_t)$ in this region (known as the ‘stopping region’). As this condition holds at the boundary, we have the following ‘value matching’ condition:

$$V(\hat{X}) = P(\hat{X}) \quad (77)$$

Together with the ODE above, these boundary conditions allow us to identify the functional form of $V(X_t)$. The simplest approach is to pick a functional form and verify that it is a solution. We consider the functional form AX^β . By substitution we can see that this satisfies the ODE provided that β is a root of the quadratic equation:

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu\beta - \rho = 0 \quad (78)$$

The two roots of this equation are:

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\frac{2\rho}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2} \quad (79)$$

$$\beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\frac{2\rho}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2} \quad (80)$$

where $\beta_1 > 1$ and $\beta_2 < 0$. As the ODE is a second-order differential equation linear in V and its derivatives, its general solution can be expressed as a linear combination of any two independent solutions. Consequently the general solution to the ODE can be expressed as:

$$V(X) = A_1X^{\beta_1} + A_2X^{\beta_2} \quad (81)$$

where A_1 and A_2 are constants. The boundary condition $V(0) = 0$ implies that $A_2 = 0$, meaning

that the solution must take the following form:

$$V(X) = A_1 X^{\beta_1} \quad (82)$$

Imposing the remaining boundary condition $V(\hat{X}) = P(\hat{X})$ gives us the following expression for A_1 :

$$A_1 = \frac{P(\hat{X})}{\hat{X}^{\beta_1}} \quad (83)$$

Substituting this into our expression for $V(X)$ shows that in the continuation region (when X_t is below the investment threshold):

$$V(X_t) = \left(\frac{B_n \hat{X}}{\rho - \mu} - I_n \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} \quad (84)$$

Recalling that $V(X_t) = P(X_t)$ in the stopping region (when X_t is above the investment threshold) this implies the following general expression for the value of the option to invest in the department's policy programme:

$$V(X_t) = \begin{cases} \left(\frac{B_n \hat{X}}{\rho - \mu} - I_n \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} & \text{if } 0 \leq X_t < \hat{X} \\ \frac{B_n X_t}{\rho - \mu} - I_n & \text{if } \hat{X} \leq X_t \end{cases} \quad (85)$$

The department's choice variable is $\hat{X}$, which is chosen to maximise the value of the option (i.e. the expected present value of the investment) in the continuation region. Taking a first-order condition with respect to $\hat{X}$ yields:

$$\hat{X} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_n}{B_n} \quad (86)$$

This tells us that the department will invest in the policy programme as soon as:⁴⁵

$$\frac{B_n X_t}{\rho - \mu} \geq \frac{\beta_1 I_n}{\beta_1 - 1} \quad (87)$$

An alternative approach to solving for the optimal $\hat{X}$ is to impose a third boundary condition, known as the ‘smooth-pasting’ condition. In the body of the thesis we prefer the method of taking a first-order condition (as above), which yields the same result. However, as the smooth-pasting condition is used often in the literature we discuss it briefly here. It can be shown that if the value function $V(X_t)$ is not smooth and continuous at the threshold $\hat{X}$, the department can do better by selecting an alternative threshold, implying that:

$$\frac{\partial V(\hat{X})}{\partial X} = \frac{\partial P(\hat{X})}{\partial X} \quad (88)$$

The simplest argument for this condition is by contradiction.⁴⁶ First suppose $V'(\hat{X}) > P'(\hat{X})$ (an upward-facing kink). Continuity implies $V(X_t) > P(X_t)$ for some $X_t > \hat{X}$, for which continuation would be preferable to termination (contradicting the definition of $\hat{X}$ as the investment threshold). Next suppose $V'(\hat{X}) < P'(\hat{X})$ (a downward-facing kink). In this case, delaying investment allows the department to observe the next step of X and choose a position on either side of the kink. As the increment of a Brownian motion is proportional to $\sqrt{dt}$ and the effect of discounting is proportional to dt , for the marginal dt the former effect is larger and consequently the average of the two points either side of the kink (i.e. the expected payoff from delaying) is preferable to the kink point itself (contradicting the definition of $\hat{X}$ as the investment threshold).

This simple case provides the basis for all analysis conducted in the body of this thesis. While we vary assumptions in relation to the benefits received under different conditions (e.g. whether another department has invested, or whether a planner has introduced a specific inducement to influence investment behaviour), these are all incorporated as adjustments to the stopping payoff $P(\hat{X})$, and therefore do not affect the form of the solution for $V(X_t)$ derived above.

⁴⁵Note that the left hand side of this condition represents the expected present value of the benefits from investment, and the right hand side is the cost of investment multiplied by a factor $\frac{\beta_1}{\beta_1 - 1}$. As $\beta_1 > 1$ this term exceeds 1, meaning that the department waits longer to invest than a simple NPV rule would suggest. This is the core result of real-options analysis — that irreversibility of investment and uncertainty of future benefit flows increase investment thresholds beyond those seen in more traditional valuation approaches.

⁴⁶As we do not rely on the smooth-pasting condition, we provide a relatively brief and intuitive explanation. A more complete proof is provided at Chapter 3 Annex C of Dixit and Pindyck (1994).

B Bargaining model — additional material

This appendix describes the first derivative of the objective function from our bargaining model (presented at Chapter 7.4) with respect to $\hat{X}$, evaluated at the socially efficient $\hat{X} = \hat{X}^{E*} = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_1+I_2}{B_1^c+B_2^c}$ and where T and $\hat{X}_2^F$ have been eliminated using the first-order condition with respect to T and the equilibrium expression for $\hat{X}_2^F$ from our basic model (respectively). The result is a complicated expression, which is the product of the following seven factors:

$$-\beta_1 B_1^c(\eta-1)I_2 + (\beta_1-1)B_2^c(\eta-1)I_1 - B_2^c\eta I_2 \quad (89)$$

$$\left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1(I_1+I_2)(\rho-\mu)} \right)^{\beta_1} \quad (90)$$

$$\left(\frac{\beta_1 B_2^c - B_2^c}{\beta_1 I_2 \rho - \beta_1 I_2 \mu} \right)^{\beta_1} - \left(\frac{B_2^c(I_1+I_2)}{I_2(B_1^c+B_2^c)} \right)^{\beta_1} \left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1(I_1+I_2)(\rho-\mu)} \right)^{\beta_1} \quad (91)$$

$$\begin{aligned} & \beta_1 ((-B_1^c)^2) (\eta-1)I_2 \left(\frac{B_2^c(I_1+I_2)}{I_2(B_1^c+B_2^c)} \right)^{\beta_1} + (B_2^c)^2 \left(I_1 \left(\beta_1(\eta-1) \left(\frac{B_2^c(I_1+I_2)}{I_2(B_1^c+B_2^c)} \right)^{\beta_1} - \eta \left(\frac{B_2^c(I_1+I_2)}{I_2(B_1^c+B_2^c)} \right)^{\beta_1} + \left(\frac{B_2^c(I_1+I_2)}{I_2(B_1^c+B_2^c)} \right)^{\beta_1} + \eta \right) \right. \\ & \quad \left. - \eta I_2 \left(\left(\frac{B_2^c(I_1+I_2)}{I_2(B_1^c+B_2^c)} \right)^{\beta_1} - 1 \right) \right) + B_1^c B_2^c \left((\eta-1)I_1 \left(\beta_1 \left(\frac{B_2^c(I_1+I_2)}{I_2(B_1^c+B_2^c)} \right)^{\beta_1} - \left(\frac{B_2^c(I_1+I_2)}{I_2(B_1^c+B_2^c)} \right)^{\beta_1} + 1 \right) \right. \\ & \quad \left. - I_2 \left(\beta_1(\eta-1) \left(\frac{B_2^c(I_1+I_2)}{I_2(B_1^c+B_2^c)} \right)^{\beta_1} + \eta \left(\left(\frac{B_2^c(I_1+I_2)}{I_2(B_1^c+B_2^c)} \right)^{\beta_1} - 1 \right) + 1 \right) \right) \end{aligned} \quad (92)$$

$$\left(\frac{\left(\beta_1 B_1^c (\eta-1) I_2 \left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} + B_2^c \left(\eta I_2 \left(\left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} - 1 \right) - (\eta-1) I_1 \left(\beta_1 \left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} - \left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} + 1 \right) + I_2 \right) \right) \left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1 (I_1+I_2)(\rho-\mu)} \right)^{\beta_1}}{B_2^c} - I_2 \left(\frac{B_2^c - \beta_1 B_2^c}{\beta_1 I_2 \mu - \beta_1 I_2 \rho} \right)^{\beta_1} \right)}{\beta_1 - 1} \right)^{-\eta} \quad (93)$$

$$\left(\left(- \frac{\left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} (\beta_1 B_1^c (\eta-1) I_2 + (\beta_1-1)(-B_2^c)(\eta-1) I_1 + B_2^c \eta I_2)}{(\beta_1-1) B_2^c} + \frac{\beta_1 (I_1+I_2)(B_1^c(\eta-1) + B_2^c \eta)}{(\beta_1-1)(B_1^c+B_2^c)} + \frac{\beta_1 B_1^c (I_1+I_2)}{(\beta_1-1)(B_1^c+B_2^c)} - \eta I_1 - \eta I_2 \right) \right. \\ \left. + \frac{\left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1 (I_1+I_2)(\rho-\mu)} \right)^{\beta_1} + \frac{((\beta_1-1) B_2^c I_1 - \beta_1 B_1^c I_2) \left(\frac{B_2^c - \beta_1 B_2^c}{\beta_1 I_2 \mu - \beta_1 I_2 \rho} \right)^{\beta_1}}{(\beta_1-1) B_2^c}}{(\beta_1-1) B_2^c} \right)^{\eta} \quad (94)$$

$$\left[B_2^c (I_1+I_2)(\mu-\rho) \left(-B_2^c \eta I_2 \left(\left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} - 1 \right) \left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1 (I_1+I_2)(\rho-\mu)} \right)^{\beta_1} \right. \right. \\ \left. - \beta_1 B_1^c I_2 \left(\eta \left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} \left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1 (I_1+I_2)(\rho-\mu)} \right)^{\beta_1} - \left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} \left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1 (I_1+I_2)(\rho-\mu)} \right)^{\beta_1} + \left(\frac{B_2^c - \beta_1 B_2^c}{\beta_1 I_2 \mu - \beta_1 I_2 \rho} \right)^{\beta_1} \right) \right. \\ \left. + B_2^c I_1 \left(\beta_1 \left(\eta \left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} \left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1 (I_1+I_2)(\rho-\mu)} \right)^{\beta_1} - \left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} \left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1 (I_1+I_2)(\rho-\mu)} \right)^{\beta_1} + \left(\frac{B_2^c - \beta_1 B_2^c}{\beta_1 I_2 \mu - \beta_1 I_2 \rho} \right)^{\beta_1} \right) \right. \right. \\ \left. \left. - \eta \left(\left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} - 1 \right) \left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1 (I_1+I_2)(\rho-\mu)} \right)^{\beta_1} + \left(\frac{B_2^c (I_1+I_2)}{I_2 (B_1^c+B_2^c)} \right)^{\beta_1} \left(\frac{(\beta_1-1)(B_1^c+B_2^c)}{\beta_1 (I_1+I_2)(\rho-\mu)} \right)^{\beta_1} - \left(\frac{B_2^c - \beta_1 B_2^c}{\beta_1 I_2 \mu - \beta_1 I_2 \rho} \right)^{\beta_1} \right) \right]^{-1} \quad (95)$$

C Addition of benefits without investment

This appendix considers the implications of amending the basic model such that a department n will receive some benefit B_n^o when it has not invested but the other department m has invested, for $n = 1, m = 2$ and $n = 2, m = 1$. We assume that $B_n^o < B_n^c$ for $n = 1, 2$; that is, the benefit flow a given department experiences when both departments have invested is greater than the benefit it receives when only the other department has invested.

Without loss of generality, assume that parameter values are such that department 2 is the follower department. With the addition of B_2^o when department 1 has invested but department 2 has not yet invested, department 2's follower objective function (i.e. assuming that department 1 has already invested) is now as follows:

$$F_2(X_t) = \begin{cases} \frac{B_2^o X_t}{\rho - \mu} + \left(\frac{(B_2^c - B_2^o) \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{B_2^o X_t}{\rho - \mu} + \frac{(B_2^c - B_2^o) X_t}{\rho - \mu} - I_2 & \text{if } \hat{X}_2^F \leq X_t \end{cases} \quad (96)$$

Note that the term $B_2^c - B_2^o$ replaces the term B_2^c in department 2's follower objective function from the basic model. This is because when department 1 has invested department 2 is already receiving a benefit flow of B_2^o , so the incremental benefit flow it gains from investing is $B_2^c - B_2^o$. Given the above objective function, the follower's optimal investment threshold is:

$$\hat{X}_2^F = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c - B_2^o} \quad (97)$$

Likewise, without loss of generality we assume department 1 is the leader department. As in the basic model, the value of department 1's policy programme immediately before investment is as follows:

$$L_1(X_t) = \begin{cases} \frac{B_1^s X_t}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1} - I_1 & \text{if } 0 \leq X_t < \hat{X}_2^F \\ \frac{B_1^c X_t}{\rho - \mu} - I_1 & \text{if } X_t \geq \hat{X}_2^F \end{cases} \quad (98)$$

Where department 1's objective function is given by $L_1(\hat{X}_1^L) \left(\frac{X}{\hat{X}_1^L} \right)^{\beta_1}$. This is the same objective

function for the leader as in the basic model, because the addition of a benefit B_2^o received by the follower does not impact the benefit received by the leader either before or after investment. Consequently, if $X \geq \hat{X}_2^F$ then the leader's optimal investment threshold is:

$$\hat{X}_1^L = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c} \quad (99)$$

If $X < \hat{X}_2^F$ the leader's optimal investment threshold is:

$$\hat{X}_1^L = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^s} \quad (100)$$

Note that in the interests of brevity the analysis here does not include explicit discussion of corner solutions, but the same considerations apply as in the basic model.

As in the basic model, assuming that there are no restrictions on which department is the leader and which is the follower, there are two types of equilibrium that may occur — a natural leader game or a waiting game. However, the conditions for a given type of equilibrium (i.e. required parameter values) and equilibrium investment timings in each of these cases are affected by the introduction of a benefit flow B_n^o for a follower department that has not yet invested:

1. **Waiting game** — as in the basic model, there are equilibria where departments will invest simultaneously (i.e. one department invests and the other follows immediately). In the basic model, for these types of equilibria investment would occur at $\hat{X} = \max\{\frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c}, \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c}\}$ as at this point both departments would be willing to invest if the other had also invested. However, with the addition of B_n^o equilibrium investment timing is given by $\hat{X} = \min\left\{\max\left\{\frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c}, \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c - B_2^o}\right\}, \max\left\{\frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c}, \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c - B_1^o}\right\}\right\}$. To understand this, consider that if $X > \max\{\frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c}, \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c}\}$ (such that both departments would be willing to invest in the basic model) but $X < \min\{\frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c - B_1^o}, \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c - B_2^o}\}$, then if one department n were to invest it would no longer be optimal for the other department m to do so, as they would already be receiving benefit B_m^o (meaning that the incremental benefit flow from investment as the follower would be $B_m^c - B_m^o$). Consequently, simultaneous investment will not occur until at least one department n is certain that the other department m will immediately follow, which in

this case occurs at $X > \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_m}{B_m^c - B_m^o}$. This type of equilibrium will occur if parameter values are such that $\min\{\max\{\frac{I_1}{B_1^c}, \frac{I_2}{B_2^c - B_2^o}\}, \max\{\frac{I_2}{B_2^c}, \frac{I_1}{B_1^c - B_1^o}\}\} < \min\{\frac{I_1}{B_1^s}, \frac{I_2}{B_2^s}\}$ is satisfied.

2. **Natural leader** — also like the basic model, there are equilibria where one department will invest knowing that the other will not immediately follow (with the second department investing later). As in the basic model, for these equilibria the leader will invest at $X = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_n}{B_n^s}$. However, the additional benefit B_m^o for the follower will delay its investment relative to the basic model; rather than investing at $X = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_m}{B_m^c}$, the follower will wait until $X = \frac{\beta_1(\rho-\mu)}{\beta_1-1} \cdot \frac{I_m}{B_m^c - B_m^o}$ to invest. Again, this is because once the leader has invested the follower already receives benefit flow B_m^o meaning that the incremental benefit flow from investment is $B_m^c - B_m^o$. This type of equilibrium will occur if parameter values are such that $\min\{\max\{\frac{I_1}{B_1^c}, \frac{I_2}{B_2^c - B_2^o}\}, \max\{\frac{I_2}{B_2^c}, \frac{I_1}{B_1^c - B_1^o}\}\} > \min\{\frac{I_1}{B_1^s}, \frac{I_2}{B_2^s}\}$ is satisfied.

Next we consider the effect of adding terms B_1^o, B_2^o to the planner's problem. As in the basic model, we must consider both an interior solution (i.e. where it is optimal for one investment to occur before the other) and a corner solution (i.e. where it is optimal for both investments occur simultaneously). Without loss of generality, assume that parameter values are such that it is socially optimal for department 1 to be the investment leader. The planner's objective function for an interior solution is:

$$\begin{aligned}
W(X_t) &= L_1(\hat{X}_1^L) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} + F_2(\hat{X}_1^L) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} \\
&= \left(\frac{B_1^s \hat{X}_1^L}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} \left(\frac{\hat{X}_1^L}{\hat{X}_2^F} \right)^{\beta_1} - I_1 \right) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} \\
&\quad + \left(\frac{B_2^o \hat{X}_1^L}{\rho - \mu} + \left(\frac{(B_2^c - B_2^o) \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{\hat{X}_1^L}{\hat{X}_2^F} \right)^{\beta_1} \right) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} \\
&= \left(\frac{B_1^s \hat{X}_1^L}{\rho - \mu} + \frac{B_2^o \hat{X}_1^L}{\rho - \mu} - I_1 \right) \left(\frac{X_t}{\hat{X}_1^L} \right)^{\beta_1} \\
&\quad + \left(\frac{(B_2^c - B_2^o) \hat{X}_2^F}{\rho - \mu} + \frac{(B_1^c - B_1^s) \hat{X}_2^F}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}_2^F} \right)^{\beta_1}
\end{aligned} \tag{101}$$

Taking first-order conditions of this objective function with respect to the choice variables $\hat{X}_1^L$,

$\hat{X}_2^F$ and solving for the optimal thresholds yields:

$$\hat{X}_1^{L*} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^s + B_2^o} \quad (102)$$

$$\hat{X}_2^{F*} = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{(B_2^c - B_2^o) + (B_1^c - B_1^s)} \quad (103)$$

The planner's objective function for a corner solution (where $\hat{X} = \hat{X}_1 = \hat{X}_2$) is:

$$\begin{aligned} W(X_t) &= L_1(\hat{X}) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} + F_2(\hat{X}) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} \\ &= \left(\frac{B_1^c \hat{X}}{\rho - \mu} - I_1 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} + \left(\frac{B_2^o \hat{X}}{\rho - \mu} + \frac{(B_2^c - B_2^o) \hat{X}}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} \\ &= \left(\frac{B_1^c \hat{X}}{\rho - \mu} - I_1 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} + \left(\frac{B_2^c \hat{X}}{\rho - \mu} - I_2 \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} \\ &= \left(\frac{(B_1^c + B_2^c) \hat{X}}{\rho - \mu} - (I_1 + I_2) \right) \left(\frac{X_t}{\hat{X}} \right)^{\beta_1} \end{aligned} \quad (104)$$

As in the basic model, taking first-order conditions and solving for the planner's optimal investment timing here yields:

$$\hat{X}^{E*} = \hat{X}_1^* = \hat{X}_2^* = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1 + I_2}{B_1^c + B_2^c} \quad (105)$$

As shown above, both the equilibrium and socially optimal outcomes of the model differ from the case where departments receive no benefits without having invested. Having identified the equilibrium and efficient behaviours the next step in our analysis (following the approach taken in the body of the thesis) would be to assess the different possible combinations of equilibrium and efficient outcomes, and consequently the possible distortions that inducements would aim to correct. It turns out that in this case there are a number of additional complexities as compared to the basic model where $B_1^o = B_2^o = 0$. We do not work through all the possible scenarios here, but make some observations that highlight the relative complexity of this extended model.

First, recall that in the basic model we were able to show that for a given set of parameter values the model had only one candidate for a social maximum — that is, if a local maximum could be achieved by one department investing before another, then this would also be a global

maximum (as a local maximum could not also be achieved by the other department investing first for the same set of parameter values — see Proposition 3). In this extended model the equivalent condition on parameter values required to avoid the possibility of multiple local maxima is $B_1^c + B_2^c > (B_1^s + B_2^o) + (B_2^s + B_1^o)$. Unlike in the basic model, satisfaction of this condition does not necessarily follow from the underlying assumptions that we have made regarding parameter values.⁴⁷ This means that unless we make further assumptions restricting the values that parameters can take it is possible to have multiple local maxima which must be compared (i.e. the maximum welfare achievable by department 1 investing first and the maximum welfare achievable by department 2 investing first).

Second, recall that in the basic model there were only three possible combinations of equilibrium and socially optimal outcome, two of which involved the departments choosing the efficient ‘type’ of equilibrium (i.e. department n investing first, or departments investing simultaneously) with inefficiencies only in the investment timing decisions. In the extended model, a wider variety of scenarios are possible in which departments select the wrong type of equilibrium from the perspective of the planner. For example, if parameter values are such that $\frac{I_1}{B_1^c + B_2^o} < \frac{I_2}{(B_2^c - B_2^o) + (B_1^c - B_1^o)}$ and $\frac{I_2}{B_2^s} < \min\{\frac{I_1}{B_1^c}, \max\{\frac{I_1}{B_1^c}, \frac{I_2}{B_2^c - B_2^o}\}, \max\{\frac{I_2}{B_2^c}, \frac{I_1}{B_1^c - B_1^o}\}\}$ then it may be that it is optimal for department 1 to invest first, but in equilibrium department 2 invests first. This indicates that in determining how to design a particular intervention a greater number of different cases must be considered.

Finally, recall that in the basic model inefficiencies in investment timing occurred only because the follower’s investment threshold was greater than the socially optimal level (while the leader’s investment threshold was less than or equal to the socially optimal level). In this extended model, additional distortions arise beyond those present in the basic model. To see this, consider for example the cases where the optimal type of equilibrium occurs (i.e. department 1 invests first, department 2 invests first or departments invest simultaneously) with inefficiencies only in investment timing decisions.⁴⁸

⁴⁷Note that there are some similarities between the requisite condition here and the equivalent condition $B_1^c + B_2^c > B_1^s + B_2^o$ for the basic model, in that the left hand side of each represent the total benefit when both departments have invested and the right hand sides represent the sum of the total benefits when only department 1 has invested and the total benefits when only department 2 has invested. However, the requisite condition here does not follow from the assumption that departments are better off when both have invested relative to when only one department has invested (i.e. $B_n^c > \max\{B_n^s, B_n^o\}$ for $n = 1, 2$).

⁴⁸We consider these cases specifically as they both have analogues that were analysed in our consideration of the basic model.

1. First, suppose that $\frac{I_n}{B_n^s + B_m^o} < \frac{I_m}{(B_m^c - B_m^o) + (B_n^c - B_n^s)}$ for $n = 1, m = 2$ or $n = 2, m = 1$, such that there are values of X for which it is optimal for department n to invest knowing that department m will not immediately follow, and suppose that $\frac{I_n}{B_n^s} < \min\{\frac{I_m}{B_m^s}, \frac{I_n}{B_n^c - B_n^o}, \frac{I_m}{B_m^c - B_m^o}\}$ such that department n will also invest first in equilibrium. Without loss of generality assume that $n = 1$ (i.e. department 1 is the leader and department 2 the follower). As in the basic model, the follower department invests late from a welfare perspective, as $\hat{X}_2^F = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{(B_2^c - B_2^o)} > \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{(B_2^c - B_2^o) + (B_1^c - B_1^s)} = \hat{X}_2^{F*}$. This arises from the follower not taking into account the impact of its investment on the leader's payoff, denoted $B_1^c - B_1^s$, a distortion that was also present in the basic model. However, there is also an additional distortion: in this case, the leader also invests late from a welfare perspective, as $\hat{X}_1^L = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^s} > \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_1^s + B_2^o} = \hat{X}_1^{L*}$. Intuitively, in this case the leader's investment also has a benefit for the follower B_2^o which was not present in the basic model, and which the leader fails to take into account in determining its investment timing.
2. Second, suppose that $\frac{I_n}{B_n^s + B_m^o} > \frac{I_m}{(B_m^c - B_m^o) + (B_n^c - B_n^s)}$ for both $n = 1, m = 2$ and $n = 2, m = 1$, such that there are no values of X for which it is optimal for either department to invest knowing that the other will not immediately follow, and also that $\min\{\frac{I_1}{B_1^c - B_1^o}, \frac{I_2}{B_2^c - B_2^o}\} < \min\{\frac{I_1}{B_1^s}, \frac{I_2}{B_2^s}\}$ such that in equilibrium departments will invest simultaneously when $\hat{X} = \min\left\{\max\left\{\frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c}, \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c - B_2^o}\right\}, \max\left\{\frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^s}, \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c - B_1^o}\right\}\right\}$. As in this case $\frac{I_1 + I_2}{B_1^c + B_2^c} < \max\{\frac{I_1}{B_1^c}, \frac{I_2}{B_2^c}\}$ and $\frac{I_n}{B_n^c} < \frac{I_n}{B_n^c - B_n^o}$ for $n = 1, 2$, the socially optimal investment threshold $\hat{X}^{E*} = \hat{X}_1^* = \hat{X}_2^* = \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1 + I_2}{B_1^c + B_2^c}$ is lower than the equilibrium investment threshold (i.e. both departments invest late from a welfare perspective). Further, note that in this case the equilibrium threshold may be higher than in the corresponding case for the basic model, as the expression for $\hat{X}$ above is weakly greater than $\max\{\frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_1}{B_1^c}, \frac{\beta_1(\rho - \mu)}{\beta_1 - 1} \cdot \frac{I_2}{B_2^c}\}$ (the corresponding equilibrium investment threshold in the basic model). Intuitively, this possible additional distortion arises from the fact that the addition of the term B_2^o raises the follower's investment threshold and consequently may delay simultaneous investment in the waiting game equilibrium (relative to the basic model).

Each of these features complicates the analysis involved in assessing a particular inducement, relative to the basic model presented in the body of the thesis. The possibility of a wider range of different possible combinations of equilibrium and social optimum increases the number of scenarios that must be considered, and the increased complexity of specific cases — including the possibility of multiple candidates for a social optimum that must be compared numerically, and the presence of additional distortions relative to the basic model — complicates the analysis of a mechanism in any given scenario (including increasing the likelihood that the model cannot be solved analytically and a numerical solution will be required). Further research could focus on extending analysis of the mechanisms we have discussed in this thesis to the more general model described here.

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